Lesson 9: Decimal Expansions of Fractions, Part 1

Student Outcomes

- Students apply knowledge of equivalent fractions, long division, and the distributive property to write the decimal expansion of fractions.

Classwork

Opening Exercises 1–2 (5 minutes)

Opening Exercises 1–2

1. a. **We know that the fraction $\frac{5}{8}$ can be written as a finite decimal because its denominator is a product of 2's. Which power of 10 will allow us to easily write the fraction as a decimal? Explain.**

   Since $8 = 2^3$ we will multiply the numerator and denominator by 5^3, which means that $2^3 \times 5^3 = 10^3$ will be the power of 10 that allows us to easily write the fraction as a decimal.

   b. **Write the equivalent fraction using the power of 10.**

$$\frac{5}{8} = \frac{5 \times 5^3}{2^3 \times 5^3} = \frac{625}{1000}$$

2. a. **We know that the fraction $\frac{17}{125}$ can be written as a finite decimal because its denominator is a product of 5's. Which power of 10 will allow us to easily write the fraction as a decimal? Explain.**

   Since $125 = 5^3$ we will multiply the numerator and denominator by 2^3, which means that $5^3 \times 2^3 = 10^3$ will be the power of 10 that allows us to easily write the fraction as a decimal.

   b. **Write the equivalent fraction using the power of 10.**

$$\frac{17}{125} = \frac{17 \times 2^3}{5^3 \times 2^3} = \frac{136}{1000}$$

Example 1 (5 minutes)

Example 1

Write the decimal expansion of the fraction $\frac{5}{8}$.

- Based on our previous work with finite decimals, we already know how to convert $\frac{5}{8}$ to a decimal. We will use this example to learn a strategy using equivalent fractions that can be applied to converting any fraction to a decimal.

- What is true about these fractions and why?

$$\frac{5}{8}, \frac{10}{16}, \frac{50}{80}$$

  - *The fractions are equivalent. In all cases, when the numerator and denominator of* $\frac{5}{8}$ *are multiplied by the same factor it produces one of the other fractions. For example,* $\frac{5\times2}{8\times2}=\frac{10}{16}$ *and* $\frac{5\times10}{8\times10}=\frac{50}{80}$.

- What would happen if we chose 10^3 as this factor? We will still produce an equivalent fraction, but note how we use the factor of 10^3 in writing the decimal expansion of the fraction.

$$\begin{aligned}\frac{5}{8} &= \frac{5\times10^3}{8}\times\frac{1}{10^3}\\ &= \frac{5000}{8}\times\frac{1}{10^3}\end{aligned}$$

- Now we use what we know about division with remainders for $\frac{5000}{8}$:

$$\begin{aligned}&= \frac{625\times8+0}{8}\times\frac{1}{10^3}\\ &= \left(625+\frac{0}{8}\right)\times\frac{1}{10^3}\\ &= 625\times\frac{1}{10^3}\\ &= \frac{625}{10^3}\\ &= 0.625\end{aligned}$$

- Because of our work with Opening Exercise 1, we knew ahead of time that using 10^3 will help us achieve our goal. However, any power of 10 would achieve the same result. Assume we used 10^5 instead. Do you think our answer would be the same?
  - *Yes, it should be the same, but I would have to do the work to check it.*

- Let's verify that our result would be the same if we used 10^5.

$$\begin{aligned}\frac{5}{8} &= \frac{5\times10^5}{8}\times\frac{1}{10^5}\\ &= \frac{500000}{8}\times\frac{1}{10^5}\\ &= \frac{62500\times8+0}{8}\times\frac{1}{10^5}\\ &= \left(62500+\frac{0}{8}\right)\times\frac{1}{10^5}\\ &= 62500\times\frac{1}{10^5}\\ &= \frac{62500}{10^5}\\ &= 0.62500\\ &= 0.625\end{aligned}$$

Using 10^5 resulted in the same answer. Now we know that we can use any power of 10 with the method of converting a fraction to a decimal.

- This process of selecting a power of 10 to use is similar to putting zeroes after the decimal point when we do the long division. You do not quite know how many zeroes you will need, and if you put extra that's ok! Using lower powers of 10 can make things more complicated. It is similar to not including enough zeroes when doing the long division. For that reason, it is better to use a higher power of 10 because we know the extra zeroes will not change the value of the fraction nor its decimal expansion.

