Lesson 28: Another Computational Method of Solving a Linear System

Student Outcomes

- Students learn the elimination method for solving a system of linear equations.
- Students use properties of rational numbers to find a solution to a system, if it exists, through computation using substitution and elimination methods.

Lesson Notes

Throughout the lesson, students are asked to verify that their solution to a system is correct by graphing the system and comparing the point of intersection to their solution. For that reason, provide graph paper for student use for both the Exercises and the Problem Set. Graphs should be provided during the presentation of the Examples so that you can discuss with students whether or not their estimated point of intersection verifies their solution.

Classwork

Discussion (5 minutes)

- In the last lesson, we saw that if a system of linear equations has a solution, it can be found without graphing. In each case, the first step was to eliminate one of the variables.
- Describe how you would solve this system algebraically: $\begin{cases} y = 3x + 5 \\ y = 8x + 3 \end{cases}$
  - *Since both equations were equal to y, we could write $3x + 5 = 8x + 3$, thereby eliminating the y from the system.*
- Describe how you would solve this system algebraically: $\begin{cases} y = 7x - 2 \\ 2y - 4x = 10 \end{cases}$
  - *We can substitute $7x - 2$ for y in the second equation, i.e., $2(7x - 2) - 4x = 10$, thereby eliminating the y again.*
- Describe how you would solve this system algebraically: $\begin{cases} x = 6y + 7 \\ x = 10y + 2 \end{cases}$
  - *Since both equations are equal to x, we could write $6y + 7 = 10y + 2$, thereby eliminating the x.*
- In this lesson, we will learn a method for solving systems that requires us to eliminate one of the variables, but in a different way from the last lesson.

Example 1 (8 minutes)

Example 1

Use what you noticed about adding equivalent expressions to solve the following system by elimination.

$$\begin{cases} 6x - 5y = 21 \\ 2x + 5y = -5 \end{cases}$$

Show students the three examples of adding integer equations together. Ask students to verbalize what they notice in the examples and to generalize what they observe. The goal is for students to see that we can add equivalent expressions and still have an equivalence.

Example 1: If $2 + 5 = 7$ and $1 + 9 = 10$, does $2 + 5 + 1 + 9 = 7 + 10$?

Example 2: If $1 + 5 = 6$ and $7 - 2 = 5$, does $1 + 5 + 7 - 2 = 6 + 5$?

Example 3: If $-3 + 11 = 8$ and $2 + 1 = 3$, does $-3 + 11 + 2 + 1 = 8 + 3$?

MP.1

- Use what you noticed about adding equivalent expressions to solve the following system by elimination:

$$\begin{cases} 6x - 5y = 21 \\ 2x + 5y = -5 \end{cases}$$

Provide students with time to attempt to solve the system by adding the equations together. Have students share their work with the class. If necessary, use the points below to support students.

- Notice that terms $-5y$ and $5y$ are opposites; that is, they have a sum of zero when added. If we were to add the equations in the system, the y would be eliminated.

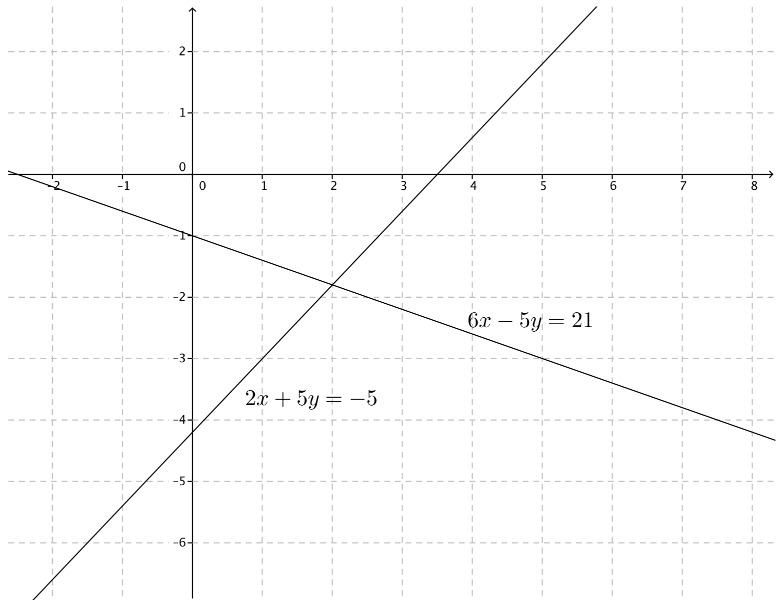
$$6x - 5y + 2x + 5y = 21 + (-5)$$
$$6x + 2x - 5y + 5y = 16$$
$$8x = 16$$
$$x = 2$$

- Just as before, now that we know what x is, we can substitute it into either equation to determine the value of y.

$$2(2) + 5y = -5$$
$$4 + 5y = -5$$
$$5y = -9$$
$$y = -\frac{9}{5}$$

The solution to the system is $\left(2, -\frac{9}{5}\right)$.

- We can verify our solution by sketching the graphs of the system.



Example 2 (5 minutes)

> **Example 2**
>
> **Solve the following system by elimination.**
>
> $$\begin{cases} -2x + 7y = 5 \\ 4x - 2y = 14 \end{cases}$$

- We will solve the following system by elimination.

$$\begin{cases} -2x + 7y = 5 \\ 4x - 2y = 14 \end{cases}$$

- In this example, it is not as obvious which variable to eliminate. It will become obvious as soon as we multiply the first equation by 2.

$$2(-2x + 7y = 5)$$
$$-4x + 14y = 10$$

Now we have the system $\begin{cases} -4x + 14y = 10 \\ 4x - 2y = 14 \end{cases}$. It is clear that when we add $-4x + 4x$, the x will be eliminated. Add the equations of this system together, and determine the solution to the system.

