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Lesson 15: Solution Sets of Two or More Equations (or  
Inequalities) Joined by “And” or “Or”

**Student Outcomes**

* Students describe the solution set of two equations (or inequalities) joined by either “and” or “or” and graph the solution set on the number line.

Classwork

Exercise 1 (6 minutes)

It may be helpful to some students to review some of the vocabulary used here, such as compound sentence (a sentence that contains at least two clauses) or declarative sentence (a sentence in the form of a statement).

Give students a few minutes to work on the exploration independently and then 1 minute to compare answers with a partner. Discuss results as a class, particularly the difference between separating the declarations by “and” and by “or.”

Exercise 1

Determine whether each claim given below is true or false.

1. Right now, I am in math class and English class. b. Right now, I am in math class or English class.

False True (assuming they are answering this in class)

1. and . d. and

True False

1. or . f. or

True False

These are all examples of declarative compound sentences.

1. When the two declarations in the sentences above were separated by “and,” what had to be true to make the statement true?

Both declarations had to be true.

1. When the two declarations in the sentences above were separated by “or,” what had to be true to make the statement true?

At least one declaration had to be true.

Discuss the following points with students:

* The word “and” means the same thing in a compound mathematical sentence as it does in an English sentence.
* If two clauses are separated by “and,” both clauses must be true for the entire compound statement to be deemed true.
* The word “or” also means a similar thing in a compound mathematical sentence as it does in an English sentence. However, there is an important distinction: In English the word “or” is commonly interpreted as the exclusive or, one condition or the other is true, but not both. In mathematics, either or both could be true.
* If two clauses are separated by “or,” one or both of the clauses must be true for the entire compound statement to be deemed true.

**Example 1 (5 minutes)**

Work through the four examples as a class.

**Example 1**

Solve each system of equations and inequalities.

|  |  |
| --- | --- |
| * 1. or   or | * 1. or |
| * 1. and   and | * 1. and .   The empty set. |

Exploratory Challenge/Exercise 2 (10 minutes)

Provide students with colored pencils and allow them a couple of minutes to complete (a) through (c). Then, stop and discuss the results.

Exercise 2

1. Using a colored pencil, graph the inequality on the number line below.
2. Using a different colored pencil, graph the inequality on the number line below.
3. Using a third colored pencil, darken the section of the number line where and .

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* In order for the compound sentence and to be true, what has to be true about ?
  + - *has to be both greater than and less than . (Students might also verbalize that it must be between and , not including the points and .)*
* On the graph, where do the solutions lie?
  + *Between and , not including the points and*

Have students list some of the solutions to the compound inequality. Make sure to include examples of integer and non-integer solutions.

* How many solutions are there to this compound inequality?
  + *An infinite number*

Introduce the abbreviated way of writing this sentence:

* Sometimes this is written as .

Use this notation to further illustrate the idea of representing all numbers ***strictly between*** and .

Allow students a couple of minutes to complete (d) *through* (f). Then, stop and discuss the results.

1. Using a colored pencil, graph the inequality on the number line below.
2. Using a different colored pencil, graph the inequality on the number line below.
3. Using a third colored pencil, darken the section of the number line where or.

****

* In order for the compound sentence or to be true, what has to be true about ?
  + *It could either be less than , or it could be greater than .*
* On the graph, where do the solutions lie?
  + *To the left of and to the right of*

Have students list solutions to the compound inequality. Make sure to include examples of integer and non-integer solutions.

* How many solutions are there to this compound inequality?
  + *Infinitely many*
* Would it be acceptable to abbreviate this compound sentence as follows: ?
  + *No.*
* Explain why not.
* *Those symbols suggest that must be greater than zero and less than at the same time, but the solution is calling for to be either less than or greater than zero.*

Allow students a couple of minutes to complete (g) *through* (i) and discuss answers.

1. Graph the compound sentence or on the number line below.



1. How could we abbreviate the sentence or ?
2. Rewrite as a compound sentence and graph the solutions to the sentence on the number line below.

or

**Example 2 (3 minutes)**

Work through Example 2 as a class.

**Example 2**

Graph each compound sentence on a number line.

* 1.  or
  2. or

****

**Rewrite as a compound sentence and graph the sentence on a number line.

and

Exercise 3 (5 minutes)

Give students a couple of minutes to read through Exercise 3 and try it independently before comparing answers with a neighbor or discussing as a class.

Exercise 3

Consider the following two scenarios. For each, specify the variable and say, “ is the width of the rectangle,” for example, and write a compound inequality that represents the scenario given. Draw its solution set on a number line.

|  |  |  |  |
| --- | --- | --- | --- |
| Scenario | Variable | Inequality | Graph |
| **a. Students are to present a persuasive speech in English class. The guidelines state that the speech must be at least minutes but not exceed minutes.** | Let length of time of the speech. | and |  |

|  |  |  |  |
| --- | --- | --- | --- |
| **b. Children and senior citizens receive a discount on tickets at the movie theater. To receive a discount, a person must be between the ages of and , including and , or years of age or older.** | Let age of moviegoer who receives a discount. | or | *Scaffolding:*   * Ask students to think of other scenarios that could be modeled using a compound inequality*.* |

Exercise 4 (10 minutes)

Give students time to work on the problems, and then allow for sharing of answers,   
possibly with a neighbor or with the class.