Example 2 (5 minutes)

> **Example 2**
>
> **Write the decimal expansion of the fraction $\frac{17}{125}$.**

- We go through the same process to convert $\frac{17}{125}$ to a finite decimal. We know from Opening Exercise 2 that we need to use 10^3 to write $\frac{17}{125}$ as a finite decimal, but from the last example we know that any power of 10 will work:

$$\frac{17}{125} = \frac{17 \times 10^3}{125} \times \frac{1}{10^3}$$

- What do we do next?
  - *Since* $17 \times 10^3 = 17{,}000$, *we need to do division with remainder for* $\frac{17000}{125}$.
- Do the division and write the next step.
  - $\frac{17000}{125} = 136$, *then* $\frac{17}{125} = \frac{136 \times 125 + 0}{125} \times \frac{1}{10^3}$

Check to make sure all students have the equation above; then instruct them to finish the work and write $\frac{17}{125}$ as a finite decimal.

$$\begin{aligned} &= 136 \times \frac{1}{10^3} \\ &= \frac{136}{10^3} \\ &= 0.136 \end{aligned}$$

Verify that students have the correct decimal; then work on Example 3.

Example 3 (7 minutes)

> **Example 3**
>
> **Write the decimal expansion of the fraction $\frac{35}{11}$.**

- Now we apply this strategy to a fraction, $\frac{35}{11}$, that is not a finite decimal. How do you know it's not a finite decimal?
  - *We know that the fraction will not be a finite decimal because the denominator is not a product of 2's and/or 5's.*
- What do you think the difference will be in our work?
  - *When we do the division with remainder, we will likely get a remainder, where the first two examples had a remainder of 0.*
- Let's use 10^6 to make sure we get enough decimal digits in order to get a good idea of what the infinite decimal is:

$$\frac{35}{11} = \frac{35 \times 10^6}{11} \times \frac{1}{10^6}$$

What do we do next?
  - *Since $35 \times 10^6 = 35{,}000{,}000$, we need to do division with remainder for $\frac{35{,}000{,}000}{11}$.*
- We need to determine what numbers make the following statement true:
$35{,}000{,}000 = \underline{\hspace{3cm}} \times 11 + \underline{\hspace{1cm}}$.
  - *$3{,}181{,}818$ and 2 would give us $35{,}000{,}000 = \underline{3{,}181{,}818} \times 11 + \underline{2}$.*
- With this information, we can continue the process:

$$\begin{aligned}
\frac{35}{11} &= \frac{3181818 \times 11 + 2}{11} \times \frac{1}{10^6} \\
&= \left(\frac{3181818 \times 11}{11} + \frac{2}{11}\right) \times \frac{1}{10^6} \\
&= \left(3181818 + \frac{2}{11}\right) \times \frac{1}{10^6} \\
&= 3181818 \times \frac{1}{10^6} + \frac{2}{11} \times \frac{1}{10^6} \\
&= \frac{3181818}{10^6} + \left(\frac{2}{11} \times \frac{1}{10^6}\right) \\
&= 3.181818 + \left(\frac{2}{11} \times \frac{1}{10^6}\right)
\end{aligned}$$

At this point we have a fairly good estimation of the decimal expansion of $\frac{35}{11}$ as 3.181818. But we need to consider the value of $\left(\frac{2}{11} \times \frac{1}{10^6}\right)$. We know that $\frac{2}{11} < 1$.

By the Basic Inequality, we know that

$$\frac{2}{11} \times \frac{1}{10^6} < 1 \times \frac{1}{10^6}$$
$$\frac{2}{11} \times \frac{1}{10^6} < \frac{1}{10^6}$$

Which means that the value of $\frac{2}{11} \times \frac{1}{10^6}$ is less than 0.000001, and we have confirmed that 3.181818 is a good estimation of the infinite decimal that is equal to $\frac{35}{11}$.