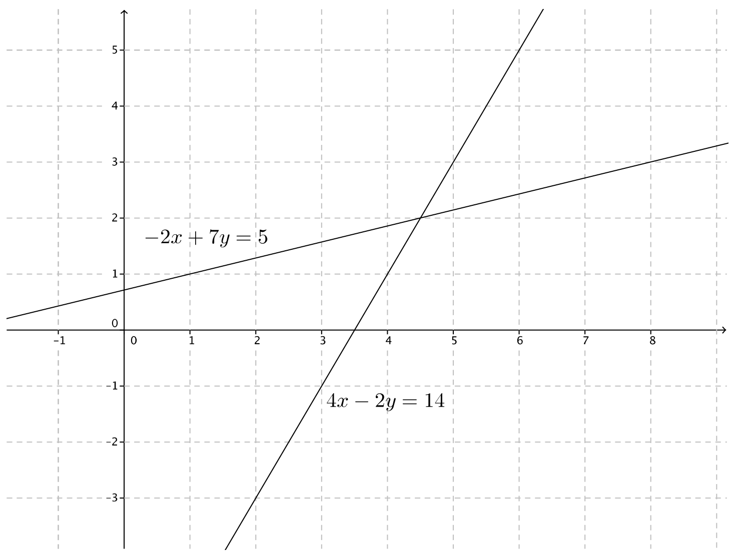
- *Sample student work:*

$$-4x + 14y + 4x - 2y = 10 + 12$$
$$14y - 2y = 24$$
$$12y = 24$$
$$y = 2$$

$$4x - 2(2) = 14$$
$$4x - 4 = 14$$
$$4x = 18$$
$$x = \frac{18}{4}$$
$$x = \frac{9}{2}$$

The solution to the system is $\left(\frac{9}{2}, 2\right)$.

- We can verify our solution by sketching the graphs of the system.



Example 3 (5 minutes)

Example 3

Solve the following system by elimination:

$$\begin{cases} \mathbf{7x - 5y = -2} \\ \mathbf{3x - 3y = 7} \end{cases}$$

MP.1

- We will solve the following system by elimination:

$$\begin{cases} 7x - 5y = -2 \\ 3x - 3y = 7 \end{cases}$$

Provide time for students to solve this system on their own before discussing it as a class.

MP.1

- In this case, it is even less obvious which variable to eliminate. On these occasions, we need to rewrite both equations. We multiply the first equation by -3 and the second equation by 7.

$$-3(7x - 5y = -2)$$
$$-21x + 15y = 6$$

$$7(3x - 3y = 7)$$
$$21x - 21y = 49$$

  Now we have the system $\begin{cases} -21x + 15y = 6 \\ 21x - 21y = 49 \end{cases}$, and it is obvious that the x can be eliminated.

- Look at the system again:

$$\begin{cases} 7x - 5y = -2 \\ 3x - 3y = 7 \end{cases}$$

  What would we do if we wanted to eliminate the y from the system?

  - *We could multiply the first equation by 3 and the second equation by -5.*

Students may say to multiply the first equation by -3 and the second equation by 5. Whichever answer is given first, ask if the second is also a possibility. Students should answer, "Yes."

Exercises 1–3 (8 minutes)

Students complete Exercises 1–3 independently.

Exercises

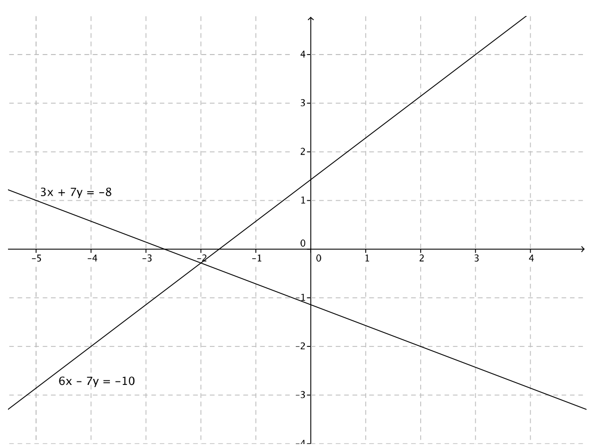
Each of the following systems has a solution. Determine the solution to the system by eliminating one of the variables. Verify the solution using the graph of the system.

1. $\begin{cases} 6x - 7y = -10 \\ 3x + 7y = -8 \end{cases}$

$$6x - 7y + 3x + 7y = -10 + (-8)$$
$$9x = -18$$
$$x = -2$$

$$3(-2) + 7y = -8$$
$$-6 + 7y = -8$$
$$7y = -2$$
$$y = -\frac{2}{7}$$

The solution is $\left(-2, -\frac{2}{7}\right)$.



