Lesson 2: Multiplying and Factoring Polynomial Expressions

Student Outcomes

- Students understand that factoring reverses the multiplication process as they find the linear factors of basic, factorable quadratic trinomials.
- Students explore squaring a binomial, factoring the difference of squares, and finding the product of a sum and difference of the same two terms.

MP.4 Throughout this lesson, students represent multiplication of binomials and factoring quadratic polynomials using geometric models.

Lesson Notes

This lesson continues to emphasize the understanding of the system and operations of polynomial expressions, specifically multiplication and factoring of polynomials (**A-APR.A.1**). Factoring quadratic expressions can unlock their secrets and reveal key features of the function to facilitate graphing. The reverse relationship between multiplication and factoring is explored, emphasizing the structure of the quadratic expressions (**A-SSE.A.2**). The lesson offers some different strategies for factoring and begins to build a toolbox for students faced with factoring quadratic expressions for which the factors may not be immediately apparent. In future lessons, students' factoring skills will be used to find values of variables that will make the polynomial expression evaluate to 0.

Classwork

Example 1 (5 minutes): Using a Table as an Aid

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Use a table to assist in multiplying $(x+7)(x+3)$.

	x	$+\ 7$
x	x^2	$7x$
$+\ 3$	$3x$	21

$x^2 + 10x + 21$

- Are there like terms in the table that can be combined?
  - *Yes, the terms in the diagonal can be added to give* $10x$.
- After combining like terms, what is the simplified product of the two binomials?
  - $x^2 + 7x + 3x + 21$ *or* $x^2 + 10x + 21$

Exercise 1 (4 minutes)

Exercise 1

Use a table to aid in finding the product of $(2x+1)(x+4)$.

	$2x$	$+$ 1
x	$2x^2$	x
$+$ 4	$8x$	4

$(2x + 1)(x+4) = 2x^2 + x + 8x + 4 = 2x^2 + 9x + 4$

Discussion (4 minutes)

- What is the constant term of the polynomial $x - 7$?

Students may respond with 7 or -7; in any case, encourage a discussion to acknowledge that the expression $x - 7$ is equivalent to the expression $x + -7$ and to recall the definition of a polynomial expression (first presented in Algebra I, Module 1 and given again here in the student materials).

POLYNOMIAL EXPRESSION: A *polynomial expression* is either:

(1) A numerical expression or a variable symbol, or

(2) The result of placing two previously generated polynomial expressions into the blanks of the addition operator (__+__) or the multiplication operator (__× __).

- What does the definition of a polynomial expression tell us, then, about the constant term of the polynomial $x - 7$?
  - That the constant term is actually -7.
- While we may write the polynomial as $x - 7$, the terms of this polynomial are actually x and -7.

Exercises 2–6 (9 minutes)

Students may benefit from doing the first few of the exercises as a group.

Exercises 2–6

Multiply the following binomials; note that every binomial given in the problems below is a polynomial in one variable, x, with a degree of one. Write the answers in standard form, which in this case will take the form $ax^2 + bx + c$, where a, b, and c are constants.

2. $(x + 1)(x-7)$

$x^2 - 6x - 7$

3. $(x+9)(x+2)$

   $x^2 + 11x + 18$

4. $(x-5)(x-3)$

   $x^2 - 8x + 15$

5. $\left(x+\frac{15}{2}\right)(x-1)$

   $x^2 + \frac{13}{2}x - \frac{15}{2}$

6. $\left(x-\frac{5}{4}\right)\left(x-\frac{3}{4}\right)$

   $x^2 - 2x + \frac{15}{16}$

Allow early finishers to record their answers to the next question independently, and then host a class discussion on the question below. Scaffold as needed until students are able to see and verbalize the patterns (as described by the sample solution below).

Describe any patterns you noticed as you worked.

All the coefficients for the x^2 term are 1. The constant term for the resulting trinomial is the product of constant terms of the two binomials. There are always two terms that are like terms and can be combined; the coefficients of those terms after they are combined is the sum of the constant terms from the two binomials.