Scaffolding:

Consider using a simpler example like $\frac{4}{3}$.

$$\begin{aligned}
\frac{4}{3} &= \frac{4 \times 10^2}{3} \times \frac{1}{10^2} \\
&= \frac{133 \times 3 + 1}{3} \times \frac{1}{10^2} \\
&= \left(\frac{133 \times 3}{3} + \frac{1}{3}\right) \times \frac{1}{10^2} \\
&= \left(133 + \frac{1}{3}\right) \times \frac{1}{10^2} \\
&= 133 \times \frac{1}{10^2} + \frac{1}{3} \times \frac{1}{10^2} \\
&= \frac{133}{10^2} + \left(\frac{1}{3} \times \frac{1}{10^2}\right) \\
&= 1.33 + \left(\frac{1}{3} \times \frac{1}{10^2}\right)
\end{aligned}$$

Example 4 (8 minutes)

Example 4

Write the decimal expansion of the fraction $\frac{6}{7}$.

- Let's write the decimal expansion of $\frac{6}{7}$. Will it be a finite or infinite decimal? How do you know?
  - *We know that the fraction will not be a finite decimal because the denominator is not a product of 2's and/or 5's.*
- We want to make sure we get enough decimal digits in order to get a good idea of what the infinite decimal is. What power of 10 should we use?
  - *Accept any power of 10 students give. Since we know it's an infinite decimal, 10^6 should be sufficient to make a good estimate of the value of $\frac{35}{11}$, but any power of 10 greater than 6 will work too. The work below uses 10^6.*
- Using 10^6 we have

$$\frac{6}{7} = \frac{6 \times 10^6}{7} \times \frac{1}{10^6}$$

  What do we do next?
  - *Since $6 \times 10^6 = 6{,}000{,}000$, we need to do division with remainder for $\frac{6{,}000{,}000}{7}$.*
- Determine which numbers make the following statement true:

  $6{,}000{,}000 =$ ____________ $\times 7 +$ ____
  - $857{,}142$ *and* 6 *would give us* $6{,}000{,}000 = \underline{857{,}142} \times 7 + \underline{6}$
- Now we know that

$$\frac{6}{7} = \frac{857142 \times 7 + 6}{7} \times \frac{1}{10^6}$$

  Finish the work to write the decimal expansion of $\frac{6}{7}$.
  - *Sample response:*

$$\begin{aligned}
\frac{6}{7} &= \left(\frac{857142 \times 7}{7} + \frac{6}{7}\right) \times \frac{1}{10^6} \\
&= \left(857142 + \frac{6}{7}\right) \times \frac{1}{10^6} \\
&= 857142 \times \frac{1}{10^6} + \left(\frac{6}{7} \times \frac{1}{10^6}\right) \\
&= \frac{857142}{10^6} + \left(\frac{6}{7} \times \frac{1}{10^6}\right) \\
&= 0.857142 + \left(\frac{6}{7} \times \frac{1}{10^6}\right)
\end{aligned}$$

- Again we can verify how good our estimate is using the Basic Inequality:

$$\frac{6}{7} < 1$$
$$\frac{6}{7} \times \frac{1}{10^6} < 1 \times \frac{1}{10^6}$$
$$\frac{6}{7} \times \frac{1}{10^6} < \frac{1}{10^6}$$

Therefore, $\frac{6}{7} \times \frac{1}{10^6} < 0.000001$ and stating that $\frac{6}{7} = 0.857142$ is a good estimate.

Exercises 3–5 (5 minutes)

Students complete Exercises 3–5 independently or in pairs. Allow students to use a calculator to check their work.

Exercises 3–5

3. a. Choose a power of ten to use to convert this fraction to a decimal: $\frac{4}{13}$. Explain your choice.

Choices will vary. The work shown below uses the factor 10^6. Students should choose a factor of at least 10^4 in order to get an approximate decimal expansion and a small remainder that will not greatly affect the value of the number.

b. Determine the decimal expansion of $\frac{4}{13}$ and verify you are correct using a calculator.

$$\frac{4}{13} = \frac{4 \times 10^6}{13} \times \frac{1}{10^6} = \frac{4,000,000}{13} \times \frac{1}{10^6}$$

$$4,000,000 = 307,692 \times 13 + 4$$

$$\begin{aligned}\frac{4}{13} &= \frac{307,692 \times 13 + 4}{13} \times \frac{1}{10^6}\\ &= \left(\frac{307,692 \times 13}{13} + \frac{4}{13}\right) \times \frac{1}{10^6}\\ &= \left(307,692 + \frac{4}{13}\right) \times \frac{1}{10^6}\\ &= 307,692 \times \frac{1}{10^6} + \left(\frac{4}{13} \times \frac{1}{10^6}\right)\\ &= \frac{307,692}{10^6} + \left(\frac{4}{13} \times \frac{1}{10^6}\right)\\ &= 0.307692 + \left(\frac{4}{13} \times \frac{1}{10^6}\right)\end{aligned}$$

The decimal expansion of $\frac{4}{13}$ is approximately 0.307692.

