Lesson 10: Converting Repeating Decimals to Fractions

Classwork

Example 1

Find the fraction that is equal to the infinite decimal $0.\overline{81}$.

Exercises 1–2

1. a. Let $x = 0.\overline{123}$. Explain why multiplying both sides of this equation by 10^3 will help us determine the fractional representation of x.

b. After multiplying both sides of the equation by 10^3, rewrite the resulting equation by making a substitution that will help determine the fractional value of x. Explain how you were able to make the substitution.

c. Solve the equation to determine the value of x.

d. Is your answer reasonable? Check your answer using a calculator.

2. Find the fraction equal to $0.\overline{4}$. Check that you are correct using a calculator.

Example 2

Find the fraction that is equal to the infinite decimal $2.13\overline{8}$.

Exercises 3–4

3. Find the fraction equal to $1.6\overline{23}$. Check that you are correct using a calculator.

4. Find the fraction equal to $2.9\overline{60}$. Check that you are correct using a calculator.

Lesson Summary

Numbers with decimal expansions that repeat are rational numbers and can be converted to fractions using a linear equation.

Example: Find the fraction that is equal to the number $0.\overline{567}$.

Let x represent the infinite decimal $0.\overline{567}$.

$\mathbf{x = 0.\overline{567}}$	
$\mathbf{10^3x = 10^3(0.\overline{567})}$	Multiply by $\mathbf{10^3}$ because there are $\mathbf{3}$ digits that repeat
$\mathbf{1000x = 567.\overline{567}}$	Simplify
$\mathbf{1000x = 567 + 0.\overline{567}}$	By addition
$\mathbf{1000x = 567 + x}$	By substitution; $\mathbf{x = 0.\overline{567}}$
$\mathbf{1000x - x = 567 + x - x}$	Subtraction Property of Equality
$\mathbf{999x = 567}$	Simplify
$\mathbf{\frac{999}{999}x = \frac{567}{999}}$	Division Property of Equality
$\mathbf{x = \frac{567}{999} = \frac{63}{111}}$	Simplify

This process may need to be used more than once when the repeating digits do not begin immediately after the decimal. For numbers such as $1.2\overline{6}$, for example.

Irrational numbers are numbers that are not rational. They have infinite decimals that do not repeat and cannot be represented as a fraction.

Problem Set

1. a. Let $x = 0.\overline{631}$. Explain why multiplying both sides of this equation by 10^3 will help us determine the fractional representation of x.
   b. After multiplying both sides of the equation by 10^3, rewrite the resulting equation by making a substitution that will help determine the fractional value of x. Explain how you were able to make the substitution.
   c. Solve the equation to determine the value of x.
   d. Is your answer reasonable? Check your answer using a calculator.

2. Find the fraction equal to $3.40\overline{8}$. Check that you are correct using a calculator.

3. Find the fraction equal to $0.\overline{5923}$. Check that you are correct using a calculator.

4. Find the fraction equal to $2.3\overline{82}$. Check that you are correct using a calculator.

5. Find the fraction equal to $0.\overline{714285}$. Check that you are correct using a calculator.

6. Explain why an infinite decimal that is not a repeating decimal cannot be rational.

7. In a previous lesson we were convinced that it is acceptable to write $0.\overline{9} = 1$. Use what you learned today to show that it is true.

8. Examine the following repeating infinite decimals and their fraction equivalents. What do you notice? Why do you think what you observed is true?

$0.\overline{81} = \frac{81}{99}$ $\quad$ $0.\overline{4} = \frac{4}{9}$ $\quad$ $0.\overline{123} = \frac{123}{999}$ $\quad$ $0.\overline{60} = \frac{60}{99}$ $\quad$ $0.\overline{9} = 1.0$