2. $\begin{cases} x - 4y = 7 \\ 5x + 9y = 6 \end{cases}$

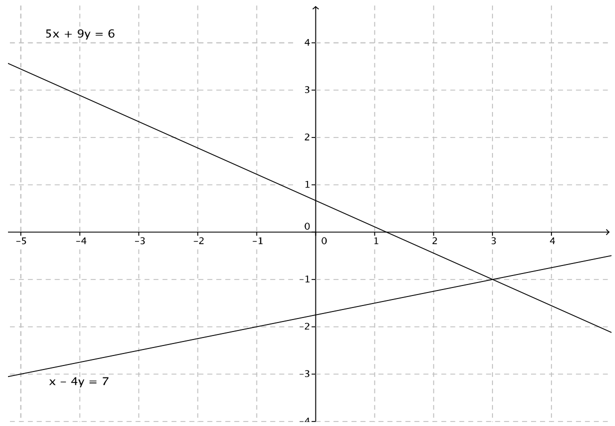
$$-5(x - 4y = 7)$$
$$-5x + 20y = -35$$

$$\begin{cases} -5x + 20y = -35 \\ 5x + 9y = 6 \end{cases}$$

$$-5x + 20y + 5x + 9y = -35 + 6$$
$$29y = -29$$
$$y = -1$$

$$x - 4(-1) = 7$$
$$x + 4 = 7$$
$$x = 3$$

The solution is $(3, -1)$.



3. $\begin{cases} 2x - 3y = -5 \\ 3x + 5y = 1 \end{cases}$

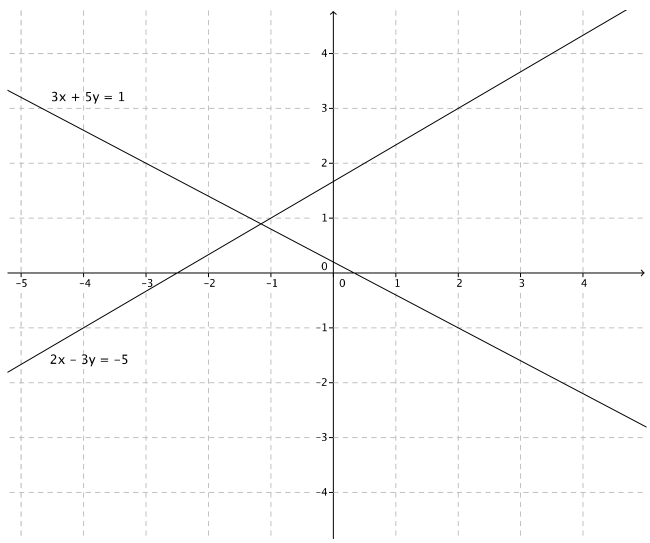
$$-3(2x - 3y = -5)$$
$$-6x + 9y = 15$$
$$2(3x + 5y = 1)$$
$$6x + 10y = 2$$

$$\begin{cases} -6x + 9y = 15 \\ 6x + 10y = 2 \end{cases}$$

$$-6x + 9y + 6x + 10y = 15 + 2$$
$$19y = 17$$
$$y = \frac{17}{19}$$

$$2x - 3\left(\frac{17}{19}\right) = -5$$
$$2x - \frac{51}{19} = -5$$
$$2x = -5 + \frac{51}{19}$$
$$2x = -\frac{44}{19}$$
$$x = -\frac{44}{38}$$
$$x = -\frac{22}{19}$$

The solution is $\left(-\frac{22}{19}, \frac{17}{19}\right)$.



Discussion (6 minutes)

- Systems of linear equations can be solved by sketching the graphs of the lines defined by the equations of the system and looking for the intersection of the lines, substitution (as was shown in the last lesson), or elimination (as was shown in this lesson). Some systems can be solved more efficiently by elimination, while others by substitution. Which method do you think would be most efficient for the following system? Explain.

$$\begin{cases} y = 5x - 19 \\ 3x + 11 = y \end{cases}$$

  - *Substitution would be the most efficient method. Since each equation is equal to y, it would be easiest to write the expressions $5x - 19$ and $3x + 11$ equal to one another; then, solve for y.*

- What method would you use for the following system? Explain.

$$\begin{cases} 2x - 9y = 7 \\ x + 9y = 5 \end{cases}$$

  - *Elimination would be the most efficient method because the terms $-9y + 9y$, when added, would eliminate the y from the equation.*

- What method would you use for the following system? Explain.

$$\begin{cases} 4x - 3y = -8 \\ x + 7y = 4 \end{cases}$$

  - *Elimination would likely be the most efficient method because we could multiply the second equation by -4 to eliminate the x from the equation.*

- What method would you use for the following system? Explain.

$$\begin{cases} x + y = -3 \\ 6x + 6y = 6 \end{cases}$$

  - *Accept any reasonable answer students provide; then, remind them that the most efficient use of time is to check to see if the system has a solution at all. Since the slopes of the graphs of these lines are parallel, this system has no solution.*

Closing (4 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know how to solve a system by eliminating one of the variables. In some cases, we will have to multiply one or both of the given equations by a constant in order to eliminate a variable.
- We know that some systems are solved more efficiently by elimination than by other methods.

Lesson Summary

Systems of linear equations can be solved by eliminating one of the variables from the system. One way to eliminate a variable is by setting both equations equal to the same variable, then writing the expressions equal to one another.

Example: Solve the system $\begin{cases} y = 3x - 4 \\ y = 2x + 1 \end{cases}$

Since both equations of the system are equal to y, then we can write and solve the equation:

$$3x - 4 = 2x + 1$$

Another way to eliminate a variable is by multiplying each term of an equation by the same constant to make an equivalent equation. Then use the equivalent equation to eliminate one of the variables and solve the system.

Example: Solve the system $\begin{cases} 2x + y = 8 \\ x + y = 10 \end{cases}$

Multiply the second equation by -2 to eliminate the x.

$$-2(x + y = 10)$$
$$-2x - 2y = -20$$

Now we have the system $\begin{cases} 2x + y = 8 \\ -2x - 2y = -20 \end{cases}$

When the equations are added together, the x is eliminated.

$$2x + y - 2x - 2y = 8 + (-20)$$
$$y - 2y = 8 + (-20)$$

Once a solution has been found, verify the solution graphically or by substitution.

Exit Ticket (4 minutes)

The graphs have been provided in the Exit Ticket in order to allow students to check their solution within the four minutes allotted.

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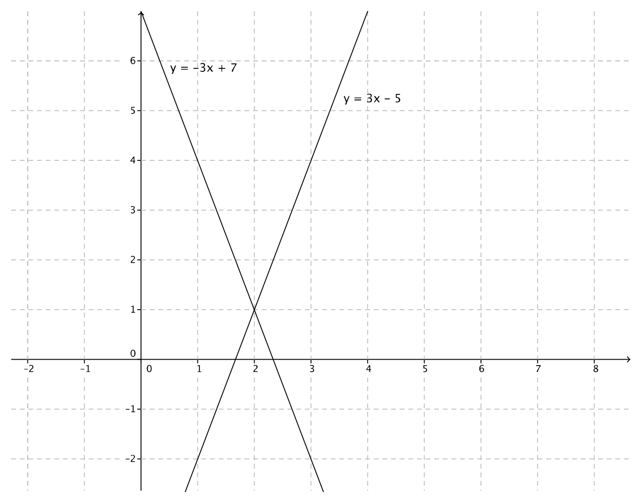
Lesson 28: Another Computational Method of Solving a Linear System

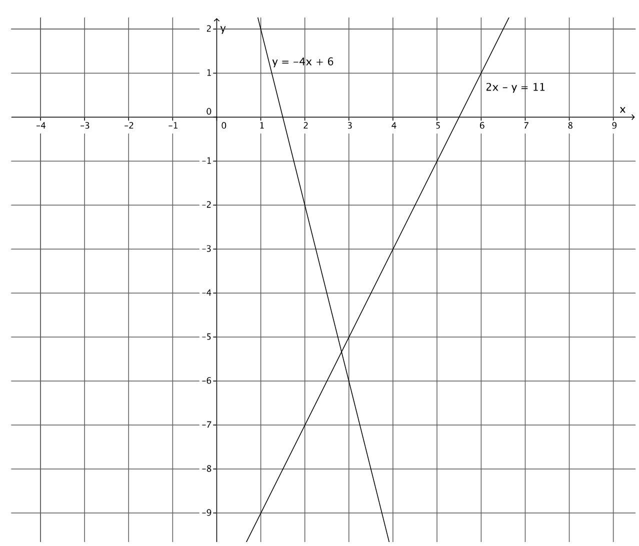
Exit Ticket

Determine the solution, if it exists, for each system of linear equations. Verify your solution on the coordinate plane.

1. $\begin{cases} y = 3x - 5 \\ y = -3x + 7 \end{cases}$



2. $\begin{cases} y = -4x + 6 \\ 2x - y = 11 \end{cases}$



Exit Ticket Sample Solutions

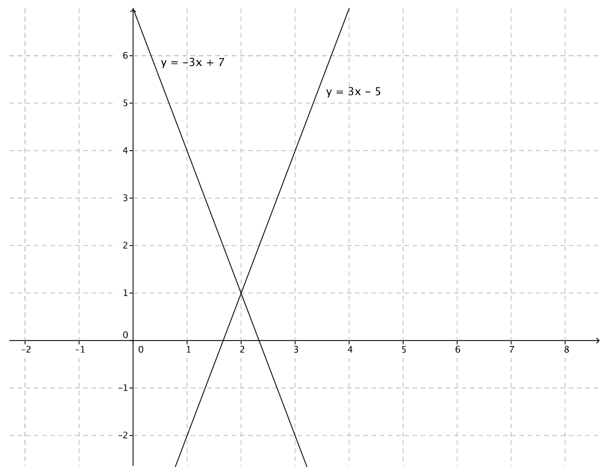
Determine the solution, if it exists, for each system of linear equations. Verify your solution on the coordinate plane.

1. $\begin{cases} y = 3x - 5 \\ y = -3x + 7 \end{cases}$

$$3x - 5 = -3x + 7$$
$$6x = 12$$
$$x = 2$$

$$y = 3(2) - 5$$
$$y = 6 - 5$$
$$y = 1$$

The solution is $(2, 1)$.

