Lesson 19: Rearranging Formulas

Student Outcomes

- Students learn to think of some of the letters in a formula as constants in order to define a relationship between two or more quantities, where one is in terms of another, for example holding V in $V = IR$ as constant, and finding R in terms of I.

Classwork

Provide an introduction to the lesson:

- Formulas that relate two or more variable symbols such as $A = lw$, $D = rt$, or $a^2 + b^2 = c^2$ arise in different applications of mathematics, science, and other areas of study. These formulas have meaning based on a situation.
- However, even without an applied setting, formulas can stand on their own as a relationship between variables.
- You can use the equation-solving techniques from earlier lessons to rearrange formulas and solve for a specific variable symbol.

Scaffolding:

- Before starting the warm-up, ask students to read the introduction and discuss other formulas they have seen in previous grades.
- As students work the warm-up and first few exercises, adjust the pacing depending on how well students are doing with Exercises 1(c) and 3.

Exercise 1 (5 minutes)

Have students work independently. Monitor their progress on part (c) and have them review and correct their solutions with a partner.

Exercise 1

Solve each equation for x. For part (c), remember a variable symbol, like a, b, and c, represents a number.

a. $2x - 6 = 10$

$x = 8$

b. $-3x - 3 = -12$

$x = 3$

c. $ax - b = c$

$$ax - b = c$$
$$ax = b + c$$
$$x = \frac{b + c}{a}$$

Exercises 2–3 (5 minutes)

Exercise 2

Compare your work in parts (a) through (c) above. Did you have to do anything differently to solve for x in part (c)?

The process to solve all three equations is the same.

Exercise 3

Solve the equation $ax - b = c$ for a. The variable symbols x, b, and c represent numbers.

Solving for a is the same process as solving for x. $a = \frac{b+c}{x}$

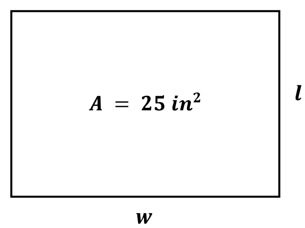
Debrief student responses to Exercises 2 and 3 as a whole class. Make sure to emphasize the points below.

- Variables are placeholders for numbers and as such have the same properties.
- When solving an equation with several variables, you use the same properties and reasoning as with single-variable equations.
- The equation in Exercise 3 holds as long as x does not equal 0 (division by 0 is undefined). Consider your result from Exercise 1(c). Does this equation hold for all values of the variables involved?
    - *No, it only holds if $a \neq 0$.*

Example 1 (5 minutes): Rearranging Familiar Formulas

Example 1: Rearranging Familiar Formulas

The area A of a rectangle is 25 in^2. The formula for area is $A = lw$.



- **If the width w is 10 inches, what is the length l?**

  $l = \frac{5}{2}$

- **If the width w is 15 inches, what is the length l?**

  $l = \frac{5}{3}$

- **Rearrange the area formula to solve for l.**

$$A = lw$$
$$\frac{A}{w} = \frac{lw}{w}$$
$$\frac{A}{w} = l \text{ or } l = \frac{A}{w}$$

- **Verify that the area formula, solved for l, will give the same results for l as having solved for l in the original area formula.**

  $A = lw$

  $l = \frac{A}{w} = \frac{25}{10} = \frac{5}{2}$

  $l = \frac{A}{w} = \frac{25}{15} = \frac{5}{3}$

MP.3

Walk students through the solution to this problem. Have them write the reasons for each step in the equation solving process on their paper. Much of the work students will do in future classes will involve rearranging formulas to highlight a variable of interest. Begin to set the stage that solving for a variable *before* you plug in values is often easier than solving after you substitute the values, especially when the numbers are not user-friendly. If time permits, give them $A = 10.356$ and $w = 5\frac{3}{11}$ and ask them to solve for the length.

Exercise 4 (7 minutes)

Have students work in small groups or with a partner. Solving these exercises two ways will help students to further understand that rearranging a formula with variables involves the same reasoning as solving an equation for a single variable.

Exercise 4

Solve each problem two ways. First, substitute the given values and solve for the given variable. Then, solve for the given variable and substitute the given values.

a. **The perimeter formula for a rectangle is $p = 2(l + w)$, where p represents the perimeter, l represents the length, and w represents the width. Calculate l when $p = 70$ and $w = 15$.**

Sample responses:

Substitute and solve. $70 = 2\ l + 15\ ,\ l = 20$

Solve for the variable first: $l = \frac{p}{2} - w$

b. **The area formula for a triangle is $A = \frac{1}{2}bh$, where A represents the area, b represents the length of the base, and h represents the height. Calculate b when $A = 100$ and $h = 20$.**

$b = \frac{2A}{h}$, $b = 10$

Have one or two students present their solutions to the entire class.

Exercise 5 (7 minutes)

The next set of exercises increases slightly in difficulty. Instead of substituting, students solve for the requested variable. Have students continue to work in groups or with a partner. If the class seems to be getting stuck, solve part of one exercise as a whole class and then have them go back to working with their partner or group.

Have students present their results to the entire class. Look for valid solution methods that arrive at the same answer using a slightly different process to isolate the variable. For part (b–ii), students may need a reminder to use the square root to "undo" the square of a number. They learned about square roots and solving simple quadratic equations in Grade 8.

