Lesson 11: Solution Sets For Equations and Inequalities

Classwork

Example 1

Consider the equation, $x^2 = 3x + 4$, where x represents a real number.

a. Are the expressions x^2 and $3x + 4$ algebraically equivalent?

b. The following table shows how we might “sift” through various values to assign to the variable symbol x in the hunt for values that would make the equation true.

x-VALUE	THE EQUATION	TRUTH VALUE
Let $x = 0$	$0^2 = 3(0) + 4$	FALSE
Let $x = 5$	$5^2 = 3(5) + 4$	FALSE
Let $x = 6$	$6^2 = 3\ \ 6\ \ + 4$	FALSE
Let $x = -7$	$(-7)^2 = 3(-7) + 4$	FALSE
Let $x = 4$	$4^2 = 3(4) + 4$	TRUE
Let $x = 9$	$9^2 = 3(9) + 4$	FALSE
Let $x = 10$	$10^2 = 3(10) + 4$	FALSE
Let $x = -8$	$(-8)^2 = 3(-8) + 4$	FALSE

Example 2

Consider the equation $7 + p = 12$.

	THE NUMBER SENTENCE	TRUTH VALUE
Let $p = 0$	$7 + 0 = 12$	FALSE
Let $p = 4$		
Let $p = 1 +\ \ \overline{2}$		
Let $p = \frac{1}{\pi}$		
Let $p = \ \ 5$		

The **solution set** of an equation written with only one variable is the set of all values one can assign to that variable to make the equation a true statement. Any one of those values is said to be a *solution to the equation.*

To *solve an equation* means to *find the solution set* for that equation.

Example 3

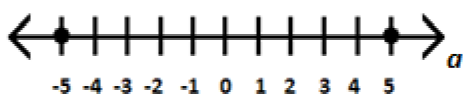
Solve for a: $a^2 = 25$.

One can describe a solution set in any of the following ways:

IN WORDS: $a^2 = 25$ has solutions 5 and -5. (That is, $a^2 = 25$ is true when $a = 5$ or $a = -5$.)

IN SET NOTATION: The solution set of $a^2 = 25$ is $-5, 5$.

IN A GRAPHICAL REPRESENTATION ON A NUMBER LINE: The solution set of $a^2 = 25$ is



In this graphical representation, a solid dot is used to indicate a point on the number line that is to be included in the solution set. (WARNING: The dot one physically draws is larger than the point it represents. One hopes that it is clear from the context of the diagram which point each dot refers to.)

How set notation works.

- The curly brackets { } indicate we are denoting a set. A set is essentially a collection of things, e.g., letters, numbers, cars, people. In this case, the things are numbers.
- From this example, the numbers -5 and 5 are called elements of the set. No other elements belong in this particular set because no other numbers make the equation $a^2 = 25$ true.
- When elements are listed, they are listed in increasing order.
- Sometimes, a set is empty; it has no elements. In which case, the set looks like { }. We often denote this with the symbol, ∅. We refer to this *as the empty set* or *the null set*.

Exercise 1

Solve for a: $a^2 = -25$. Present the solution set in words, in set notation, and graphically.

Exercise 2

Depict the solution set of $7 + p = 12$ in words, in set notation, and graphically.

Example 4

Solve $\frac{x}{x} = 1$ for x, over the set of positive real numbers. Depict the solution set in words, in set notation, and graphically.

x-VALUE	THE EQUATION	TRUTH VALUE
Let $x = 2$	$\frac{2}{2} = 1$	TRUE
Let $x = 7$	$\frac{7}{7} = 1$	TRUE
Let $x = 0.01$	$\frac{0.01}{0.01} = 1$	TRUE
Let $x = 562\frac{2}{3}$	$\frac{562\frac{2}{3}}{562\frac{2}{3}} = 1$	TRUE
Let $x = 10^{100}$	$\frac{10^{100}}{10^{100}}$	TRUE
Let $x = \pi$	$\frac{\pi}{\pi} = 1$	TRUE

Exercise 3

Solve $\frac{x}{x} = 1$ for x, over the set of all non-zero real numbers. Describe the solution set in words, in set notation, and graphically.

Example 5

Solve for x: $x(3 + x) = 3x + x^2$.

Exercise 4

Solve for α: $\alpha + \alpha^2 = \alpha(\alpha + 1)$. Describe carefully the reasoning that justifies your solution. Describe the solution set in words, in set notation, and graphically.

An **identity** is an equation that is always true.

Exercise 5

Identify the properties of arithmetic that justify why each of the following equations has a solution set of all real numbers:

a. $2x^2 + 4x = 2\ x^2 + 2x$

b. $2x^2 + 4x = 4x + 2x^2$

c. $2x^2 + 4x = 2x(2 + x)$

Exercise 6

Create an expression for the right side of each equation such that the solution set for the equation will be all real numbers. (There is more than one possibility for each expression. Feel free to write several answers for each one.)

a. $2x - 5$ = ______________

b. $x^2 + x$ = ______________

c. $4 \cdot x \cdot y \cdot z\ =$ ______________

d. $x + 2\ ^2$ = ______________

Example 6

Solve for w, $w + 2 > 4$.

