Lesson 9

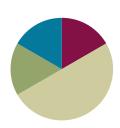
Objective: Add fractions making like units numerically.

Suggested Lesson Structure



- Application Problem (10 minutes)
- Concept Development (30 minutes)
- Student Debrief (10 minutes)

Total Time (60 minutes)



Fluency Practice (10 minutes)

Adding and Subtracting Fractions with Like Units 4.NF.3a

(1 minute)

• Sprint: Add and Subtract Fractions with Like Units 4.NF.3a

(9 minutes)

Adding and Subtracting Fractions with Like Units (1 minute)

Note: This quick fluency activity reviews adding and subtracting like units mentally.

- T: I'll say an addition or subtraction sentence. You say the answer. 2 fifths + 1 fifth.
- S: 3 fifths.
- T: 2 fifths 1 fifth.
- S: 1 fifth.
- T: 2 fifths + 2 fifths.
- S: 4 fifths.
- T: 2 fifths 2 fifths.
- S: Zero.
- T: 3 fifths + 2 fifths.
- S: 1.
- T: I'm going to write an addition sentence. You say whether it is true or false.
- T: (Write $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$.)
- S: True.
- T: (Write $\frac{3}{7} + \frac{3}{7} = \frac{6}{14}$.)
- S: False.



Provide written equations alongside the oral presentation. Colored response cards (green = true and red = false) can help scaffold responses to the statement, "Tell me if it's true or false." This statement might also be simplified to "Is it right?" to which English language learners may respond "yes" or "no."



Lesson 9: Date:



- T: Say the answer that makes this addition sentence true.
- S: 3 sevenths + 3 sevenths = 6 sevenths.

T: (Write
$$\frac{5}{9} + \frac{2}{9} = \frac{7}{18}$$
.)

- T: True or false?
- S: False.
- T: Say the answer that makes this addition sentence true.
- S: 5 ninths + 2 ninths = 7 ninths.

T: (Write
$$\frac{5}{9} + \frac{4}{9} = 1$$
.)

- T: True or false?
- S: True.
- T: Great work. You're ready for your Sprint!

Sprint: Add and Subtract Fractions with Like Units (9 minutes)

Materials: (S) Add and Subtract Fractions with Like Units Sprint

Note: This Sprint solidifies adding and subtracting fractions with like units and lays the groundwork for more advanced work with fractions.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

The Application Problem may feel like review for some students. Consider extending it by asking, "If Hannah keeps to this training pattern, how many days will it take her to reach a distance of 2 miles?"

You might also task students with generating other questions that could be asked about the story. For example:

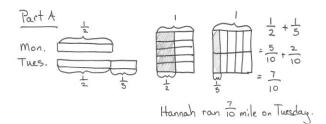
- How far did Hannah run in 5 days?
- How much farther did Hannah run than her friend on Tuesday?
- How much farther did Hannah run on day 10 than day 1?

If students offer a question for which there is insufficient information, ask how the problem could be altered for their question to be answered.

Application Problem (10 minutes)

Hannah and her friend are training to run in a 2-mile race. On Monday, Hannah ran $\frac{1}{2}$ mile. On Tuesday, she ran $\frac{1}{5}$ mile farther than she ran on Monday.

- a. How far did Hannah run on Tuesday?
- b. If her friend ran $\frac{3}{4}$ mile on Tuesday, how many miles did the girls run in all on Tuesday?
- T: Use the RDW (read, draw, write) process to solve this problem with your partner.
- S: (Read, draw, and write an equation, as well as a sentence to answer the question.)
- T: (Debrief the problem.) Could you use the same units to answer Problems 1 (a) and (b)? Why or why not?







Lesson 9: Date:



NOTES ON

MULTIPLE MEANS

OF ENGAGEMENT:

The Turn and Talk strategy allows

opportunity to practice academic

language in a relatively low-stakes

students to formulate responses before

setting. It also allows time for all

Consider pairing English language

learners with each other at first, and then shift the language composition of

groups over the course of the year.

sharing with the whole class.