Exercise 5

Rearrange each formula to solve for the specified variable. Assume no variable is equal to 0.

a. **Given $A = P(1 + rt)$,**

i. **Solve for P.**

$$P = \frac{A}{1 + rt}$$

ii. **Solve for t.**

$$t = \frac{A}{P} - 1\ \div r$$

b. Given $K = \frac{1}{2}mv^2$,

i. Solve for m.

$$m = \frac{2K}{v^2}$$

ii. Solve for v.

$$v = \pm\ \overline{\frac{2K}{m}}$$

Example 2 (10 minutes): Comparing Equations with One Variable to Those With More Than One Variable

Demonstrate how to reverse the distributive property (factoring) to solve for x. Start by solving the related equation OR solve both equations at the same time, one line at a time. Encourage students to make notes justifying their reasoning on each step. Continue emphasizing that the process is the same because variables are just numbers whose values have yet to be assigned.

Scaffolding:

- In Example 2, some students may not need to solve the problems in the right column first. Others may need to start there before solving the literal equations in the left column.

Example 2

Equation Containing More Than One Variable	Related Equation
Solve $ax + b = d - cx$ for x. $ax + cx + b = d$ $ax + cx = d - b$ $x\ \ a + c\ \ = d - b$ $x = \frac{d-b}{a+c}$	Solve $3x + 4 = 6 - 5x$ for x. $3x + 5x + 4 = 6$ $3x + 5x = 6 - 4$ $x\ \ 3 + 5\ \ = 2$ $8x = 2$ $x = \frac{2}{8} = \frac{1}{4}$
Solve for x. $\frac{ax}{b} + \frac{cx}{d} = e$ $bd\ \ \frac{ax}{b} + \frac{cx}{d}\ \ = bd\ \ e$ $dax + cbx = bde$ $x\ \ da + cb\ \ = bde$ $x = \frac{bde}{da+cb}$	Solve $\frac{2x}{5} + \frac{x}{7} = 3$ for x. $\frac{2x}{5} + \frac{x}{7} = 3$ $35\ \ \frac{2x}{5} + \frac{x}{7}\ \ = 35\ \ 3$ $14x + 5x = 105$ $x\ \ 14 + 5\ \ = 105$ $19x = 105$ $x = \frac{105}{19}$

Closing (2 minutes)

Review the Lesson Summary and close with these questions.

- How is rearranging formulas the same as solving equations that contain a single variable symbol?
- How is rearranging formulas different from solving equations that contain a single variable symbol?

As you wrap up, make sure students understand that while there is essentially no difference when rearranging formulas, it may seem more difficult and the final answers may appear more complicated because you cannot combine the variables into a single numerical expression like you can when you add, subtract, multiply, or divide numbers in the course of solving a typical equation.

> **Lesson Summary**
>
> **The properties and reasoning used to solve equations apply regardless of how many variables appear in an equation or formula. Rearranging formulas to solve for a specific variable can be useful when solving applied problems.**

Exit Ticket (4 minutes)

Name ____ Date____________________

Lesson 19: Rearranging Formulas

Exit Ticket

Given the formula $x = \frac{1+a}{1-a}$,

1. Solve for a when $x = 12$.

2. Rearrange the formula to solve for a.

Exit Ticket Sample Responses

Given the formula $= \frac{1+a}{1-a}$,

1. Solve for a when $x = 12$.

$$12 = \frac{1+a}{1-a}$$
$$12\ \ 1-a\ \ = 1+a$$
$$12 - 12a = 1 + a$$
$$11 = 13a$$
$$\frac{11}{13} = a$$

2. Rearrange the formula to solve for a.

$$x = \frac{1+a}{1-a}$$
$$x\ \ 1-a\ \ = 1+a$$
$$x - xa = 1 + a$$
$$x - 1 = a + xa$$
$$x - 1 = a(1+x)$$
$$\frac{x-1}{1+x} = a$$

Problem Set Sample Solutions

For Problems 1–8, solve for x.

1. $ax + 3b = 2f$

   $x = \frac{2f - 3b}{a}$

2. $rx + h = sx - k$

   $x = \frac{h+k}{s-r}$

3. $3px = 2q(r - 5x)$

   $x = \frac{2qr}{3p + 10q}$

4. $\frac{x+b}{4} = c$

   $x = 4c - b$

5. $\frac{x}{5} - 7 = 2q$

   $x = 10q + 35$

6. $\frac{x}{6} - \frac{x}{7} = ab$

   $x = 42ab$

7. $\frac{x}{m} - \frac{x}{n} = \frac{1}{p}$

   $x = \frac{nm}{n - m\ \ p}$

8. $\frac{3ax+2b}{c} = 4d$

   $x = \frac{4cd - 2b}{3a}$

9. Solve for m.

   $t = \frac{ms}{m+n}$

   $m = \frac{nt}{s-t}$

10. Solve for u.

    $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

    $u = \frac{vf}{v-f}$

11. Solve for s.

    $A = s^2$

    $s = \pm\ \overline{A}$

12. Solve for h.

    $V = \pi r^2 h$

    $h = \frac{V}{\pi r^2}$

13. Solve for m.

$T = 4\sqrt{m}$

$m = \frac{T^2}{16}$

14. Solve for d.

$F = G\frac{mn}{d^2}$

$d = \pm\sqrt{\frac{Gmn}{F}}$

15. Solve for y.

$ax + by = c$

$y = \frac{c - ax}{b}$

16. Solve for b_1.

$A = \frac{1}{2}h\ b_1 + b_2$

$b_1 = \frac{2A}{h} - b_2$

17. The science teacher wrote three equations on a board that relate velocity, v, distance traveled, d, and the time to travel the distance, t, on the board.

$v = \frac{d}{t}$ $\quad t = \frac{d}{v}$ $\quad d = vt$

Would you need to memorize all three equations or could you just memorize one? Explain your reasoning.

You could just memorize $d = vt$ since the other two equations are obtained from this one by solving for v and t.