Lesson 4: More Examples of Functions

Student Outcomes

- Students examine and recognize real-world functions as discrete functions, such as the cost of a book.
- Students examine and recognize real-world functions as continuous functions, such as the temperature of a pot of cooling soup.

Classwork

Discussion (5 minutes)

- In the past couple of lessons, we looked at several linear functions and the numbers that are assigned by the functions in the form of a table.

Table A:

Bags of candy (x)	1	2	3	4	5	6	7	8
Cost (y)	$\$1.25$	$\$2.50$	$\$3.75$	$\$5.00$	$\$6.25$	$\$7.50$	$\$8.75$	$\$10.00$

Table B:

Number of seconds (x)	0.5	1	1.5	2	2.5	3	3.5	4
Distance traveled in feet (y)	4	16	36	64	100	144	196	256

- In Table A, the context was purchasing bags of candy. In Table B, it was the distance traveled by a moving object. Examine the tables. What are the differences between these two situations?

Provide time for students to discuss the differences between the two tables and share their thoughts with the class. Then continue with the discussion below.

- For the function in Table A, we said that the rule that described the function was $y = 1.25x$, where $x \geq 0$.
- Why did we restrict x to numbers equal to or greater than 0?
  - *We restricted x to numbers equal to or greater than 0 because you cannot purchase -1 bags of candy, for example.*
- If we assume that only a whole number of bags can be sold because a bag cannot be opened up and divided into fractional parts, then we need to be more precise about our restriction on x. Specifically, we must say that x is a positive integer, or $x \geq 0$. Now, it is clear that only $0, 1, 2, 3$, etc., bags can be sold, as opposed to 1.25 bags or 5.7 bags.

- With respect to Table B, the rule that describes this function was $y = 16x^2$. Does this problem require the same restrictions on x as the previous problem? Explain.
  - *We should state that x must be a positive number because x represents the amount of time traveled, but we do not need to say that x must be a positive integer. The intervals of time do not need to be in whole seconds; the distance can be measured at fractional parts of a second.*
- We describe these different functions as discrete and continuous. When only positive integers make sense for the input of a function, like the bags of candy example, we say that it is a discrete rate problem. When there are no gaps in the values of the input—for example, fractional values of time—we say that it is a continuous rate problem. In terms of functions, we see the difference reflected in the input values of the function. We cannot do problems of motion using the concept of unit rate without discussing the meaning of constant speed.

Scaffolding:

The definition of *discrete* is individually separate or distinct. Knowing this can help students understand why we call certain rates discrete rates.

Example 1 (6 minutes)

This is another example of a discrete rate problem.

Example 1

If 4 copies of the same book cost $\$256.00$, what is the unit rate for the book?

The unit rate is $\frac{256}{4}$ or $\$64.00$ per book.

- The total cost is a function of the number of books that are purchased. That is, if x is the cost of a book and y is the total cost, then $y = 64x$.
- What cost does the function assign to 3 books? 3.5 books?
  - *For 3 books: $y = 64(3)$; the cost of 3 books is $\$192.00$.*
  - *For 3.5 books: $y = 64(3.5)$; the cost of 3.5 books is $\$224.00$.*
- We can use the rule that describes the cost function to determine the cost of 3.5 books, but does it make sense?
  - *No. You cannot buy half of a book.*
- Is this a discrete rate problem or a continuous rate problem? Explain.
  - *This is a discrete rate problem because you cannot buy a fraction of a book; only a whole number of books can be purchased.*

Example 2 (2 minutes)

This is an example of a continuous rate problem examined in the last lesson.

- Let's revisit a problem that we examined in the last lesson.

Example 2

Water flows from a faucet at a constant rate. That is, the volume of water that flows out of the faucet is the same over any given time interval. If 7 gallons of water flow from the faucet every 2 minutes, determine the rule that describes the volume function of the faucet.

- We said then that the rule that describes the volume function of the faucet is $y = 3.5x$, where y is the volume of water in gallons that flows from the faucet and x is the number of minutes the faucet is on.
- What limitations are there on x and y?
  - *Both x and y should be positive numbers because they represent time and volume.*
- Would this rate be considered discrete or continuous? Explain.
  - *This rate is continuous because we can assign any positive number to x, not just positive integers.*

Example 3 (8 minutes)

This is a more complicated example of a continuous rate problem.

Example 3

You have just been served freshly made soup that is so hot that it cannot be eaten. You measure the temperature of the soup, and it is $210°\text{F}$. Since $212°\text{F}$ is boiling, there is no way it can safely be eaten yet. One minute after receiving the soup, the temperature has dropped to $203°\text{F}$. If you assume that the rate at which the soup cools is linear, write a rule that would describe the rate of cooling of the soup.

