Lesson 16: Solving and Graphing Inequalities Joined by "And" or "Or"

Student Outcomes

- Students solve two inequalities joined by "and" or "or" and then graph the solution set on the number line.

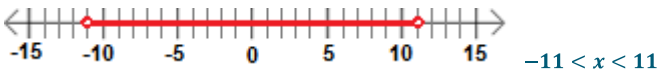## Classwork

Exercise 1 (5 minutes)

Do parts (a)–(c) and as much of (d)–(e) as time permits depending on the level of your students. You can also present the challenge problem given below if time allows.
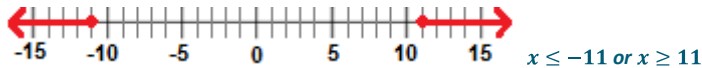
Exercise 1

a. Solve $w^2 = 121$, for w. Graph the solution on a number line.

b. Solve $w^2 < 121$, for w. Graph the solution on a number line and write the solution set as a compound inequality.

$-11 < x < 11$

c. Solve $w^2 \geq 121$ for w. Graph the solution on a number line and write the solution set as a compound inequality.

$x \leq -11 \text{ or } x \geq 11$

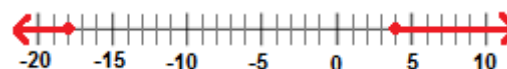d. Quickly solve $(x + 7)^2 = 121$, for x. Graph the solution on a number line.

e. Use your work from part (d) to quickly graph the solution on a number line to each inequality below.

   i. $(x + 7)^2 < 121$

   ii. $(x + 7)^2 \geq 121$

Extension

Use the following to challenge students who finish early.

a. Poindexter says that $a + b$ ^{2} equals $a^2 + 2ab + b^2$. Is he correct?

b. Solve $x^2 + 14x + 49 < 121$, for x. Present the solution graphically on a number line.

> *Scaffolding:*
> - Remind students of their experience from the previous lesson. For a statement separated by "and" to be true, BOTH statements must be true. If it is separated by "or," at least one statement must be true.

Exercises 2–3 (6 minutes)

Give students 4 minutes to work on Exercises 2 and 3. Then, discuss the results as a class. Students are applying their knowledge from the previous lesson to solve an unfamiliar type of problem.

Exercise 2

Consider the compound inequality $-5 < x < 4$
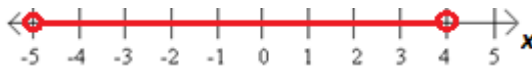
a. Rewrite the inequality as a compound statement of inequality.

$x > -5 \text{ and } x < 4$

b. Write a sentence describing the possible values of x.

x can be any number between -5 and 4.

c. Graph the solution set on the number line below.

Exercise 3

Consider the compound inequality $-5 < 2x + 1 < 4$.

a. Rewrite the inequality as a compound statement of inequality.

$2x + 1 > -5 \text{ and } 2x + 1 < 4$

b. Solve each inequality for x. Then, write the solution to the compound inequality.

$x > -3 \text{ and } x < \frac{3}{2} \text{ OR } -3 < x < \frac{3}{2}$

c. Write a sentence describing the possible values of x.

x can be any number between -3 and $\frac{3}{2}$.

d. Graph the solution set on the number line below.

Review Exercise 3 with students to demonstrate how to solve it without rewriting it.

- A friend of mine suggested I could solve the inequality as follows. Is she right?

$$-5 \; < \; 2x + 1 \; < \; 4$$
$$-5 - 1 \; < \; 2x + 1 - 1 \; < \; 4 - 1$$
$$-6 \; < \; 2x \; < \; 3$$
$$-3 \; < \; x \; < \; \frac{3}{2}$$

MP.3

Encourage students to articulate their thoughts and scrutinize each other's reasoning.

Point out to students that solving the two inequalities did not require any new skills. They are solved just as they learned in previous lessons.

Have students verify their solution by filling in a few test values.

Remind students that the solution can be written two ways: $x > -3$ and $x < \frac{3}{2}$

OR $-3 < x < \frac{3}{2}$.

> *Scaffolding:*
> - Remind students that when an inequality is multiplied or divided by a negative number, the direction of the inequality changes.

Exercises 4–5 (5 minutes)

Give students 4 minutes to work on Exercises 4 and 5. Then, review the results as a class. Again, point out to students that solving the two inequalities did not require any new skills. They are solved just as they learned in previous lessons. Have students verify their solutions by filling in a few test values.

