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Lesson 18: Equations Involving a Variable Expression in the Denominator

**Student Outcomes**

* Students interpret equations like as two equations “” and “” joined by “and.” Students find the solution set for this new system of equations.

Classwork

Opening Exercise (5 minutes)

Allow students time to complete the warm up and then discuss the results.

Opening Exercise

Nolan says that he checks the answer to a division problem by performing multiplication. For example, he says that is correct because is, and is correct because is.

1. Using Nolan’s reasoning, explain why there is no real number that is the answer to the division problem .

There is no number such that .

1. Quentin says that . What do you think?

**MP.3**

While it is true that , the problem is that by that principle could equal any number.

1. Mavis says that the expression has a meaningful value for whatever value one chooses to assign to. Do you agree?

No, the expression does not have a meaningful value when .

1. Bernoit says that the expression always has the value for whichever value one assigns to . Do you agree?

The expression does equal for all values of except .

Note that the problem with is that too many numbers pass Nolan’s criterion! Have students change to a different number. It still passes Nolan’s multiplication check. Like , it is a problematic notion. For this reason, we want to disallow the possibility of ever dividing by zero.

Point out that an expression like is really accompanied with the clause “under the assumption the denominator is not zero.” So, should be read as a compound statement:

and OR and

Exercises 1–2 (5 minutes)

*Scaffolding:*

* Remind students of the difference between dividing by a number and dividing a number by .

Give students a few minutes to complete the problems individually. Then, elicit answers from students.

Exercises 1–2

1. Rewrite as a compound statement.

and

1. Consider .
   1. Is it permissible to let in this expression?

Yes, .

* 1. Is it permissible to let in this expression?

No, is not permissible.

* 1. Give all the values of that are *not* permissible in this expression.

, ,

**Examples 1–2 (10 minutes)**

Work through the examples as a whole class.

Example 1

Consider the equation .

1. Rewrite the equation into a system of equations.

and and

1. Solve the equation for , excluding the value(s) of that lead to a denominator of zero.

and and solution set:

Example 2

Consider the equation

1. Rewrite the equation into a system of equations.

*and*

1. Solve the equation for , excluding the value(s) of that lead to a denominator of zero.

and solution set:

Emphasize the process of recognizing a rational equation as a system composed of the equation itself and the excluded value(s) of . For Example 1, this is really the compound statement:

and and

* By the properties of equality, we can multiply through by non-zero quantities. Within this compound statement, and are nonzero, so we may write and and , which is equivalent toand and .
* All three declarations in this compound statement are true for . This is the solution set.

In Example 2, remind students of the previous lesson on solving equations involving factored expressions. Students will need to factor out the common factor and then apply the zero-product property.

* What happens in Example 2 when we have and ? Both declarations cannot be true. What can we say about the solution set of the equation?
  + *There is no solution.*

Exercises 3–11 (15 minutes)

Allow students time to complete the problems individually. Then, have students compare their work with another student. Make sure that students are setting up a system of equations as part of their solution.

Exercises 3–11

Rewrite each equation into a system of equations excluding the value(s) of that lead to a denominator of zero; then, solve the equation for .

1. 4. 5.

and and and

6. 7. 8.

and and and and

No solution

9. 10.

and and

No solution All real numbers except

1. A baseball player’s batting average is calculated by dividing the number of times a player got a hit by the total number of times the player was at bat. It is expressed as a decimal rounded to three places. After the first ten games of the season, Samuel had hits off of “at bats.”
   1. What is his batting average after the first ten games?
   2. How many hits in a row would he need to get to raise his batting average to above ?

He would need hits in a row to be above .

* 1. How many “at bats” in a row without a hit would result in his batting average dropping below ?

If he went “at bats” in a row without a hit, he would be below .

Ask:

* What was the difference between Exercises and ? How did that affect the solution set?
* Work through Exercises as a class.

Closing (5 minutes)

Ask these questions after going over the exercises.

* When an equation has a variable in the denominator, what must be considered?
* When the solution to the equation is also an excluded value of , then what is the solution set to the equation?

Exit Ticket (5 minutes)

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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Exit Ticket

1. Rewrite the equation as a system of equations. Then, solve for .
2. Write an equation that would have the restriction

Exit Ticket Sample Solutions

1. Rewrite the equation as a system of equations. Then, solve for .

and

1. Write an equation that would have the restriction .

sample answer

Problem Set Sample Solutions

1. Consider the equation . Is permissible? Which values of are excluded? Rewrite as a system of equations.

Yes, is permissible. The excluded values are , , and . The system is

and and and and .

1. Rewrite each equation as a system of equations excluding the value(s) of that lead to a denominator of zero. Then solve the equation for .

System: and ; solution set:

System: and ; solution set:

System: and ; solution set:

System: and and ; solution set:

System: and and ; solution set:

1. Ross wants to cut a -foot rope into two pieces so that the length of the first piece divided by the length of the second piece is .
   1. Let represent the length of the first piece. Write an equation that represents the relationship between the pieces as stated above.
   2. **What values of are not permissible in this equation? Describe within the context of the problem, what  
      situation is occurring if were to equal this value(s). Rewrite as a system of equations to exclude the value(s).**

is not a permissible value because it would mean the rope is still intact. System: and

* 1. **Solve the equation to obtain the lengths of the two pieces of rope. (Round to the nearest tenth if necessary.)**

First piece is feet long; second piece is feet long.

1. Write an equation with the restrictions , , and .

Answers will vary. Sample equation:

1. Write an equation that has no solution.

Answers will vary. Sample equation: