|  |
| --- |
|  |

Lesson 10: Converting Repeating Decimals to Fractions

Student Outcomes

* Students know the intuitive reason why every repeating decimal is equal to a fraction. Students convert a decimal expansion that eventually repeats into a fraction.
* Students know that the decimal expansions of rational numbers repeat eventually.
* Students understand that irrational numbers are numbers that are not rational. Irrational numbers cannot be represented as a fraction and have infinite decimals that never repeat.

Classwork

**Discussion (4 minutes)**

* We have just seen that every fraction (therefore every rational number) is equal to a repeating decimal, and we have learned strategies for determining the decimal expansion of fractions. Now we must learn how to write a repeating decimal as a fraction.
* We begin by noting a simple fact about finite decimals: Given a finite decimal, such as , if we multiply the decimal by we get . That is, when we multiply by a power of , in this case , the decimal point is moved places to the right, i.e.,

This is true because of what we know about the Laws of Exponents:

* We have discussed in previous lessons that we treat infinite decimals as finite decimals in order to compute with them. For that reason, we will now apply the same basic fact we observed about finite decimals to infinite decimals. That is,

We will use this fact to help us write infinite decimals as fractions.

Example 1 (10 minutes)

**Example 1**

Find the fraction that is equal to the infinite decimal

* We want to find the fraction that is equal to the infinite decimal .
* We let .

Allow students time to work in pairs or small groups to write the fraction equal to Students should recognize that the preceding discussion has something to do with this process and should be an entry point for finding the solution. They should also recognize that since we let , an equation of some form will lead them to the fraction. Give them time to make sense of the problem. Make a plan for finding the fraction, and then attempt to figure it out.

MP.1

* Since , we will multiply both sides of the equation by and then solve for . We will multiply by because there are two decimal digits that repeat immediately following the decimal point.

Ordinarily we would finish solving for by dividing both sides of the equation by . Do you see why that is not a good plan for this problem?

* *If we divide both sides by , we would get, which does not really show us that the repeating decimal is equal to a fraction (rational number) because the repeating decimal is still in the numerator.*
* We know that is the same as . Then by substitution, we have

**MP.7**

How can we rewrite in a useful way using the fact that ?

* *We can rewrite as because represents the repeating decimal block*

Now we can solve for to find the fraction that represents the repeating decimal :

* Therefore, the repeating decimal .

Have students verify that we are correct using a calculator.

Exercises 1–2 (5 minutes)

Students complete Exercises 1–2 in pairs. Allow students to use a calculator to check their work.

Exercises 1–2

1. a. Let Explain why multiplying both sides of this equation by will help us determine the

fractional representation of

When we multiply both sides of the equation by , on the right side we will have This is helpful because we will be able to subtract the repeating decimal from both sides by subtracting .

* 1. After multiplying both sides of the equation by , rewrite the resulting equation by making a substitution that will help determine the fractional value of . Explain how you were able to make the substitution.

Since we let , we can substitute the repeating decimal with .

* 1. Solve the equation to determine the value of
  2. Is your answer reasonable? Check your answer using a calculator.

Yes, my answer is reasonable and correct. It is reasonable because the denominator cannot be expressed as a product of ’s and ’s; therefore, I know that the fraction must represent an infinite decimal. It is also reasonable because the decimal value is closer to than to , and the fraction is also closer to than to It is correct because the division of using a calculator is .

1. Find the fraction equal to Check that you are correct using a calculator.

***Let***

Example 2 (6 minutes)

**Example 2**

**Find the fraction that is equal to the infinite decimal.**

* We want to find the fraction that is equal to the infinite decimal Notice that this time there is just one digit that repeats, but it is three places to the right of the decimal point. If we let by what power of should we multiply? Explain.
  + *The goal is to multiply by a power of so that the only remaining decimal digits are those that repeat. For that reason, we should multiply by .*
* We let , and multiply both sides of the equation by

This time, we cannot simply subtract from each side. Explain why.

* + *Subtracting in previous problems allowed us to completely remove the repeating decimal. This time, , not just*
* What we will do now is treat as a separate, mini-problem. Determine the fraction that is equal to
  + *Let*
* Now that we know that , we will go back to our original problem:

Exercises 3–4 (6 minutes)

Students complete Exercises 3–4 independently or in pairs. Allow students to use a calculator to check their work.