Exercise 7

a. Solve for B: $B^2 \geq 9$. Describe the solution set using a number line.

b. What is the solution set to the statement: "Sticks of lengths 2 yards, 2 yards, and L yards make an isosceles triangle"? Describe the solution set in words and on a number line.

Lesson Summary

The **solution set** of an equation written with only one variable symbol is the set of all values one can assign to that variable to make the equation a true number sentence. Any one of those values is said to be a *solution to the equation.*

To *solve an equation* means to *find the solution set* for that equation.

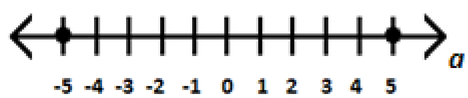
One can describe a solution set in any of the following ways:

IN WORDS: $a^2 = 25$ has solutions 5 and -5. (That is, $a^2 = 25$ is true when $a = 5$ or $a = -5$.)

IN SET NOTATION: The solution set of $a^2 = 25$ is $-5, 5$.

It is awkward to express the set of infinitely many numbers in set notation. In these cases we can use the notation: variable symbol number type a description . For example x real $x > 0$ reads, "x is a real number where x is greater than zero." The symbol $\mathbb{R}$ can be used to indicate all real numbers.

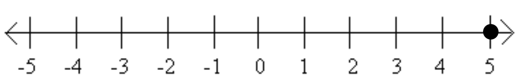
IN A GRAPHICAL REPRESENTAION ON A NUMBER LINE: The solution set of $a^2 = 25$ is as follows:

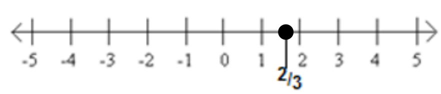


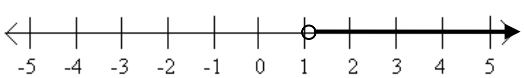
In this graphical representation, a solid dot is used to indicate a point on the number line that is to be included in the solution set. (WARNING: The dot one physically draws is larger than the point it represents! One hopes that it is clear from the context of the diagram which point each dot refers to.)

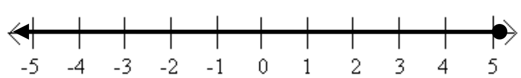
Problem Set

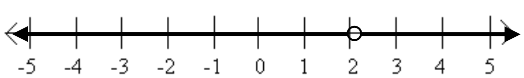
For each solution set graphed below, (a) describe the solution set in words, (b) describe the solution set in set notation, and (c) write an equation or an inequality that has the given solution set.

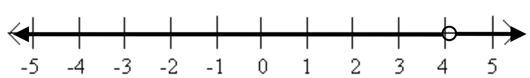
1. 

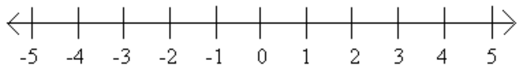
2. 

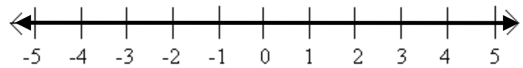
3. 

4. 

5. 

6. 

7. 

8. 

Fill in the chart below.

	SOLUTION SET IN WORDS	SOLUTION SET IN SET NOTATION	GRAPH
9. $z = 2$			
10. $z^2 = 4$			
11. $4z \neq 2$			
12. $z - 3 = 2$			
13. $z^2 + 1 = 2$			
14. $z = 2z$			
15. $z > 2$			
16. $z - 6 = z - 2$			
17. $z - 6 < -2$			
18. $4\ z - 1\ \ > 4z - 4$			

For Problems 19–24, answer the following: Are the two expressions algebraically equivalent? If so, state the property (or properties) displayed. If not, state why (the solution set may suffice as a reason) and change the equation, ever so slightly, e.g., touch it up, to create an equation whose solution set is all real numbers.

19. $x(4 - x^2) = (-x^2 + 4)x$

20. $\frac{2x}{2x} = 1$

21. $(x - 1)(x + 2) + (x - 1)(x - 5) = (x - 1)(2x - 3)$

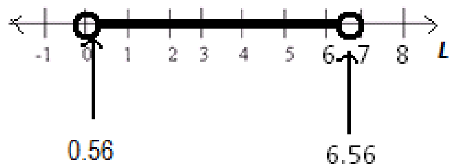
22. $\frac{x}{5} + \frac{x}{3} = \frac{2x}{8}$

23. $x^2 + 2x^3 + 3x^4 = 6x^9$

24. $x^3 + 4x^2 + 4x = x\ \ x + 2\ \ ^2$

25. Solve for w: $\frac{6w+1}{5} \neq 2$. Describe the solution set in set notation.

26. Edwina has two sticks, one 2 yards long and the other 2 meters long. She is going to use them, with a third stick of some positive length, to make a triangle. She has decided to measure the length of the third stick in units of feet.
   a. What is the solution set to the statement: "Sticks of lengths 2 yards, 2 meters, and L feet make a triangle"? Describe the solution set in words and through a graphical representation.



   b. What is the solution set to the statement: "Sticks of lengths 2 yards, 2 meters, and L feet make an isosceles triangle"? Describe the solution set in words and through a graphical representation.
   c. What is the solution set to the statement: "Sticks of lengths 2 yards, 2 meters, and L feet make an equilateral triangle"? Describe the solution set in words and through a graphical representation.