The temperature of the soup dropped $7°\text{F}$ in one minute. Assuming the cooling continues at the same rate, then if y is the number of degrees that the soup drops after x minutes, $y = 7x$.

Scaffolding:

The more real you can make this, the better. Consider having a cooling cup of soup, coffee, or tea with a digital thermometer available for students to observe.

- We want to know how long it will be before the temperature of the soup is at a more tolerable temperature of $147°\text{F}$. The difference in temperature from $210°\text{F}$ to $147°\text{F}$ is $63°\text{F}$. For what number x will our function assign 63?
  - *$63 = 7x$; then, $x = 9$. Our function assigns 63 to 9.*
- Recall that we assumed that the cooling of the soup would be linear. However, that assumption appears to be incorrect. The data in the table below shows a much different picture of the cooling soup.

Time	Temperature
after 2 minutes	196
after 3 minutes	190
after 4 minutes	184
after 5 minutes	178
after 6 minutes	173
after 7 minutes	168
after 8 minutes	163
after 9 minutes	158

Our function led us to believe that after 9 minutes the soup would be safe to eat. The data in the table shows that it is still too hot.

- What do you notice about the change in temperature from one minute to the next?
  - *For the first few minutes, minute 2 to minute 5, the temperature decreased $6°\text{F}$ each minute. From minute 5 to minute 9, the temperature decreased just $5°\text{F}$ each minute.*

- Since the rate of cooling at each minute is not linear, then this function is said to be a nonlinear function. In fact, the rule that describes the cooling of the soup is

$$y = 70 + 140\left(\frac{133}{140}\right)^{x},$$

where y is the temperature of the soup after x minutes.
- Finding a rule that describes a function like this one is something you will spend more time on in high school. In this module, the nonlinear functions we work with will be much simpler. The point is that nonlinear functions exist, and in some cases, we cannot think of mathematics as computations of simply numbers. In fact, some functions cannot be described with numbers at all.
- Would this function be described as discrete or continuous? Explain.
  - *This function is continuous because we could find the temperature of the soup for any fractional time x, as opposed to just integer intervals of time.*

Example 4 (6 minutes)

Example 4

Consider the following function: There is a function G so that the function assigns to each input, the number of a particular player, an output, the player's height. For example, the function G assigns to the input 1 an output of $5'11''$.

1	$5'11''$
2	$5'4''$
3	$5'9''$
4	$5'6''$
5	$6'3''$
6	$6'8''$
7	$5'9''$
8	$5'10''$
9	$6'2''$

- The function G assigns to the input 2 what output?
  - *The function G would assign the height $5'4''$ to the player 2.*
- Could the function G also assign to the player 2 a second output value of $5'6''$? Explain.
  - *No. The function assigns height to a particular player. There is no way that a player can have two different heights.*
- Can you think of a way to describe this function using a rule? Of course not. There is no formula for such a function. The only way to describe the function would be to list the assignments shown in part in the table.
- Can we classify this function as discrete or continuous? Explain.
  - *This function would be described as discrete because the input is a particular player, and the output is the player's height. A person is one height or another, not two heights at the same time.*
- This function is an example of a function that cannot be described by numbers or symbols, but it is still a function.

Exercises 1–3 (10 minutes)

Exercises 1–3

1. A linear function has the table of values below related to the number of buses needed for a field trip.

Number of students (x)	35	70	105	140
Number of buses (y)	1	2	3	4

a. Write the linear function that represents the number of buses needed, y, for x number of students.

$$y = \frac{1}{35}x$$

b. Describe the limitations of x and y.

Both x and y must be positive whole numbers. The symbol x represents students, so we cannot have 1.2 students. Similarly, y represents the number of buses needed, so we cannot have a fractional number of buses.

c. Is the function discrete or continuous?

The function is discrete.

d. The entire eighth-grade student body of 321 students is going on a field trip. What number of buses does our function assign to 321 students? Explain.

$$y = \frac{1}{35}(321)$$
$$y = \frac{321}{35}$$
$$y = 9.1714\ldots$$
$$y \approx 9.2$$

Ten buses will be needed for the field trip. The function gives us an assignment of about 9.2, which means that 9.2 buses would be needed for the field trip, but we need a whole number of buses. Nine buses means some students will be left behind, so 10 buses will be needed to take all 321 students on the trip.

e. Some seventh-grade students are going on their own field trip to a different destination, but just 180 are attending. What number does the function assign to 180? How many buses will be needed for the trip?

$$y = \frac{1}{35}(180)$$
$$y = 5.1428\ldots$$
$$y \approx 5.1$$

Six buses will be needed for the field trip.

f. What number does the function assign to 50? Explain what this means and what your answer means.

$$y = \frac{1}{35}(50)$$
$$y = 1.4285\ldots$$
$$y \approx 1.4$$

The question is asking us to determine the number of buses needed for 50 students. The function assigns approximately 1.4 to 50. The function tells us that we need 1.4 buses for 50 students, but it makes more sense to say we need 2 buses because you cannot have 1.4 buses.