4. Write the decimal expansion of $\frac{1}{11}$. Verify you are correct using a calculator.

$$\frac{1}{11} = \frac{1 \times 10^6}{11} \times \frac{1}{10^6} = \frac{1,000,000}{11} \times \frac{1}{10^6}$$

$$1,000,000 = 90,909 \times 11 + 1$$

$$\begin{aligned}\frac{1}{11} &= \frac{90,909 \times 11 + 1}{11} \times \frac{1}{10^6}\\ &= \left(\frac{90,909 \times 11}{11} + \frac{1}{11}\right) \times \frac{1}{10^6}\\ &= \left(90,909 + \frac{1}{11}\right) \times \frac{1}{10^6}\\ &= 90,909 \times \frac{1}{10^6} + \left(\frac{1}{11} \times \frac{1}{10^6}\right)\\ &= \frac{90,909}{10^6} + \left(\frac{1}{11} \times \frac{1}{10^6}\right)\\ &= 0.0909090 + \left(\frac{1}{11} \times \frac{1}{10^6}\right)\end{aligned}$$

The decimal expansion of $\frac{1}{11}$ is approximately 0.0909090.

5. **Write the decimal expansion of $\frac{19}{21}$. Verify you are correct using a calculator.**

$$\frac{19}{21} = \frac{19 \times 10^8}{21} \times \frac{1}{10^8}$$
$$= \frac{1900000000}{21} \times \frac{1}{10^8}$$

$$1,900,000,000 = 90,476,190 \times 21 + 10$$

$$\frac{19}{21} = \frac{90476190 \times 21 + 10}{21} \times \frac{1}{10^8}$$
$$= \left(\frac{990476190 \times 21}{21} + \frac{10}{21}\right) \times \frac{1}{10^8}$$
$$= \left(90476190 + \frac{10}{21}\right) \times \frac{1}{10^8}$$
$$= 90476190 \times \frac{1}{10^8} + \left(\frac{10}{21} \times \frac{1}{10^8}\right)$$
$$= \frac{90476190}{10^8} + \left(\frac{10}{21} \times \frac{1}{10^8}\right)$$
$$= 0.90476190 + \left(\frac{10}{21} \times \frac{1}{10^8}\right)$$

The decimal expansion of $\frac{19}{21}$ *is approximately* 0.90476190.

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know how to write the decimal expansion for any fraction.
- Using what we know about equivalent fractions, we can multiply a fraction by a power of 10 large enough to give us enough decimal digits to estimate the decimal expansion of a fraction.
- We know that the amount we do not include in the decimal expansion is a very small amount that will not change the value of the number in any meaningful way.

Lesson Summary

Multiplying a fraction's numerator and denominator by the same power of 10 to determine its decimal expansion is similar to including extra zeroes to the right of a decimal when using the long division algorithm. The method of multiplying by a power of 10 reduces the work to whole number division.

Example: We know that the fraction $\frac{5}{3}$ has an infinite decimal expansion because the denominator is not a product of 2's and/or 5's. Its decimal expansion is found by the following procedure:

$\frac{5}{3} = \frac{5 \times 10^2}{3} \times \frac{1}{10^2}$	**Multiply numerator and denominator by 10^2**
$= \frac{166 \times 3 + 2}{3} \times \frac{1}{10^2}$	**Rewrite the numerator as a product of a number multiplied by the denominator**
$= \left(\frac{166 \times 3}{3} + \frac{2}{3}\right) \times \frac{1}{10^2}$	**Rewrite the first term as a sum of fractions with the same denominator**
$= \left(166 + \frac{2}{3}\right) \times \frac{1}{10^2}$	**Simplify**
$= \frac{166}{10^2} + \left(\frac{2}{3} \times \frac{1}{10^2}\right)$	**Use the Distributive Property**
$= 1.66 + \left(\frac{2}{3} \times \frac{1}{10^2}\right)$	**Simplify**
$= 166 \times \frac{1}{10^2} + \frac{2}{3} \times \frac{1}{10^2}$	**Simplify the first term using what you know about place value**

Notice that the value of the remainder, $\left(\frac{2}{3} \times \frac{1}{10^2}\right) = \frac{2}{300} = 0.00\overline{6}$, is quite small and does not add much value to the number. Therefore, 1.66 is a good estimate of the value of the infinite decimal for the fraction $\frac{5}{3}$.