Exercises 3–4

1. Find the fraction equal to . Check that you are correct using a calculator.

|  |  |  |
| --- | --- | --- |
| Let | Let |  |

1. Find the fraction equal to . Check that you are correct using a calculator.

|  |  |  |
| --- | --- | --- |
| ***Let*** | Let |  |

**Discussion (4 minutes)**

* What we have observed so far is that when an infinite decimal repeats, it can be written as a fraction, which means that it is a rational number. Do you think infinite decimals that do not repeat are rational as well? Explain.

Provide students time to discuss with a partner before sharing their thoughts with the class.

* *Considering the work from this lesson, it does not seem reasonable that an infinite decimal that does not repeat can be expressed as a fraction. We would not have a value that we could set for and use to compute in order to find the fraction. For those reasons, we do not believe that an infinite decimal that does not repeat is a rational number.*
* Infinite decimals that do not repeat are *irrational numbers*, that is, when a number is not equal to a rational number, it is irrational. What we will learn next is how to use rational approximation to determine the approximate decimal expansion of an irrational number.

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

* We know that to work with infinite decimals we must treat them as finite decimals.
* We know how to use our knowledge of powers of and linear equations to write an infinite decimal that repeats as a fraction.
* We know that every decimal that eventually repeats is a rational number.

Lesson Summary

Numbers with decimal expansions that repeat are rational numbers and can be converted to fractions using a linear equation.

Example: Find the fraction that is equal to the number .

Let represent the infinite decimal .

|  |  |
| --- | --- |
|  | Multiply by because there are digits that repeat  Simplify  By addition  By substitution;  Subtraction Property of Equality  Simplify  Division Property of Equality  Simplify |

This process may need to be used more than once when the repeating digits do not begin immediately after the decimal. For numbers such as , for example.

Irrational numbers are numbers that are not rational. They have infinite decimals that do not repeat and cannot be represented as a fraction.

Exit Ticket (5 minutes)

Name Date

Lesson 10: Converting Repeating Decimals to Fractions

Exit Ticket

1. Find the fraction equal to .
2. Find the fraction equal to .

Exit Ticket Sample Solutions

1. Find the fraction equal to .

Let .

1. Find the fraction equal to .

|  |  |  |
| --- | --- | --- |
| Let | Let |  |

Problem Set Sample Solutions

1. a. Let . Explain why multiplying both sides of this equation by will help us determine the

fractional representation of .

When we multiply both sides of the equation by , on the right side we will have . This is helpful because we will be able to subtract the repeating decimal from both sides by subtracting .

* 1. After multiplying both sides of the equation by ,rewrite the resulting equation by making a substitution that will help determine the fractional value of . Explain how you were able to make the substitution.

Since we let , we can substitute the repeating decimal with .

* 1. Solve the equation to determine the value of .
  2. Is your answer reasonable? Check your answer using a calculator.

Yes, my answer is reasonable and correct. It is reasonable because the denominator cannot be expressed as a product of ’s and ’s; therefore, I know that the fraction must represent an infinite decimal. Also the number is closer to than , and the fraction is also closer to than . It is correct because the division using the calculator is

1. Find the fraction equal to Check that you are correct using a calculator.

|  |  |  |
| --- | --- | --- |
| Let | Let |  |

1. Find the fraction equal to . Check that you are correct using a calculator.

Let

1. Find the fraction equal to . Check that you are correct using a calculator.

|  |  |  |
| --- | --- | --- |
| Let | Let |  |

1. Find the fraction equal to . Check that you are correct using a calculator.

Let

1. Explain why an infinite decimal that is not a repeating decimal cannot be rational.

Infinite decimals that do repeat can be expressed as a fraction and are therefore rational. The method we learned today to write a repeating decimal as a rational number cannot be applied to infinite decimals that do not repeat. The method requires that we let represent the repeating part of the decimal. If the number has a decimal expansion that does not repeat, we cannot express the number as a fraction, i.e., a rational number.

1. In a previous lesson we were convinced that it is acceptable to write . Use what you learned today to show that it is true.

Let

1. Examine the following repeating infinite decimals and their fraction equivalents. What do you notice? Why do you think what you observed is true?

In each case, the fraction that represents the infinite decimal has a numerator that is exactly the repeating part of the decimal and a denominator comprised of ’s. Specifically, the denominator has the same number of digits of ’s as the number of digits that repeat. For example, has two repeating decimal digits, so the denominator has two ’s. Since we know that , we can make the assumption that repeating ’s, like could be expressed as , meaning that the fraction is almost , which would then be expressed as