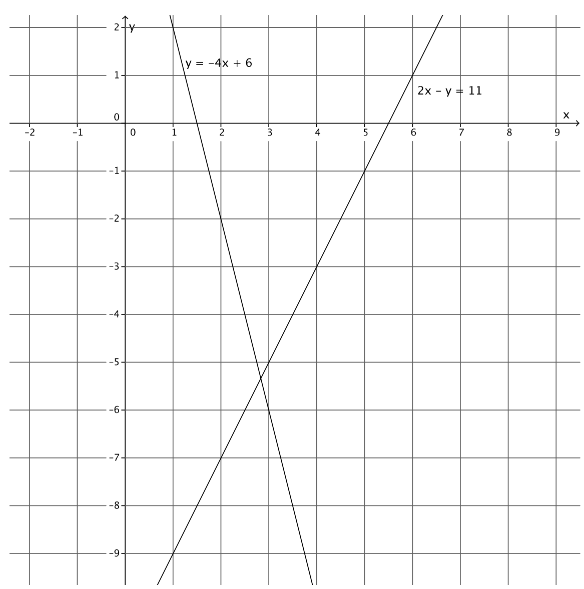


2. $\begin{cases} y = -4x + 6 \\ 2x - y = 11 \end{cases}$

$$2x - (-4x + 6) = 11$$
$$2x + 4x - 6 = 11$$
$$6x = 17$$
$$x = \frac{17}{6}$$

$$y = -4\left(\frac{17}{6}\right) + 6$$
$$y = -\frac{34}{3} + 6$$
$$y = -\frac{16}{3}$$

The solution is $\left(\frac{17}{6}, -\frac{16}{3}\right)$.



Problem Set Sample Solutions

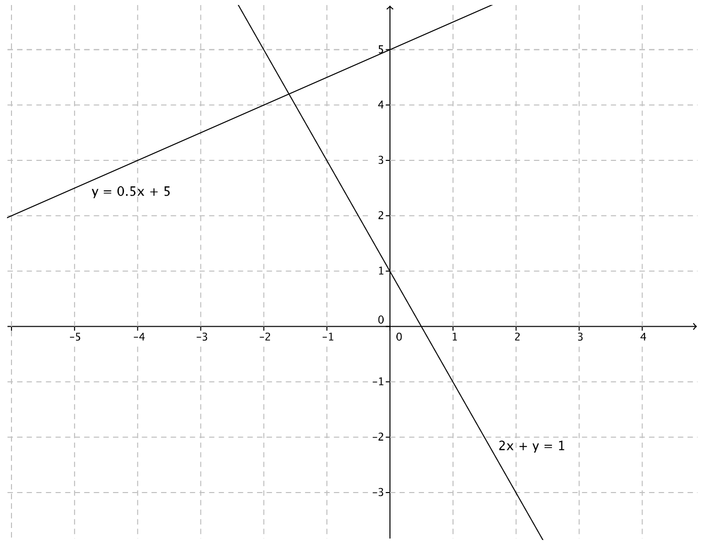
Determine the solution, if it exists, for each system of linear equations. Verify your solution on the coordinate plane.

1. $\begin{cases} \frac{1}{2}x + 5 = y \\ 2x + y = 1 \end{cases}$

$$2x + \frac{1}{2}x + 5 = 1$$
$$\frac{5}{2}x + 5 = 1$$
$$\frac{5}{2}x = -4$$
$$x = -\frac{8}{5}$$

$$2\left(-\frac{8}{5}\right) + y = 1$$
$$-\frac{16}{5} + y = 1$$
$$y = \frac{21}{5}$$

The solution is $\left(-\frac{8}{5}, \frac{21}{5}\right)$.

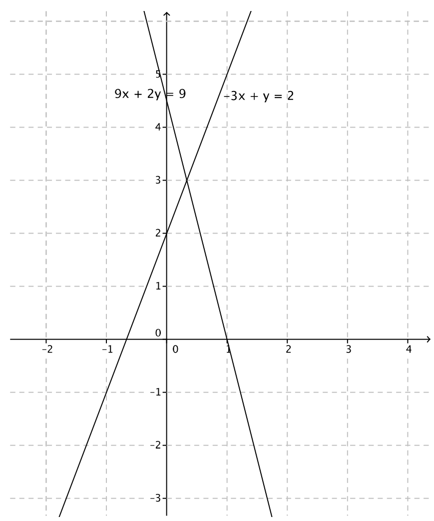


2. $\begin{cases} 9x + 2y = 9 \\ -3x + y = 2 \end{cases}$

$$3(-3x + y = 2)$$
$$-9x + 3y = 6$$
$$\begin{cases} 9x + 2y = 9 \\ -9x + 3y = 6 \end{cases}$$
$$9x + 2y - 9x + 3y = 15$$
$$5y = 15$$
$$y = 3$$

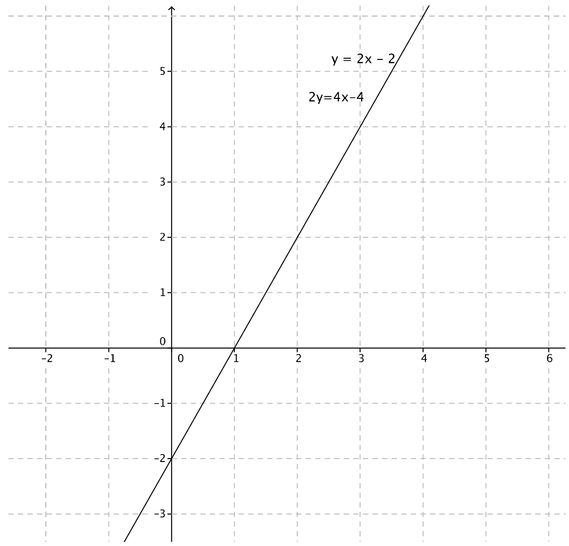
$$-3x + 3 = 2$$
$$-3x = -1$$
$$x = \frac{1}{3}$$

The solution is $\left(\frac{1}{3}, 3\right)$.



3. $\begin{cases} y = 2x - 2 \\ 2y = 4x - 4 \end{cases}$

These equations define the same line. Therefore, this system will have infinitely many solutions.