- If the coefficients of one of the x-terms in the binomials above were not 1, would the observations that we just made still hold true?
  - *No.*
- Can you give an example that proves these patterns would not hold in that case?

Optionally, note that:

- The trinomial that results from the multiplication in Exercise 6 is factorable over the rationals.

Exercises 7–10 (8 minutes)

- A polynomial expression of degree 2 is often referred to as a *quadratic expression*. Why do you suppose that is? What does the prefix "quad" have to do with a polynomial of degree 2?
- The term quadratic relates to quadrangles (rectangles and squares); quadratic expressions and equations are useful for solving problems involving quadrangles. Further, we will come to learn about a quadrangle method for working with quadratic expressions and equations called *completing the square*.

Scaffolding:

Choose one or more of the exercises, and use a tabular model to reinforce the conceptual understanding of the use of the distributive property to multiply polynomials.

- In this module, we will spend a substantial amount of time exploring quadratic expressions in one variable.

You might decide to begin these exercises with guidance and slowly remove support, moving the students toward independence.

Exercises 7–10

Factor the following quadratic expressions.

7. $x^2 + 8x + 7$

   $(x + 7)(x + 1)$

8. $m^2 + m - 90$

   $(m + 10)(m - 9)$

9. $k^2 - 13k + 40$

   $(k - 8)(k - 5)$

10. $v^2 + 99v - 100$

    $(v - 1)(v + 100)$

Have students check their results by multiplying to see if the product is the original quadratic expression.

Example 3 (5 minutes): Quadratic Expressions

Example 3: Quadratic Expressions

If the leading coefficient for a quadratic expression is not 1, the first step in factoring should be to see if all the terms in the expanded form have a common factor. Then, after factoring out the greatest common factor, it may be possible to factor again.

For example, to factor to $2x^3 - 50x$ completely, begin by finding the GCF.
The GCF of the expression is $2x$: $2x(x^2 - 25)$.
Now, factor the difference of squares: $2x(x - 5)(x + 5)$.

Another example: Follow the steps to factor $-16t^2 + 32t + 48$ completely.

a. **First, factor out the GCF. (Remember: When you factor out a negative number, all the signs on the resulting factor will change.)**

   The GCF is -16. Hint: Do not leave the negative 1 as the leading coefficient. Factor it out with the 16.

   $-16(t^2 - 2t - 3)$

b. **Now look for ways to factor further. (Notice the quadratic expression will factor.)**

   $-16(t - 3)(t + 1)$

Scaffolding:

Strategically grouping or pairing students for these examples might help struggling students to get support and emerging students to deepen their understanding.

- Are all the factors prime?
  - *No, -16 is not prime, but the other factors, $(t-3)$ and $(t+1)$, are prime, so we can say that the polynomial has been factored completely over the integers.*

Closing (2 minutes)

- A deep understanding of factoring quadratic trinomials relies on a full and deep understanding of multiplication of binomials and the reverse relationship between the two.

Lesson Summary

Multiplying binomials is an application of the distributive property; each term in the first binomial is distributed over the terms of the second binomial.

The area model can be modified into a tabular form to model the multiplication of binomials (or other polynomials) that may involve negative terms.

When factoring trinomial expressions (or other polynomial expressions), it is useful to look for a GCF as your first step.

Do not forget to look for these special cases:

- **The square of a binomial**
- **The product of the sum and difference of two expressions.**

Exit Ticket (8 minutes)

Name ____ Date____________________

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Exit Ticket

1. Factor completely: $2a^2 + 6a + 18$

2. Factor completely: $5x^2 - 5$

3. Factor completely: $3t^3 + 18t^2 - 48t$

4. Factor completely: $4n - n^3$

Exit Ticket Sample Solutions

1. **Factor completely:** $2a^2 + 6a + 18$

   Factor out the GCF: $2(a^2 + 3a + 9)$

2. **Factor completely:** $5x^2 - 5$

   Factor out the GCF: $5(x^2 - 1)$

   Now, factor the difference of perfect squares: $5(x+1)(x-1)$

3. **Factor completely:** $3t^3 + 18t^2 - 48t$

   The GCF of the terms is $3t$.

