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Lesson 17: Equations Involving Factored Expressions

Student Outcomes

* Students learn that equations of the form have the same solution set as two equations joined by “or:” or Students solve factored or easily factorable equations.

Classwork

Exercise 1 (5 minutes)

Allow students a few minutes to complete only (a) through (d) of Exercise 1, either individually or in pairs.

Exercise 1

Solve each equation for .

* 2. Demanding Dwight insists that you give him two solutions to the following equation:

Can you provide him with two solutions?

**MP.7**

**&**

**MP.8**

* 1. Demanding Dwight now wants FIVE solutions to the following equation:

Can you provide him with five solutions?

Do you think there might be a sixth solution?

There are exactly solutions.

Discussion (5 minutes)

* If I told you that the product of two numbers is , could you tell me anything about the two numbers?
* Would the numbers have to be and ?
* Would both numbers have to be smaller than ?
* Would they both have to be positive?
* Is there much at all you could say about the two numbers.
  + *Not really. They have to have the same sign is about all we can say.*
* If I told you that the product of two numbers is zero, could you tell me anything about the two numbers?
  + *At least one of the numbers must be zero.*

**MP.7**

**&**

**MP.8**

* How could we phrase this mathematically?
  + *If , then either or or*
* This is known as the **zero-product property**.
* What if the product of three numbers is zero? What if the product of seven numbers is zero?
  + *If any product of numbers is zero, at least one of the terms in that product is zero.*

Exercise 1 (continued) (2 minutes)

Give students a few minutes to complete (e) and (f) and elicit student responses.

*Scaffolding:*

* + Give early finishers this challenge: Write a factored equation that has the solution: .

Consider the equation .

1. Rewrite the equation as a compound statement.

or

1. Find the two solutions to the equation.

**Examples 1–2 (5 minutes)**

Work the two examples as a class.

Example 1

Solve , for .

*Scaffolding:*

* Remind students of the practice of applying the distribution property “backwards” that they saw in the Lesson 6 Problem Set. This practice is called factoring.

Example 2

Solve for .

Lead a discussion about the application of the distributive property, in the form of factoring polynomial expressions, when solving the equations in these two examples.

**MP.6**

Students may want to divide both sides by . Remind them that is an unknown quantity that could be positive, negative, or zero. These cases need to be handled separately to get the correct answer. Here we will take a more familiar approach in the solution process, factoring.

Continue to emphasize the idea of rewriting the factored equation as a compound statement. Do not let students skip this step!

Exercises 2–7 (7 minutes)

Give students time to work on the problems individually. As students finish, have them work the problems on the board.

Answers are below.

Exercises 2–7

1. 3. 4.

1. 6. 7.

**Example 3 (3 minutes)**

Example 3

Consider the equation . Lulu chooses to multiply through by and gets the   
answer . But Poindexter points out that is also an answer, which Lulu missed.

* 1. What’s the problem with Lulu’s approach?

You cannot multiply by because could equal , which means that you would be dividing by .

* 1. Use factoring to solve the original equation for .

Work through the responses as a class.

* Emphasize the idea that multiplying by is a problem when equals .

Exercises 8–11 (10 minutes)

Give students time to work on Exercises 8–11 in pairs. Then, elicit student responses.

Remind students of the danger of multiplying both sides by a variable expression.

Exercises 8–11

1. Use factoring to solve the equation for : .

1. Solve each of the following for :

*Suggestion for Early Finishers:*

* The problems seen in Exercise 9 are often called the difference of two squares. Give early finishers this challenge:
  1. b.



* 1. Verify: . b. Verify: .

* 1. Verify: .

* 1. Solve for : . e. Solve for : .

1. A string inches long is to be laid out on a table-top to make a rectangle of perimeter inches. Write the width of the rectangle as inches. What is an expression for its length? What is an expression for its area? What value for gives an area of largest possible value? Describe the shape of the rectangle for this special value of .

Length: area:

The largest area is when . In this case, the rectangle is a square with length and width both equal to

Discuss the results of Exercise 10.

Work through Exercise 11 as a class, explaining why gives the largest area.

* Since as gets larger, gets smaller until at which point the area is zero. So the domain of for this problem is .
* How can we change the domain if we don’t want to allow zero area?
  + *You can leave the end of the interval open if you don’t want to allow zero area.*

Closing (3 minutes)

Elicit student responses. Students should make notes of responses in the Lesson Summary rectangle.

* If the product of numbers is zero, what do we know about the numbers? At least one of them must equal .
* What is the danger of dividing both sides of an equation by a variable factor? What should be done instead?

Lesson Summary

The zero-product property says that if , then either or or .

Exit Ticket (5 minutes)

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 17: Equations Involving Factored Expressions

Exit Ticket

1. Find the solution set to the equation .
2. Determine if each statement is true or false. If the statement is false, explain why or show work proving that it is false.
   1. If , then .
   2. If , then .

Exit Ticket Solutions

1. Find the solution set to the equation

solution set:

1. Determine if each statement is true or false. If the statement is false, explain why or show work proving that it is false.
   1. If , then .

True.

* 1. If , then .

False, could equal or could equal .

Problem Set Solutions

1. Find the solution set of each equation:

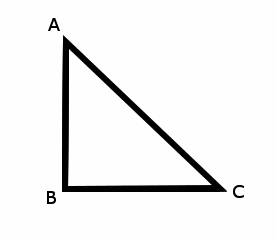
1. Solve , for .
2. Solve , for . What solution do you lose if you simply divide by to get ?

or . The lost solution is. We assumed was not zero when we divided by ; therefore, our solution was only complete for values not equal to .

1. The square of a number plus times the number is equal to . What is the number?

Solve , for to obtain or .

1. In the right triangle shown below, the length of side AB is , the length of side BC is , and the length of the hypotenuse AC is . Use this information to find the length of each side. (Use the Pythagorean Theorem to get an equation, and solve for .)



Use the Pythagorean Theorem to get the equation . This is equivalent to , and the solutions are and . Choose since represents a length, and the lengths are

AB:

BC: AC:

1. Using what you learned in this lesson, create an equation that has and as its only solutions.