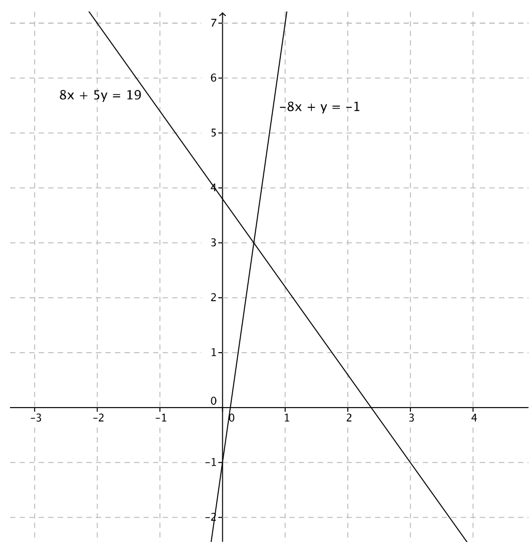


4. $\begin{cases} 8x + 5y = 19 \\ -8x + y = -1 \end{cases}$

$$8x + 5y - 8x + y = 19 - 1$$
$$5y + y = 18$$
$$6y = 18$$
$$y = 3$$

$$8x + 5(3) = 19$$
$$8x + 15 = 19$$
$$8x = 4$$
$$x = \frac{1}{2}$$

The solution is $\left(\frac{1}{2}, 3\right)$.

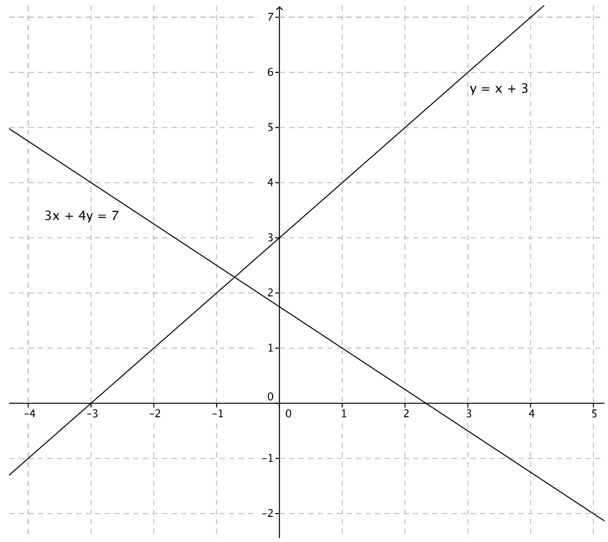


5. $\begin{cases} x+3=y \\ 3x+4y=7 \end{cases}$

$$3x+4(x+3)=7$$
$$3x+4x+12=7$$
$$7x+12=7$$
$$7x=-5$$
$$x=-\frac{5}{7}$$

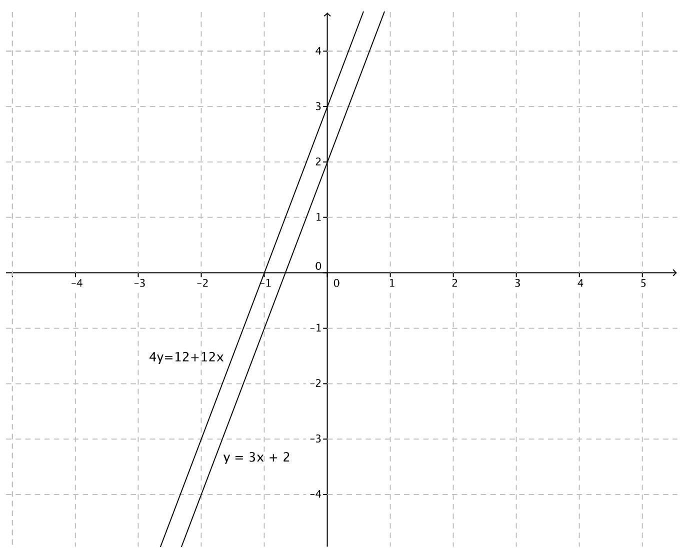
$$-\frac{5}{7}+3=y$$
$$\frac{16}{7}=y$$

The solution is $\left(-\frac{5}{7},\frac{16}{7}\right)$.



6. $\begin{cases} y=3x+2 \\ 4y=12+12x \end{cases}$

The equations graph as distinct lines. The slopes of these two equations are the same, and the y-intercepts are different, which means they graph as parallel lines. Therefore, this system will have no solutions.



7. $\begin{cases} 4x - 3y = 16 \\ -2x + 4y = -2 \end{cases}$

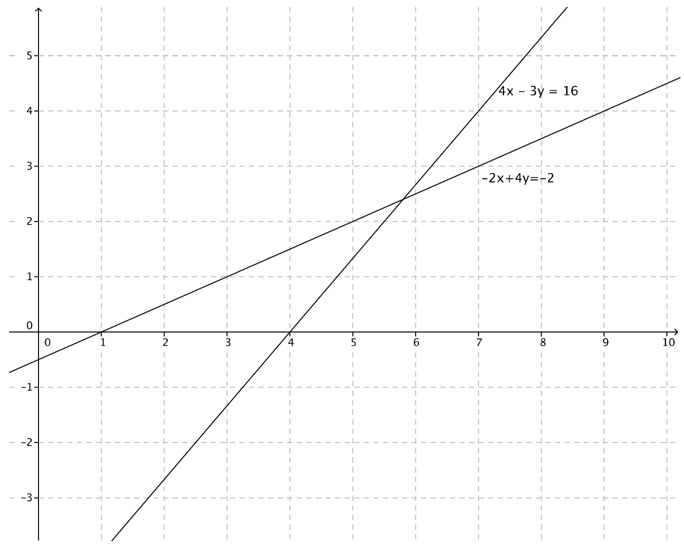
$2(-2x + 4y = -2)$
$-4x + 8y = -4$

$\begin{cases} 4x - 3y = 16 \\ -4x + 8y = -4 \end{cases}$

$$4x - 3y - 4x + 8y = 16 - 4$$
$$-3y + 8y = 12$$
$$5y = 12$$
$$y = \frac{12}{5}$$