   Factor out $3t$: $3t^3 + 18t^2 - 48t = 3t(t^2 + 6t - 16)$

   To factor further, find the pair of integers whose product is -16 and whose sum is $+6$.

   $(+8)(-2) = -16$ and $(+8) + (-2) = 6$, so the factors will have -2 and $+8$.

   So, the final factored form is: $3t(t+8)(t-2)$.

4. **Factor completely:** $4n - n^3$

   Factor out the GCF: $n(4 - n^2)$

   Then factor the difference of squares: $n(2-n)(2+n)$

Problem Set Sample Solutions

1. **Factor these trinomials as the product of two binomials, and check your answer by multiplying.**

   a. $x^2 + 3x + 2$

      The pair of integers whose product is $+2$ and whose sum is $+3$ are $+1$ and $+2$.

      So, the factored form is $(x+1)(x+2)$.

      Check: $(x+1)(x+2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$

   b. $x^2 - 8x + 15$

      The pair of integers whose product is $+15$ and whose sum is -8 are -3 and -5.

      So, the factored form is $(x-3)(x-5)$.

      Check: $(x-3)(x-5) = x^2 - 5x - 3x + 15 = x^2 - 8x + 15$

   c. $x^2 + 8x + 15$

      The pair of integers whose product is $+15$ and whose sum is $+8$ are $+3$ and $+5$.

      So, the factored form is $(x+3)(x+5)$.

      Check: $(x+3)(x+5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15$

Scaffolding:

Use area models to reinforce the connection between quadratic expressions and rectangles.

- Example for part (a): We use the larger square to represent the x^2 (x by x square units), three 1 by x smaller rectangles, and two 1 by 1 unit squares.

x^2	x	x
x	1	1

Ask: What are the dimensions of this rectangle?

Answer: $(x+2)$ *by* $(x+1)$

Factor completely.

d. $4m^2 - 4n^2$

The GCF of the terms is 4.

Factor out 4: $4(m^2 - n^2)$

Factor the difference of squares: $4(m - n)(m + n)$

e. $-2x^3 - 2x^2 + 112x$

The GCF of the terms is $-2x$.

Factor out $-2x$: $-2x^3 - 2x^2 + 112x = -2x(x^2 + x - 56)$

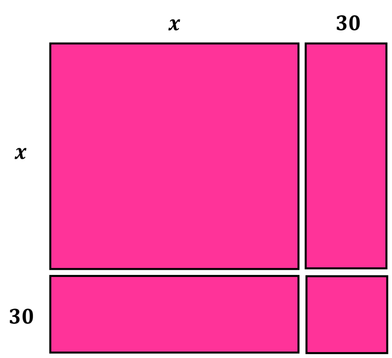
Factor the quadratic trinomial: $-2x(x - 7)(x + 8)$

f. $y^8 - 81x^4$

Factor the difference of squares: $y^8 - 81x^4 = (y^4 + 9x^2)(y^4 - 9x^2)$

Factor the difference of squares: $(y^4 + 9x^2)(y^2 + 3x)(y^2 - 3x)$

2. **The parking lot at Gene Simon's Donut Palace is going to be enlarged so that there will be an additional 30 ft. of parking space in the front of the lot and an additional 30 ft. of parking space on the side of the lot. Write an expression in terms of x that can be used to represent the area of the new parking lot.**



We know that the original parking lot is a square. We can let x equal the length of each side. We can represent each side of the new parking lot as $x + 30$. Using the area formula for a square, $\text{Area} = s^2$, we can represent this as $(x + 30)^2$.

$$\begin{aligned}(x + 30)^2 &= (x + 30)(x + 30)\\ &= x^2 + 60x + 900\end{aligned}$$

Explain how your solution is demonstrated in the area model.

The original square in the upper left corner is x by x, which results in an area of x^2 square units; each smaller rectangle is 30 by x, which results in an area of $30x$ square units; there are 2 of them, giving a total of $60x$ square units. The smaller square is 30 by 30 square units, which results in an area of 900 square units. That gives us the following expression for the area of the new parking lot: $x^2 + 60x + 900$.