Lesson 26: Recursive Challenge Problem—The Double and Add 5 Game

The *double and add 5* game is *loosely* related to the Collatz conjecture—an *unsolved* conjecture in mathematics named after Lothar Collatz, who first proposed the problem in 1937. The conjecture includes a recurrence relation, *triple and add 1,* as part of the problem statement. A worthwhile activity for you and your class is to read about the conjecture online.

Classwork

**Example 1**

Fill in the *doubling and adding*  below:

|  |  |
| --- | --- |
| Number | Double and add 5 |
|  |  |
|  | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| \_\_\_\_\_\_ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| \_\_\_\_\_\_ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| \_\_\_\_\_\_ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

Exercise 1

Complete the tables below for the given starting number.

|  |  |
| --- | --- |
| Number | Double and add 5 |
|  | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| \_\_\_\_\_\_ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| \_\_\_\_\_\_ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

|  |  |
| --- | --- |
| Number | Double and add 5 |
|  | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| \_\_\_\_\_\_ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| \_\_\_\_\_\_ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

Exercise 2

Given a starting number, double it and add 5 to get the result of round 1. Double the result of Round 1 and add 5, and so on. The goal of the game is to find the smallest starting whole number that produces a result of 100 or greater in three rounds or fewer.

Exercise 3

Using a generic initial value, , and the recurrence relation, , for , find a formula for in terms of .

**Vocabulary**

**Sequence:** A *sequence* can be thought of as an ordered list of elements. The elements of the list are called the *terms of the sequence.*

For example, (P, O, O, L) is a sequence that is different than (L, O, O, P). Usually the terms are *indexed* (and therefore ordered) by a subscript starting at either or : ,, , , …. The “…” symbol indicates that the pattern described is regular, that is, the next term is , and the next is , and so on. In the first example, is the first term, is the second term, and so on. Both finite and infinite sequences exist everywhere in mathematics. For example, the infinite decimal expansion of can be represented as the sequence, (, , , , …).

**Recursive Sequence:** An example of a *recursive sequence* is a sequence that is defined by (1) specifying the values of one or more initial terms and (2) having the property that the remaining terms satisfy a recurrence relation that describes the value of a term based upon an algebraic expression in numbers, previous terms, or the index of the term.

The sequence generated by initial term, , and recurrence relation, , is the sequence (,,,,, …). Another example, given by the initial terms, , and recurrence relation, , generates the famed *Fibonacci sequence* (, ,,, , …).

Problem Set

1. Write down the first 5 terms of the recursive sequences defined by the initial values and recurrence relations below:
   1. and , for ,
   2. and , for ,
   3. and , for ,
   4. and , for ,
   5. and , for ,
   6. and , for ,
   7. and , for ,
   8. and , for ,
   9. and , for ,
2. Look at the sequences you created in Problems 1(b) through 1(d). How would you define a recursive sequence that generates multiples of ?
3. Look at the sequences you created in problems 1(e) through 1(g). How would you define a recursive sequence that generates powers of ?
4. The following recursive sequence was generated starting with an initial value of , and the recurrence relation , for. Fill in the blanks of the sequence

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1. For the recursive sequence generated by initial value, , and recurrence relation, , for , find a formula for ,,, in terms of . Describe in words what this sequence is generating.
2. For the recursive sequence generated by initial value, , and recurrence relation, , for , find a formula for ,,, in terms of .