English language learners the

S: No. There's no easy way to change fourths to tenths.

Note: This Application Problem reviews addition of fractions with unlike denominators, using visual models as learned in earlier lessons. This pictorial strategy lays the foundation for a more abstract strategy for making like units introduced in this lesson's Concept Development.

Concept Development (30 minutes)

Materials: (S) Personal white board

Problem 1

- T: How did you decide to use tenths in the first part of our Application Problem? Turn and talk.
- S: We can draw a rectangle and split it using the other unit. → Since we had halves and fifths, we drew two parts, and then split them into 5 parts each. That made 10 parts for the halves. That meant the fifths were each 2 smaller units, too.
- T: Turn and talk. What happened to the number of units we selected when we split our rectangle?
- S: Instead of one part, now we have five. → The number of selected parts is five times more. → The total number of parts is now 10.
- T: What happened to the size of the units?
- S: The units got smaller.
- T: Let me record what I hear you saying. Does this equation say the same thing?

(Record the following equation.)

$$\left(\frac{1\times5}{2\times5}\right) = \frac{5}{10}$$
 5 times as many selected units.
5 times as many units in the whole.

- S: Yes
- T: Write an equation like mine to explain what happened to the fifths.
- T: (Circulate and listen.) Jennifer, will you share for us?
- S: The number of parts we had doubled. The units are half as big as before, but there are twice as many of them.

$$\left(\frac{1\times2}{5\times2}\right) = \frac{2}{10}$$
 Number of parts doubled or 2 times as many parts.

Number of units in whole doubled or twice as many parts in the whole.



COMMON

Lesson 9: Date: Add fractions making like units numerically. 10/29/14



3.C.18

T: Then, of course, we could add the two fractions together. (Write the equation as shown below.)

$$\left(\frac{1\times5}{2\times5}\right) + \left(\frac{1\times2}{5\times2}\right) = \frac{5}{10} + \frac{2}{10} = \frac{7}{10}$$

- T: Are there other units we could have used to make these denominators the same? In other words, do 2 and 5 have other common multiples?
- S: Yes. We could have used 20ths, 30ths, or 50ths.
- T: If we had used 20ths, how many slices would we need to change $\frac{1}{2}$? To change $\frac{1}{5}$? Turn and talk. Draw a model on your personal board, if necessary.
- S: (Discuss.)
- T: Let's hear your ideas.
- S: 10 slices for half. → Ten times as many units in the whole, and 10 times as many units that were selected. → 4 slices for fifths. → 4 times as many selected units and 4 times as many units in the whole. The units are smaller in size.
- T: Let's record that on our boards in equation form. (Write the equation as shown below.)

$$\left(\frac{1\times10}{2\times10}\right) + \left(\frac{1\times4}{5\times4}\right)$$
$$= \frac{10}{20} + \frac{4}{20}$$
$$= \frac{14}{20}$$

- T: $ls \frac{14}{20}$ the same amount as $\frac{7}{10}$?
- S: Yes, they are equivalent. $\rightarrow \frac{7}{10}$ is simplified.
- T: Express $\frac{1}{2} + \frac{1}{5}$ using another unit. Show your thinking with an equation.
- S: (Draw and write appropriate representations.)
- T: Who used the smallest unit? Who used the largest unit? Who had the least or most units in their whole? Turn and talk.
- S: (Share.)
- T: Please share your findings.
- S: I used 30ths, so I had to multiply both the numerator and denominator of $\frac{1}{2}$ by 15, and multiply both parts of $\frac{1}{5}$ by 6. That's $\frac{21}{30}$ in all. \rightarrow I used 50ths. I had smaller units in my whole, so I needed 25 to make $\frac{1}{2}$ and 10 to make $\frac{1}{5}$. That's $\frac{35}{50}$ in all.
- T: (Write $\frac{1}{2} + \frac{1}{5} = \frac{7}{10} = \frac{14}{20} = \frac{21}{30} = \frac{35}{50}$.) Look at this equation. What do the types of units we used have in common?