2. A linear function has the table of values below related to the cost of movie tickets.

Number of tickets (x)	3	6	9	12
Total cost (y)	$\$27.75$	$\$55.50$	$\$83.25$	$\$111.00$

a. Write the linear function that represents the total cost, y, for x tickets purchased.

$$y = \frac{27.75}{3}x$$
$$y = 9.25x$$

b. Is the function discrete or continuous? Explain.

The function is discrete. You cannot have half of a movie ticket; therefore, it must be a whole number of tickets, which means it is discrete.

c. What number does the function assign to 4? What do the question and your answer mean?

It is asking us to determine the cost of buying 4 tickets. The function assigns 37 to 4. The answer means that 4 tickets will cost $\$37.00$.

3. A function produces the following table of values.

Input	Output
Banana	Yellow
Cherry	Red
Orange	Orange
Tangerine	Orange
Strawberry	Red

a. Can this function be described by a rule using numbers? Explain.

No. Much like the example with the players and their heights, this function cannot be described by numbers or a rule. There is no number or rule that can define the function.

b. Describe the assignment of the function.

The function assigns to each fruit the color of its skin.

c. State an input and the assignment the function would give to its output.

Answers will vary. Accept an answer that satisfies the function; for example, the function would assign red to the input of tomato.

Closing (4 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know that not all functions are linear and, moreover, not all functions can be described by numbers.
- We know that linear functions can have discrete rates and continuous rates.
- We know that discrete functions are those where only integer inputs can be used in the function for the inputs to make sense. An example of this would be purchasing 3 books compared to 3.5 books.
- We know that continuous functions are those whose inputs are any numbers of an interval, including fractional values, as an input. An example of this would be determining the distance traveled after 2.5 minutes of walking.

Lesson Summary

Not all functions are linear. In fact, not all functions can be described using numbers.

Linear functions can have discrete rates and continuous rates.

A function that can have only integer inputs is called a *discrete function*. For example, when planning for a field trip, it only makes sense to plan for a whole number of students and a whole number of buses, not fractional values of either.

***Continuous functions* are those whose inputs are any numbers of an interval, including fractional values–for example, determining the distance a person walks for a given time interval. The input, which is time in this case, can be in minutes, fractions of minutes, or decimals of minutes.**

Exit Ticket (4 minutes)

Name ____ Date ____________________

Lesson 4: More Examples of Functions

Exit Ticket

1. A linear function has the table of values below related to the cost of a certain tablet.

Number of tablets (x)	17	22	25
Total cost (y)	$\$10,183.00$	$\$13,178.00$	$\$14,975.00$

a. Write the linear function that represents the total cost, y, for x number of tablets.

b. Is the function discrete or continuous? Explain.

c. What number does the function assign to 7? Explain.

2. A function produces the following table of values.

Serious	Adjective
Student	Noun
Work	Verb
They	Pronoun
And	Conjunction
Accurately	Adverb

a. Describe the function.

b. What part of speech would the function assign to the word *continuous*?

Exit Ticket Sample Solutions

1. A linear function has the table of values below related to the cost of a certain tablet.

Number of tablets (x)	17	22	25
Total cost (y)	$\$10,183.00$	$\$13,178.00$	$\$14,975.00$

a. Write the linear function that represents the total cost, y, for x number of tablets.

$$y = \frac{10,183}{17}x$$
$$y = 599x$$

b. Is the function discrete or continuous? Explain.

The function is discrete. You cannot have half of a tablet; therefore, it must be a whole number of tablets, which means it is discrete.

c. What number does the function assign to 7? Explain.

The function assigns $4,193$ to 7, which means that the cost of 7 tablets would be $\$4,193.00$.

2. A function produces the following table of values.

Serious	Adjective
Student	Noun
Work	Verb
They	Pronoun
And	Conjunction
Accurately	Adverb

a. Describe the function.

The function assigns to each input a word that is a part of speech.

b. What part of speech would the function assign to the word *continuous*?