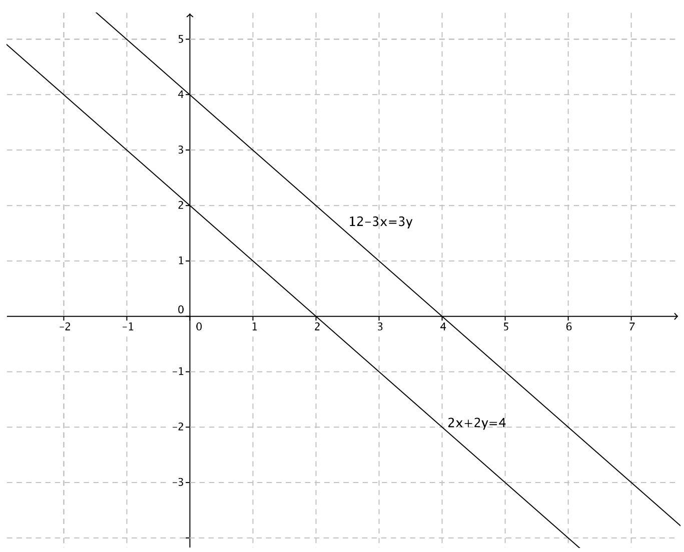
$$4x - 3\left(\frac{12}{5}\right) = 16$$
$$4x - \frac{36}{5} = 16$$
$$4x = \frac{116}{5}$$
$$x = \frac{29}{5}$$

The solution is $\left(\frac{29}{5}, \frac{12}{5}\right)$.



8. $\begin{cases} 2x + 2y = 4 \\ 12 - 3x = 3y \end{cases}$

The equations graph as distinct lines. The slopes of these two equations are the same, and the y-intercepts are different, which means they graph as parallel lines. Therefore, this system will have no solutions.

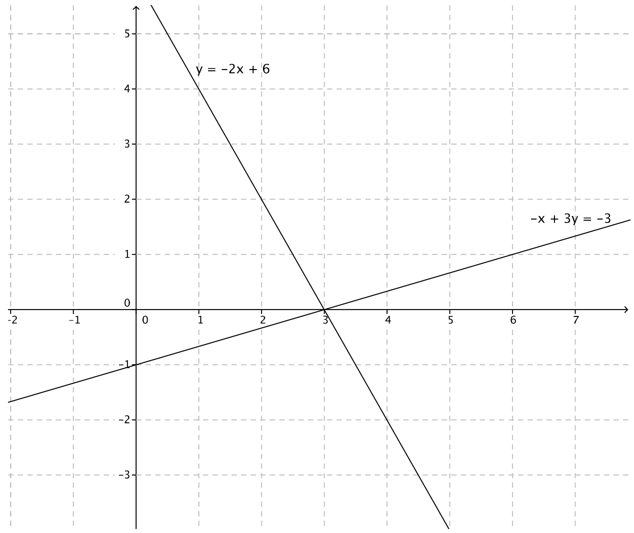


9. $\begin{cases} y = -2x + 6 \\ 3y = x - 3 \end{cases}$

$$3(y = -2x + 6)$$
$$3y = -6x + 18$$
$$\begin{cases} 3y = -6x + 18 \\ 3y = x - 3 \end{cases}$$
$$-6x + 18 = x - 3$$
$$18 = 7x - 3$$
$$21 = 7x$$
$$\frac{21}{7} = x$$
$$x = 3$$

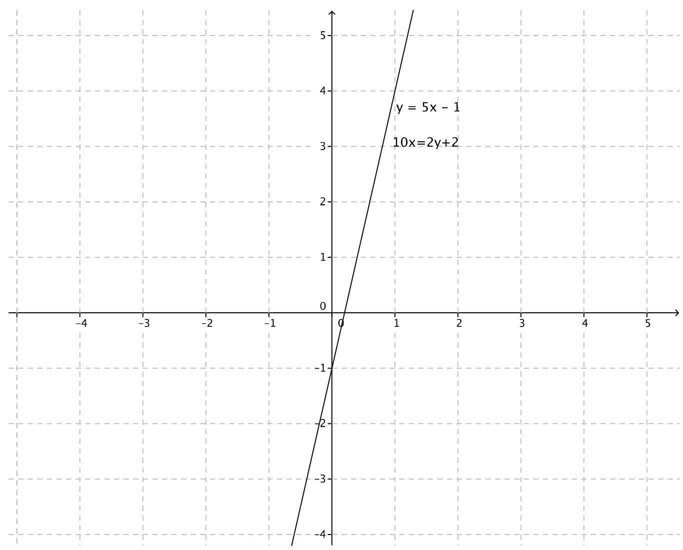
$$y = -2(3) + 6$$
$$y = -6 + 6$$
$$y = 0$$

The solution is $(3, 0)$.