Lesson 9: Date:



- S: All of the units are smaller than halves and fifths. → All are common multiples of 2 and 5. → All are multiples of ten.
- T: Will the new unit always be a multiple of the original units? Think about this question as we solve the next problem.

Problem 2:
$$\frac{1}{2} + \frac{2}{3}$$

- T: (Write $\frac{1}{2} + \frac{2}{3}$.) How does this problem compare with our first problem?
- S: It's still adding ¹/₂ to something else. → The first problem was two unit fractions. → This problem only has one unit fraction. → We were adding an amount less than half to ¹/₂ in the first problem, but ²/₃ is more than half.

$$\left(\frac{1\times3}{2\times3}\right) + \left(\frac{2\times2}{3\times2}\right)$$

$$= \frac{3}{6} + \frac{4}{6}$$

$$= \frac{7}{6} \text{ or } |\frac{1}{6}|$$

- T: Great observations! What predictions can you make about the sum? For the units we use, what changes?
- S: The sum should be a fraction greater than one. \rightarrow We won't use most of the units from before.
- T: How can you be sure?
- S: We are adding half and more than half. \rightarrow Only one of our units from before is a multiple of 3.
- T: Imagine the rectangle that helps you find a like unit. Record an equation that explains what you saw in your mind's eye. (Circulate and observe.)
- T: Show your equation, and explain it.
- S: (Display equation.) I used sixths. My equation shows that for $\frac{1}{2}$, the number of pieces tripled, and the units in the whole tripled too. For $\frac{2}{3}$, the number of parts doubled, and so did the units in the whole.
- T: Was our prediction about the answer correct?
- S: Yes! The sum is greater than one!
- T: Did anyone use another unit to find the sum? (Record sums on the board as students respond.)
- T: Do these units follow the pattern we saw in our earlier work? Let's keep looking for evidence as we work.

Problem 3: $\frac{5}{9} + \frac{5}{6}$

- T: Compare this problem to the others. Turn and talk.
- S: My partner and I see different things. I think this problem is like Problem 2 because the addends are more than half. But my partner says this one is like Problem 1 because the numerators are the same, even though they are not unit



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

While the focus of this lesson is the transition between pictorial and abstract representations of like units, allow students to continue to use the rectangular fraction model from previous lessons as a scaffold for writing and solving equations.

$$\frac{5}{9} + \frac{5}{6}$$

$$= \left(\frac{5 \times 2}{9 \times 2}\right) + \left(\frac{5 \times 3}{6 \times 3}\right)$$

$$= \frac{10}{18} + \frac{15}{18}$$

$$= \frac{25}{18} = \frac{18}{18} + \frac{7}{18} = \left[\frac{7}{8}\right]$$



Lesson 9: Date:



fractions. \rightarrow This one is different because you can find a larger unit than the one you would use if you multiplied 6 and 9. \rightarrow 18ths work as a like or common unit, and is a larger unit than 54ths.

T: Find the sum. Use an equation to show your thinking.

Follow a similar procedure to Problems 1 and 2 to debrief the solution.

Problem 4:
$$\frac{2}{3} + \frac{1}{4} + \frac{1}{2}$$

- T: This problem has three addends. Does this affect our approach to solving?
- No. We still have to find a like unit. It has to be a multiple of all three denominators.
- T: Find the sum using an equation. (Debrief as above.)
- T: To summarize, what patterns have you observed about the like
- $= \left(\frac{2 \times 4}{3 \times 4}\right) + \left(\frac{1 \times 3}{4 \times 3}\right) + \left(\frac{1 \times 6}{2 \times 6}\right)$ $=\frac{8}{12}+\frac{3}{12}+\frac{6}{12}$ $=\frac{17}{12}=\frac{12}{12}+\frac{5}{12}=\frac{5}{12}$

 $\frac{2}{3} + \frac{1}{4} + \frac{1}{2}$

All the new units we found are common multiples of our original units.

