Lesson 23: Solution Sets to Simultaneous Equations

Classwork

Opening Exercise

Here is a system of two linear equations. Verify that the solution to this system is $(3,4)$.

Equation A1: $y = x + 1$

Equation A2: $y = -2x + 10$

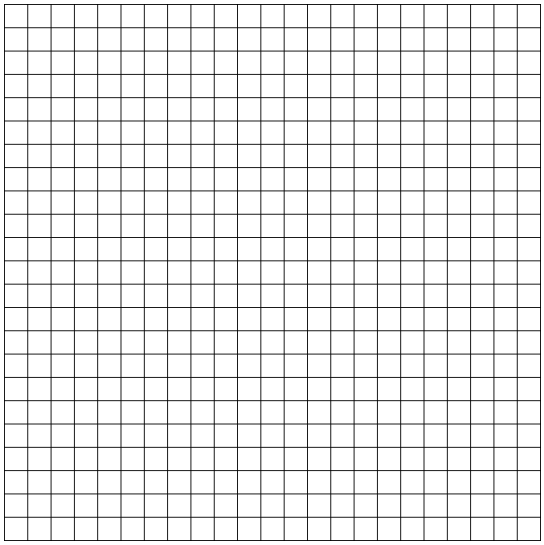
Exploratory Challenge

a. Write down another system of two linear equations whose solution is $(3, 4)$. This time make sure both linear equations have a positive slope.

b. Verify that the solution to this system of two linear equations is $3,4$.

c. Graph equation B1 and B2.

d. Are either B1 or B2 equivalent to the original A1 or A2? Explain your reasoning.



e. Add A1 and A2 to create a new equation C1. Then, multiply A1 by 3 to create a new equation C2. Why is the solution to this system also $(3, 4)$? Explain your reasoning.

The following system of equations was obtained from the original system by adding a multiple of equation A2 to equation A1.

Equation D1: $y = x + 1$

Equation D2: $3y = -3x + 21$

f. What multiple of A2 was added to A1 to create D2?

g. What is the solution to the system of two linear equations formed by D1 and D2?

h. Is D2 equivalent to the original A1 or A2? Explain your reasoning.

i. Start with equation A1. Multiply it by a number of your choice and add the result to equation A2. This creates a new equation E2. Record E2 below to check if the solution is $(3,4)$.

Equation E1: $y = x + 1$

Equation E2:

Example 1: Why Does the Elimination Method Work?

Solve this system of linear equations algebraically.

ORIGINAL SYSTEM	NEW SYSTEM	SOLUTION
$2x + y = 6$		
$x - 3y = -11$		

Exercises 1–2

1. Explain a way to create a new system of equations with the same solution as the original that eliminates variable y from one equation. Then determine the solution.

ORIGINAL SYSTEM	NEW SYSTEM	SOLUTION
$2x + 3y = 7$		
$x - y = 1$		

2. Explain a way to create a new system of equations with the same solution as the original that eliminates variable x from one equation. Then determine the solution.

ORIGINAL SYSTEM	NEW SYSTEM	SOLUTION
$2x + 3y = 7$		
$x - y = 1$		

Problem Set

Try to answer the following without solving for x and y first:

1. If $3x + 2y = 6$ and $x + y = 4$, then
   a. $2x + y = ?$ b. $4x + 3y = ?$

2. You always get the same solution no matter which two of the four equations you choose from Problem 1 to form a system of two linear equations. Explain why this is true.

3. Solve the system of equations $\begin{aligned} y &= \frac{1}{4}x \\ y &= -x + 5 \end{aligned}$ by graphing. Then, create a new system of equations that has the same solution. Show either algebraically or graphically that the systems have the same solution.

4. Without solving the systems, explain why the following systems must have the same solution.
   System (i): $\begin{aligned} 4x - 5y &= 13 \\ 3x + 6y &= 11 \end{aligned}$ System (ii): $\begin{aligned} 8x - 10y &= 26 \\ x - 11y &= 2 \end{aligned}$

Solve each system of equations by writing a new system that eliminates one of the variables.

5. $\begin{aligned} 2x + y &= 25 \\ 4x + 3y &= 9 \end{aligned}$

6. $\begin{aligned} 3x + 2y &= 4 \\ 4x + 7y &= 1 \end{aligned}$