Exercise 4

Given $x < -3$ or $x > -1$

    a.   **What must be true in order for the compound inequality to be a true statement?**

        One of the statements must be true, so either x has to be less than -3, or it has to be greater than -1. (In this case it is not possible that both are true.)

    b.   **Write a sentence describing the possible values of x.**

        x can be any number that is less than -3 or any number that is greater than -1.

    c.   **Graph the solution set on the number line below.**
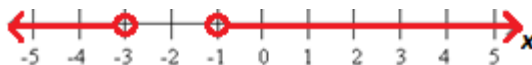
Exercise 5

Given $x + 4 < 6$ or $x - 1 > 3$
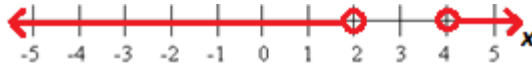
    a.   **Solve each inequality for x. Then, write the solution to the compound inequality.**

        $x < 2$ or $x > 4$

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b. Write a sentence describing the possible values of x.

x can be any number that is less than 2 or any number that is greater than 4.

c. Graph the solution set on the number line below.

Exercise 6 (14 minutes)

Have students work the exercises individually, with partners, or with small groups. Circulate around the room monitoring progress and offering guidance as needed. Make sure students are attending to the detail of correctly using open and closed endpoints.

Exercise 6

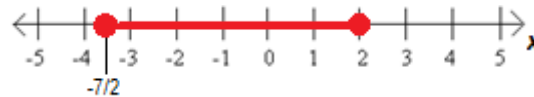Solve each compound inequality for x and graph the solution on a number line.

a. $x + 6 < 8$ and $x - 1 > -1$

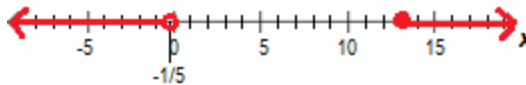$x < 2$ *and* $x > 0 \rightarrow 0 < x < 2$

b. $-1 \leq 3 - 2x \leq 10$

$x \geq -\dfrac{7}{2}$ *and* $x \leq 2 \rightarrow -\dfrac{7}{2} \leq x \leq 2$

c. $5x + 1 < 0$ or $8 \leq x - 5$

$x < -\dfrac{1}{5}$ *or* $x \geq 13$

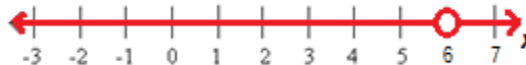d. $10 > 3x - 2$ or $x = 4$

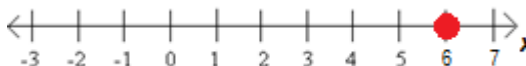$x < 4$ *or* $x = 4 \rightarrow x \leq 4$

e. $x - 2 < 4$ or $x - 2 > 4$

$x < 6$ *or* $x > 6 \rightarrow x \neq 6$

f. $x - 2 \leq 4$ and $x - 2 \geq 4$

$x = 6$

Debrief the exercise with the following questions:

▪ Look at the solution to part (f) closely. Remind students that both statements must be true. Therefore, the solution is only $x = 6$.

▪ How would the solution to part (f) change if the "and" was an "or"? Let this discussion lead in to Exercise 7.
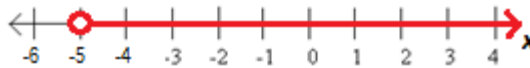
Exercise 7 (9 minutes)

Have students work in groups to answer the questions. Students are exploring variations of previously seen problems. After completing the exercises, ask students to articulate how the problems differed from most of the other examples seen thus far.

MP.1

> **Exercise 7**
>
> Solve each compound inequality for x and graph the solution on a number line. Pay careful attention to the inequality symbols and the "and" or "or" statements as you work.
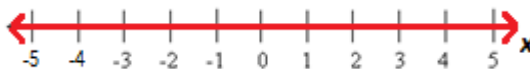>
> a. $1 + x > -4$ or $3x - 6 > -12$
>
> $x > -5$
>
> b. $1 + x > -4$ or $3x - 6 < -12$
>
>   *x can be any real number.*
>
> c. $1 + x > 4$ and $3x - 6 < -12$
>
>   *$x > 3$ and $x < -2$*
>
>   *No solution (empty set) since there are no numbers that satisfy both statements.*

Have early finishers explore further with the following:

- Is it possible to write a problem separated by "or" that has no solution?
- Is it possible to have a problem separated by "and" that has a solution set consisting of all real numbers?

Closing (2 minutes)

For the first problem, students may have written the solution as $x > -5$ or $x > -2$. Look at the graph as a class and remind them that the solution is the set of all of the numbers included in either of the two solution sets (or the union of the two sets). Lead them to the idea that the solution is $x > -5$.

For the second problem, the two graphs overlap and span the entire number line. Lead them to the idea that the solution is all real numbers. Have students fill in a few test values to verify that any number will work.