10. $\begin{cases} y = 5x - 1 \\ 10x = 2y + 2 \end{cases}$

These equations define the same line. Therefore, this system will have infinitely many solutions.

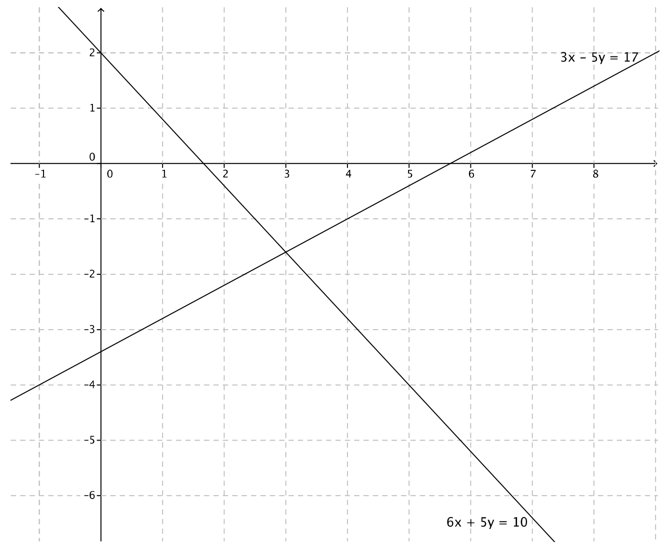


11. $\begin{cases} 3x - 5y = 17 \\ 6x + 5y = 10 \end{cases}$

$$3x - 5y + 6x + 5y = 17 + 10$$
$$9x = 27$$
$$x = 3$$

$$3(3) - 5y = 17$$
$$9 - 5y = 17$$
$$-5y = 8$$
$$y = -\frac{8}{5}$$

The solution is $\left(3, -\frac{8}{5}\right)$.

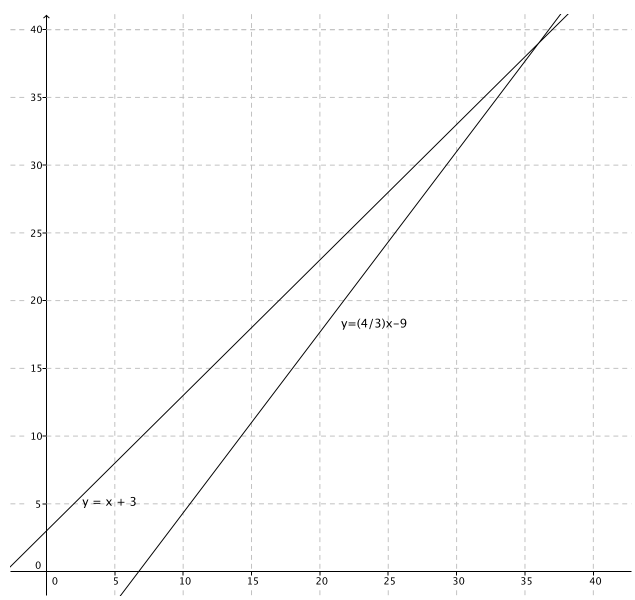


12. $\begin{cases} y = \frac{4}{3}x - 9 \\ y = x + 3 \end{cases}$

$$\frac{4}{3}x - 9 = x + 3$$
$$\frac{1}{3}x - 9 = 3$$
$$\frac{1}{3}x = 12$$
$$x = 36$$

$$y = 36 + 3$$
$$y = 39$$

The solution is $(36, 39)$.

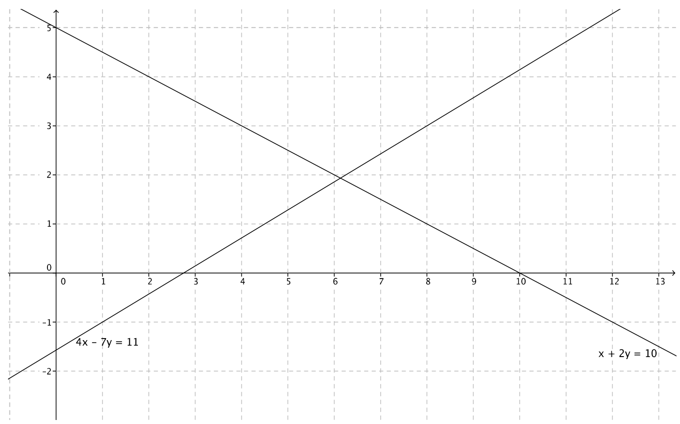


13. $\begin{cases} 4x - 7y = 11 \\ x + 2y = 10 \end{cases}$

$$-4(x + 2y = 10)$$
$$-4x - 8y = -40$$
$$4x - 7y - 4x - 8y = 11 - 40$$
$$-15y = -29$$
$$y = \frac{29}{15}$$

$$x + 2\left(\frac{29}{15}\right) = 10$$
$$x + \frac{58}{15} = 10$$
$$x = \frac{92}{15}$$

The solution is $\left(\frac{92}{15}, \frac{29}{15}\right)$*.*



14. $\begin{cases} 21x + 14y = 7 \\ 12x + 8y = 16 \end{cases}$

The slopes of these two equations are the same, and the y-intercepts are different, which means they graph as parallel lines. Therefore, this system will have no solution.