Exit Ticket (5 minutes)

Name ______________________________ Date ______________

Lesson 9: Decimal Expansions of Fractions, Part 1

Exit Ticket

1. Write the decimal expansion of $\frac{823}{40}$.

2. Write the decimal expansion of $\frac{48}{21}$.

Exit Ticket Sample Solutions

1. Write the decimal expansion of $\frac{823}{40}$.

$$\begin{aligned}\frac{23}{40} &= \frac{823 \times 10^3}{40} \times \frac{1}{10^3}\\ &= \frac{823000}{40} \times \frac{1}{10^3}\\ &= \frac{20575 \times 40 + 1}{40} \times \frac{1}{10^3}\\ &= \left(20575 + \frac{1}{40}\right) \times \frac{1}{10^3}\\ &= 20575 \times \frac{1}{10^3} + \frac{1}{40} \times \frac{1}{10^3}\\ &= \frac{20575}{10^3} + \left(\frac{1}{40} \times \frac{1}{10^3}\right)\\ &= 20.575\end{aligned}$$

The decimal expansion of $\frac{823}{40}$ *is approximately* 20.575.

2. Write the decimal expansion of $\frac{48}{21}$.

$$\begin{aligned}\frac{48}{21} &= \frac{48 \times 10^6}{21} \times \frac{1}{10^6}\\ &= \frac{48000000}{21} \times \frac{1}{10^6}\end{aligned}$$

$$48,000,000 = 2,285,714 \times 21 + 6$$

$$\begin{aligned}\frac{48}{21} &= \frac{2285714 \times 21 + 6}{21} \times \frac{1}{10^6}\\ &= \left(\frac{2285714 \times 21}{21} + \frac{6}{21}\right) \times \frac{1}{10^6}\\ &= \left(2285714 + \frac{6}{21}\right) \times \frac{1}{10^6}\\ &= 2285714 \times \frac{1}{10^6} + \left(\frac{6}{21} \times \frac{1}{10^6}\right)\\ &= \frac{2285714}{10^6} + \left(\frac{6}{21} \times \frac{1}{10^6}\right)\\ &= 2.285714 + \left(\frac{6}{21} \times \frac{1}{10^6}\right)\end{aligned}$$

The decimal expansion of $\frac{48}{21}$ *is approximately* 2.285714.

Problem Set Sample Solutions

1. a. Choose a power of ten to convert this fraction to a decimal: $\frac{4}{11}$. Explain your choice.

Choices will vary. The work shown below uses the factor 10^6. *Students should choose a factor of at least* 10^4 *in order to get an approximate decimal expansion and notice that the decimal expansion repeats.*

b. Determine the decimal expansion of $\frac{4}{11}$ and verify you are correct using a calculator.

$$\frac{4}{11} = \frac{4 \times 10^6}{11} \times \frac{1}{10^6} = \frac{4000000}{11} \times \frac{1}{10^6}$$

$$4,000,000 = 363,636 \times 11 + 4$$

$$\begin{aligned}\frac{4}{11} &= \frac{363636 \times 11 + 4}{11} \times \frac{1}{10^6} \\ &= \left(\frac{363636 \times 11}{11} + \frac{4}{11}\right) \times \frac{1}{10^6} \\ &= \left(363636 + \frac{4}{11}\right) \times \frac{1}{10^6} \\ &= 363636 \times \frac{1}{10^6} + \left(\frac{4}{11} \times \frac{1}{10^6}\right) \\ &= \frac{363636}{10^6} + \left(\frac{4}{11} \times \frac{1}{10^6}\right) \\ &= 0.363636 + \left(\frac{4}{11} \times \frac{1}{10^6}\right)\end{aligned}$$

The decimal expansion of $\frac{4}{11}$ *is approximately* 0.363636.