For the third problem, the two graphs do not overlap. Remind them that the solution set is only the values that are in both of the individual solution sets. There is no number that will make both statements true. Lead them to the idea that there is no solution.

Read the questions at the end of the exploration and give students a few minutes to summarize their thoughts on the work in Exercise 7 independently. Call for a few volunteers to read their solutions.

Exit Ticket (4 minutes)

Name _____   Date_____

Lesson 16: Solving and Graphing Inequalities Joined by "And" or "Or"

Exit Ticket

1. Solve each compound inequality for x and graph the solution on a number line.

   a. $9 + 2x < 17$ or $7 - 4x < -9$

   b. $6 \leq \frac{x}{2} \leq 11$

2.

   a.   Give an example of a compound inequality separated by "or" that has a solution of all real numbers.

   b.   Take the example from (a) and change the "or" to an "and." Explain why the solution set is no longer all real numbers. Use a graph on a number line as part of your explanation.

Exit Ticket Sample Solutions

1. Solve each compound inequality for x and graph the solution on a number line.

    a. $9 + 2x < 17$ or $7 - 4x < -9$

        $x < 4$ or $x > 4$ or $x \neq 4$

    b. $6 \leq \frac{x}{2} \leq 11$

        $12 \leq x \leq 22$

2.

    a. Give an example of a compound inequality separated by "or" that has a solution of all real numbers.

        Sample response: $x > 0$ or $x < 2$

    b. Take the example from (a) and change the "or" to an "and." Explain why the solution set is no longer all real numbers. Use a graph on a number line as part of your explanation.

        Sample response: $x > 0$ and $x < 2$

        In the first example, only one of the inequalities needs to be true to make the compound statement true. Any number selected is either greater than 0 or less than 2 or both. In the second example, both inequalities must be true to make the compound statement true. This restricts the solution set to only numbers between 0 and 2.

Problem Set Sample Solutions

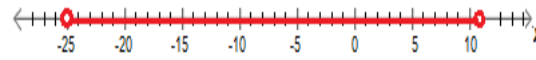Solve each inequality for x and graph the solution on a number line.
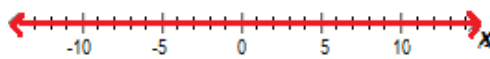
1. $x - 2 < 6$ or $\frac{x}{3} > 4$

    $x < 8$ or $x > 12$
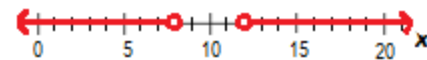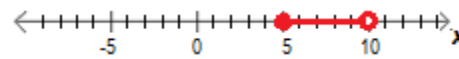
2. $-6 < \frac{x+1}{4} < 3$

    $-25 < x < 11$

3. $5x \leq 21 + 2x$ or $3x + 1 \geq 24$

    $x \leq 7$ or $x \geq 7$ → all real numbers

4. $5x + 2 \geq 27$ and $3x - 1 < 29$

    $x \geq 5$ and $x < 10$ → $5 \leq x < 10$
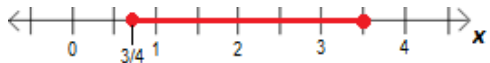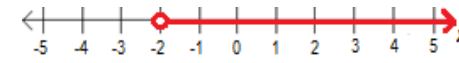
5. $0 \leq 4x - 3 \leq 11$

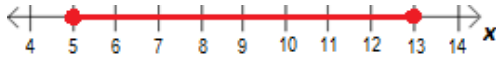   *$\frac{3}{4} \leq x \leq \frac{7}{2}$*

6. $2x > 8$ or $-2x < 4$

    $x > 4$ or $x > -2$ → $x > -2$
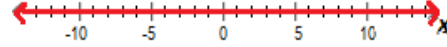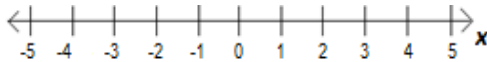
7. $8 \geq -2(x - 9) \geq -8$

   $5 \leq x \leq 13$

8. $4x + 8 > 2x - 10 \ or \ \frac{1}{3}x - 3 < 2$

   $x > -9 \ or \ x < 15 \ \rightarrow \ all \ real \ numbers$

9. $7 - 3x < 16$ and $x + 12 < -8$

   $x > -3 \ and \ x < -20 \ \rightarrow \ no \ solution$

10. If inequalities question 8 were joined by "and" instead of "or," what would the solution set become?

   $-9 < x < 15$

11. If the inequalities in question 9 were joined by "or" instead of "and," what would the solution set become?

   $x > -3 \ or \ x < -20$