Exercise 4

Determine if each sentence is true or false. Explain your reasoning.

1. and . b. or

True True

Solve each system and graph the solution on a number line.

1. or d. or

Graph the solution set to each compound inequality on a number line.

1. or f.





Write a compound inequality for each graph.

1. h.







or

1. A poll shows that a candidate is projected to receive of the votes. If the margin for error is plus or minus , write a compound inequality for the percentage of votes the candidate can expect to get.

let percentage of votes

1. Mercury is one of only two elements that is liquid at room temperature. Mercury is non-liquid for temperatures less than °F or greater than °F. Write a compound inequality for the temperatures at which mercury is non-liquid.

Let temperatures (in degrees F) for which mercury is non-liquid or

As an extension, students can come up with ways to alter parts (a) and (b) to make them false compound statements. Share several responses.

Ask the following:

* What would be a more concise way of writing the sentence for part (e)?

*Scaffolding:*

* The other element that is liquid at room temperature is bromine. Students could be asked to look up the temperatures at which bromine is non-liquid and write a similar compound inequality.
* For part (f), list some numbers that are solutions to the inequality.
* What is the largest possible value of ?

**MP.2**

* What is the smallest possible value of ?
  + This is tougher to answer. can be infinitely close to but cannot equal zero. Therefore, there is no absolute smallest value for in this case.

For parts (i) and (j), make sure students specify what the variable they choose represents.

Closing (2 minutes)

Lead a conversation on the idea that in math, as in English, it is important that we are precise in our use of language and that we are able to read (and comprehend) and write mathematical sentences. Ask students to give examples to justify why the precision is important in math, and why it is important in English.

Reinforce that, in mathematical sentences, like in English sentences, a compound sentence separated by

**MP.6**

AND is true if both clauses are true.

OR is true if at least one of the clauses is true.

Lesson Summary

In mathematical sentences, like in English sentences, a compound sentence separated by

AND is true if \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

OR is true if \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Exit Ticket (5 minutes)

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 15: Solution Sets of Two or More Equations (or Inequalities) Joined by “And” or “Or”

Exit Ticket

* 1. Solve the system and graph the solution set on a number line.

or

* 1. Write a different system of equations that would have the same solution set.

1. Swimming pools must have a certain amount of chlorine content. The United States standard for safe levels of chlorine in swimming pools is at least part per million and no greater than parts per million. Write a compound inequality for the acceptable range of chlorine levels.
2. Consider each of the following compound sentences:

and or

Does the change of word from “and” to “or” change the solution set?

Use number-line graphs to support your answer.

Exit Ticket Sample Solutions

* 1. Solve the system and graph the solution set on a number line.

or

or



* 1. Write a different system of equations that would have the same solution set.

sample answer: or

1. Swimming pools must have a certain amount of chlorine content. The United States standard for safe levels of chlorine in swimming pools is at least part per million and no greater than parts per million. Write a compound inequality for the acceptable range of chlorine levels.

Let chlorine level in a swimming pool (in parts per million)

1. Consider each of the following compound sentences:

and or

Does the change of word from “and” to “or” change the solution set?

Use number-line graphs to support your answer.

For the first sentence, both statements must be true, so x can only equal values that are both greater than and less than . For the second sentence, only one statement must be true, so must be greater than or less than . This means can equal any number on the number line.



Problem Set Sample Solutions

1. Consider the inequality .
   1. Rewrite the inequality as a compound sentence.

and

* 1. Graph the inequality on a number line.



* 1. How many solutions are there to the inequality? Explain.

There are an infinite number of solutions. can be any value between and , which includes the integer values of and as well as non-integer values. The set of numbers between and is infinite.

* 1. What are the largest and smallest possible values for ? Explain.

There is no absolute largest or absolute smallest value for . can be infinitely close to or to but cannot equal either value.

* 1. If the inequality is changed to , then what are the largest and smallest possible values for ?

In this case, we can define the absolute maximum value to be and the absolute minimum value to be .

Write a compound inequality for each graph.

1.  *or*  3.  *or* , which can be written as

Write a single or compound inequality for each scenario.

1. The scores on the last test ranged from to .

scores on last test

1. To ride the roller coaster, one must be at least feet tall.

height (in feet) to ride the roller coaster

1. Unsafe body temperatures are those lower than 96°F or above 104°F.

body temperature (in degrees F) that are unsafe or

Graph the solution(s) to each of the following on a number line.

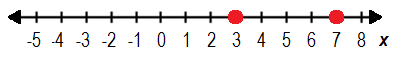
1. and



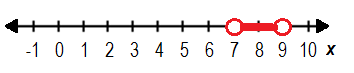
1. and



1. or



1. or



1. and



1. or