We don't always have to multiply the original units to find a common multiple. \rightarrow You can skip-count by the largest common unit, or like unit, to find smaller common or like units.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

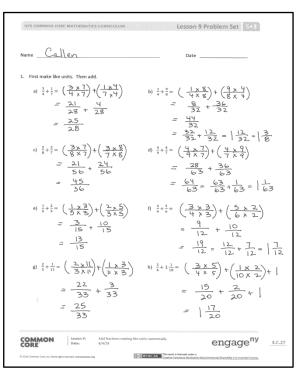
Lesson Objective: Add fractions making like units numerically.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

T: Check your answers with your partner. Please do not change any of them. (Allow time for



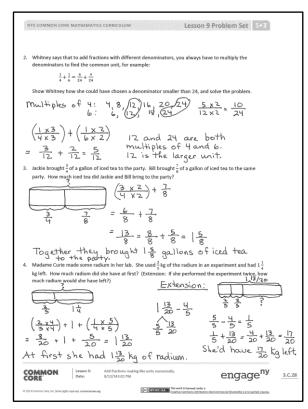


Lesson 9: Date:



students to confer.)

- T: Did you notice any patterns in the sums on the Problem Set?
- S: The answers in the first column were all less than a whole. → The answers in the second column were more than a whole.
- T: I noticed that Problem 1(b) is different from all the other problems. Can you explain how it is different?
- S: 1(b) is different because I only had to change the unit of one fraction to be like the other one.
 → One unit is a multiple of the other. → Eighths can be made out of fourths. None of the others were like that.
- T: Student A, please share your answer and your partner's answer to Problem 1(b).
- S: I got 1 and $\frac{3}{8}$, but Student B got 1 and $\frac{12}{32}$.
- T: Class, is it a problem that Student A's and Student B's answers to 1(b) are different?
- S: No. It is the same amount. They just used different units. → You don't always have to multiply.



MP.7

- T: Did this situation come up more often in some problems than others?
- S: Yes. It happened more in Problems 1 (f) and (h).
- T: Why?
- S: Multiplying the units together in these didn't give us the largest unit they had in common. → I could find a smaller common multiple than just multiplying them together. → I skip-counted by the smaller denominator until I got to a multiple of the other denominator. → If I multiplied them together, I could simplify the answer I got to use a larger unit.
- T: How can these observations help you answer Problem 2?
- S: Problem 2 was like Problems 1 (f) and (h). There was a larger unit in common. → You can slice the units by the same number to get a common unit.
- T: Terrific insights! Put them to use as you complete your Exit Ticket.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.



Lesson 9: Date:



A

Correct____

	Add or subtract.				
1	$\frac{1}{5} + \frac{1}{5} =$	/	23	$\frac{1}{9} + \frac{1}{9} + \frac{1}{9} =$	/
2	$\frac{1}{10} + \frac{5}{10} =$	/	24	$\frac{1}{9} + \frac{3}{9} + \frac{1}{9} =$	/
3	$\frac{1}{10} + \frac{7}{10} =$	/	25	$\frac{4}{9} - \frac{1}{9} - \frac{3}{9} =$	/
4	$\frac{2}{5} + \frac{2}{5} =$	/	26	$\frac{1}{4} + \frac{2}{4} + \frac{1}{4} =$	/
5	$\frac{5}{10} - \frac{4}{10} =$	/	27	$\frac{1}{8} + \frac{3}{8} + \frac{2}{8} =$	/
6	$\frac{3}{5} - \frac{1}{5} =$	/	28	$\frac{5}{12} + \frac{1}{12} + \frac{5}{12} =$	/
7	$\frac{3}{10} + \frac{3}{10} =$	/	29	$\frac{2}{9} + \frac{3}{9} + \frac{2}{9} =$	/
8	$\frac{4}{5} - \frac{1}{5} =$	/	30	$\frac{3}{10} - \frac{3}{10} + \frac{3}{10} =$	/
9	$\frac{1}{4} + \frac{1}{4} =$	/	31	$\frac{3}{5} - \frac{1}{5} - \frac{1}{5} =$	/
10	$\frac{1}{4} + \frac{2}{4} =$	/	32	$\frac{1}{6} + \frac{2}{6} =$	/
11	$\frac{3}{12} - \frac{2}{12} =$	/	33	$\frac{3}{12} + \frac{4}{12} =$	1
12	$\frac{1}{4} + \frac{3}{4} =$	1	34	$\frac{3}{12} + \frac{6}{12} =$	/
13	$\frac{1}{12} + \frac{1}{12} =$	/	35	$\frac{4}{8} + \frac{2}{8} =$	/
14	$\frac{1}{3} + \frac{1}{3} =$	/	36	$\frac{4}{12} + \frac{1}{12} =$	/
15	$\frac{3}{12} - \frac{2}{12} =$	/	37	$\frac{1}{5} + \frac{3}{5} =$	/
16	$\frac{5}{12} + \frac{6}{12} =$	1	38	$\frac{2}{5} + \frac{2}{5} =$	1
17	$\frac{7}{12} + \frac{4}{12} =$	/	39	$\frac{1}{6} + \frac{2}{6} =$	/
18	$\frac{4}{6} - \frac{1}{6} =$	1	40	$\frac{5}{12} - \frac{3}{12} = $	/
19	$\frac{1}{6} + \frac{2}{6} =$, <i>L</i>	41	$\frac{7}{15} - \frac{2}{15} =$	/
20	$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} =$	1	42	$\frac{7}{15} - \frac{3}{15} =$	/
21	$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} =$	/	43	$\frac{11}{15} - \frac{2}{15} =$	/
22	$\frac{1}{12} + \frac{1}{12} + \frac{1}{12} =$	1	44	$\frac{2}{15} + \frac{4}{15} =$	/