The function would assign the word adjective to the word continuous.

Problem Set Sample Solutions

1. A linear function has the table of values below related to the total cost for gallons of gas purchased.

Number of gallons (x)	5.4	6	15	17
Total cost (y)	$\$19.71$	$\$21.90$	$\$54.75$	$\$62.05$

a. Write the linear function that represents the total cost, y, for x gallons of gas.

$$y = 3.65x$$

b. Describe the limitations of x and y.

Both x and y must be positive rational numbers.

c. Is the function discrete or continuous?

The function is continuous.

d. What number does the function assign to 20? Explain what your answer means.

$$y = 3.65(20)$$
$$y = 73$$

The function assigns 73 to 20. It means that if 20 gallons of gas are purchased, it will cost $\$73.00$.

2. A function has the table of values below. Examine the information in the table to answer the questions below.

Input	Output
one	3
two	3
three	5
four	4
five	4
six	3
seven	5

a. Describe the function.

The function assigns to each input, a word, the number of letters in the word.

b. What number would the function assign to the word *eleven*?

The function would assign the number 6 to the word eleven.

3. A linear function has the table of values below related to the total number of miles driven in a given time interval in hours.

Number of hours driven (x)	3	4	5	6
Total miles driven (y)	141	188	235	282

a. Write the linear function that represents the total miles driven, y, for x number of hours.

$$y = \frac{141}{3}x$$
$$y = 47x$$

b. Describe the limitations of x and y.

Both x and y must be positive rational numbers.

c. Is the function discrete or continuous?

The function is continuous.

d. What number does the function assign to 8? Explain what your answer means.

$$y = 47(8)$$
$$y = 376$$

The function assigns 376 to 8. The answer means that 376 miles are driven in 8 hours.

e. Use the function to determine how much time it would take to drive 500 miles.

$$500 = 47x$$
$$\frac{500}{47} = x$$
$$10.63829\ldots = x$$
$$10.6 \approx x$$

It would take about 10.6 hours to drive 500 miles.

4. A function has the table of values below that gives temperatures at specific times over a period of 8 hours.

12:00 p.m.	$92°F$
1:00 p.m.	$90.5°F$
2:00 p.m.	$89°F$
4:00 p.m.	$86°F$
8:00 p.m.	$80°F$

a. Is the function a linear function? Explain.

Yes, it is a linear function. The change in temperature is the same over each time interval. For example, the temperature drops $1.5°F$ from 12:00 to 1:00 and 1:00 to 2:00. The temperature drops $3°F$ from 2:00 to 4:00, which is the same as $1.5°F$ each hour and $6°F$ over a 4-hour period of time, which is also $1.5°F$ per hour.

b. **Describe the limitations of x and y.**

The input is a particular time of the day, and y is the temperature. The input cannot be negative but could be intervals that are fractions of an hour. The output could potentially be negative because it can get that cold.

c. **Is the function discrete or continuous?**

The function is continuous. The input can be any interval of time, including fractional amounts.

d. **Let y represent the temperature and x represent the number of hours from 12:00 p.m. Write a rule that describes the function of time on temperature.**

$$y = 92 - 1.5x$$

e. **Check that the rule you wrote to describe the function works for each of the input and output values given in the table.**

At 12:00, 0 hours have passed since 12:00; then, $y = 92 - 1.5(0) = 92$.

At 1:00, 1 hour has passed since 12:00; then, $y = 92 - 1.5(1) = 90.5$.

At 2:00, 2 hours have passed since 12:00; then, $y = 92 - 1.5(2) = 89$.

At 4:00, 4 hours have passed since 12:00; then, $y = 92 - 1.5(4) = 86$.

At 8:00, 8 hours have passed since 12:00; then, $y = 92 - 1.5(8) = 80$.

f. **Use the function to determine the temperature at 5:30 p.m.**

At 5:30, 5.5 hours have passed since 12:00; then $y = 92 - 1.5(5.5) = 83.75$.

The temperature at 5:30 will be $83.75°F$.

g. **Is it reasonable to assume that this function could be used to predict the temperature for 10:00 a.m. the following day or a temperature at any time on a day next week? Give specific examples in your explanation.**

No. The function can only predict the temperature for as long as the temperature is decreasing. At some point, the temperature will rise. For example, if we tried to predict the temperature for a week from 12:00 p.m. when the data was first collected, we would have to use the function to determine what number it assigns to 168 because 168 would be the number of hours that pass in the week. Then we would have

$$y = 92 - 1.5(168)$$
$$y = -160,$$

which is an unreasonable prediction for the temperature.