2. Write the decimal expansion of $\frac{5}{13}$. Verify you are correct using a calculator.

$$\frac{5}{13} = \frac{5 \times 10^6}{13} \times \frac{1}{10^6} = \frac{5000000}{13} \times \frac{1}{10^6}$$

$$5,000,000 = 384,615 \times 13 + 5$$

$$\begin{aligned}\frac{5}{13} &= \frac{384615 \times 13 + 5}{13} \times \frac{1}{10^6} \\ &= \left(\frac{384615 \times 13}{13} + \frac{5}{13}\right) \times \frac{1}{10^6} \\ &= \left(384615 + \frac{5}{13}\right) \times \frac{1}{10^6} \\ &= 384615 \times \frac{1}{10^6} + \left(\frac{5}{13} \times \frac{1}{10^6}\right) \\ &= \frac{384615}{10^6} + \left(\frac{5}{13} \times \frac{1}{10^6}\right) \\ &= 0.384615 + \left(\frac{5}{13} \times \frac{1}{10^6}\right)\end{aligned}$$

The decimal expansion of $\frac{5}{13}$ *is approximately* 0.384615.

3. Write the decimal expansion of $\frac{23}{39}$. Verify you are correct using a calculator.

$$\frac{23}{39} = \frac{23 \times 10^6}{39} \times \frac{1}{10^6} = \frac{23000000}{39} \times \frac{1}{10^6}$$

$$23,000,000 = 589,743 \times 39 + 23$$

$$\begin{aligned}\frac{23}{39} &= \frac{589743 \times 39 + 23}{39} \times \frac{1}{10^6} \\ &= \left(\frac{589743 \times 39}{39} + \frac{23}{39}\right) \times \frac{1}{10^6} \\ &= \left(589743 + \frac{23}{39}\right) \times \frac{1}{10^6} \\ &= 589743 \times \frac{1}{10^6} + \left(\frac{23}{39} \times \frac{1}{10^6}\right) \\ &= \frac{589743}{10^6} + \left(\frac{23}{39} \times \frac{1}{10^6}\right) \\ &= 0.589743 + \left(\frac{23}{39} \times \frac{1}{10^6}\right)\end{aligned}$$

The decimal expansion of $\frac{23}{39}$ *is approximately* 0.589743.

4. Tamer wrote the decimal expansion of $\frac{3}{7}$ as 0.418571, but when he checked it on a calculator it was 0.428571. Identify his error and explain what he did wrong.

$$\frac{3}{7} = \frac{3\times 10^6}{7} \times \frac{1}{10^6} = \frac{3000000}{7} \times \frac{1}{10^6}$$

$$3,000,000 = 418,571 \times 7 + 3$$

$$\begin{aligned}\frac{3}{7} &= \frac{418571 \times 7 + 3}{7} \times \frac{1}{10^6}\\ &= \left(\frac{418571 \times 7}{7} + \frac{3}{7}\right) \times \frac{1}{10^6}\\ &= \left(418571 + \frac{3}{7}\right) \times \frac{1}{10^6}\\ &= 418571 \times \frac{1}{10^6} + \left(\frac{3}{7} \times \frac{1}{10^6}\right)\\ &= \frac{418571}{10^6} + \left(\frac{3}{7} \times \frac{1}{10^6}\right)\\ &= 0.418571 + \left(\frac{3}{7} \times \frac{1}{10^6}\right)\end{aligned}$$

Tamer did the division with remainder incorrectly. He wrote that $3,000,000 = 418,571 \times 7 + 3$ when actually $3,000,000 = 428,571 \times 7 + 3$. This error led to his decimal expansion being incorrect.

5. Given that

$$\frac{6}{7} = 0.857142 + \left(\frac{6}{7} \times \frac{1}{10^6}\right)$$

Explain why 0.857142 is a good estimate of $\frac{6}{7}$.

When you consider the value of $\left(\frac{6}{7} \times \frac{1}{10^6}\right)$, then it is clear that 0.857142 is a good estimate of $\frac{6}{7}$. We know that $\frac{6}{7} < 1$. By the Basic Inequality, we also know that $\frac{6}{7} \times \frac{1}{10^6} < 1 \times \frac{1}{10^6}$, which means that $\frac{6}{7} \times \frac{1}{10^6} < 0.000001$. That is such a small value that it will not affect the estimate of $\frac{6}{7}$ in any real way.