add and subtract fractions with like units



Lesson 9: Date:



В	Improvement	# Correct	
Add or subtract.			

	Add or subtract.				
1	$\frac{1}{2} + \frac{1}{2} =$	/	23	$\frac{1}{12} + \frac{6}{12} + \frac{2}{12} =$	/
2	$\frac{2}{8} + \frac{1}{8} =$	/	24	$\frac{4}{12} + \frac{3}{12} + \frac{3}{12} =$	/
3	$\frac{2}{8} + \frac{3}{8} =$	/	25	$\frac{8}{12} - \frac{4}{12} - \frac{4}{12} =$	/
4	$\frac{2}{12} - \frac{1}{12} =$	/	26	$\frac{1}{10} + \frac{2}{10} + \frac{4}{10} =$	/
5	$\frac{5}{12} + \frac{2}{12} =$	/	27	$\frac{1}{10} + \frac{1}{10} + \frac{6}{10} =$	/
6	$\frac{4}{8} + \frac{3}{8} =$	/	28	$\frac{4}{6} + \frac{1}{6} + \frac{1}{6} =$	/
7	$\frac{4}{8} - \frac{3}{8} =$	/	29	$\frac{2}{12} + \frac{3}{12} + \frac{4}{12} =$	/
8	$\frac{1}{8} + \frac{5}{8} =$	1	30	$\frac{2}{10} + \frac{4}{10} + \frac{4}{10} =$	/
9	$\frac{3}{4} - \frac{1}{4} =$	/	31	$\frac{3}{10} + \frac{1}{10} + \frac{2}{10} =$	/
10	$\frac{3}{6} - \frac{3}{6} =$	/	32	$\frac{4}{6} - \frac{2}{6} =$	1
11	$\frac{3}{9} + \frac{3}{9} =$	1	33	$\frac{3}{12} - \frac{2}{12} =$	/
12	$\frac{2}{3} + \frac{1}{3} =$	/	34	$\frac{2}{3} + \frac{1}{3} =$	/
13	$\frac{6}{9} - \frac{4}{9} =$	/	35	$\frac{2}{4} + \frac{1}{4} =$	/
14	$\frac{5}{9} - \frac{3}{9} =$	/	36	$\frac{3}{12} + \frac{2}{12} =$	/
15	$\frac{2}{9} + \frac{2}{9} =$	/	37	$\frac{1}{5} + \frac{2}{5} =$	1
16	$\frac{1}{12} + \frac{3}{12} =$	/	38	$\frac{4}{5} - \frac{4}{5} =$	1
17	$\frac{5}{12} - \frac{4}{12} =$	1	39	$\frac{5}{12} - \frac{1}{12} =$	1
18	$\frac{9}{12} - \frac{6}{12} =$	/	40	$\frac{6}{8} + \frac{2}{8} =$	1
19	$\frac{6}{10} - \frac{4}{10} =$	/	41	$\frac{2}{8} + \frac{2}{8} + \frac{2}{8} =$	1
20	$\frac{2}{8} + \frac{2}{8} + \frac{2}{8} =$	/	42	$\frac{9}{10} - \frac{7}{10} - \frac{1}{10} =$	/
21	$\frac{1}{10} + \frac{1}{10} + \frac{1}{10} =$	/	43	$\frac{2}{10} + \frac{5}{10} + \frac{2}{10} =$	/
22	$\frac{7}{10} - \frac{2}{10} - \frac{4}{10} =$	/	44	$\frac{9}{12} - \frac{1}{12} - \frac{4}{12} =$	/

add and subtract fractions with like units



Lesson 9: Date:



Date _____

1. First make like units, then add.

a.
$$\frac{3}{4} + \frac{1}{7} =$$

b.
$$\frac{1}{4} + \frac{9}{8} =$$

c.
$$\frac{3}{8} + \frac{3}{7} =$$

d.
$$\frac{4}{9} + \frac{4}{7} =$$

e.
$$\frac{1}{5} + \frac{2}{3} =$$

f.
$$\frac{3}{4} + \frac{5}{6} =$$

g.
$$\frac{2}{3} + \frac{1}{11} =$$

h.
$$\frac{3}{4} + 1\frac{1}{10} =$$



Lesson 9: Date:



2. Whitney says that to add fractions with different denominators, you always have to multiply the denominators to find the common unit; for example:

$$\frac{1}{4} + \frac{1}{6} = \frac{6}{24} + \frac{4}{24}$$

Show Whitney how she could have chosen a denominator smaller than 24, and solve the problem.

3. Jackie brought $\frac{3}{4}$ of a gallon of iced tea to the party. Bill brought $\frac{7}{8}$ of a gallon of iced tea to the same party. How much iced tea did Jackie and Bill bring to the party?

4. Madame Curie made some radium in her lab. She used $\frac{2}{5}$ kg of the radium in an experiment and had $1\frac{1}{4}$ kg left. How much radium did she have at first? (Extension: If she performed the experiment twice, how much radium would she have left?)



Lesson 9: Date:



Name _____ Date _____

Make like units, then add.

a.
$$\frac{1}{6} + \frac{3}{4} =$$

b.
$$1\frac{1}{2} + \frac{2}{5} =$$



Lesson 9: Date:



Date _____

1. Make like units, then add.

a.
$$\frac{3}{5} + \frac{1}{3} =$$

b.
$$\frac{3}{5} + \frac{1}{11} =$$

c.
$$\frac{2}{9} + \frac{5}{6} =$$

d.
$$\frac{2}{5} + \frac{1}{4} + \frac{1}{10} =$$

e.
$$\frac{1}{3} + \frac{7}{5} =$$

f.
$$\frac{5}{8} + \frac{7}{12} =$$

g.
$$1\frac{1}{3} + \frac{3}{4} =$$

h.
$$\frac{5}{6} + 1\frac{1}{4} =$$



2. On Monday, Ka practiced guitar for $\frac{2}{3}$ of one hour. When she finished, she practiced piano for $\frac{3}{4}$ of one hour. How much time did Ka spend practicing instruments on Monday?

3. Ms. How bought a bag of rice for dinner. She used $\frac{3}{5}$ kg of the rice, and still had $2\frac{1}{4}$ kg left. How heavy was the bag of rice that Ms. How bought?

4. Joe spends $\frac{2}{5}$ of his money on a jacket and $\frac{3}{8}$ of his money on a shirt. He spends the rest on a pair of pants. What fraction of his money does he use to buy the pants?



Lesson 9: Date:

