Lesson 8: Linear Equations in Disguise

Student Outcomes

- Students rewrite and solve equations that are not obviously linear equations using properties of equality.

Lesson Notes

In this lesson, students learn that some equations that may not look like linear equations are, in fact, linear. This lesson on solving rational equations is included because of the types of equations students will see in later topics of this module related to slope. Students will recognize these equations as proportions. It is not necessary to refer to these types of equations as equations that contain rational expressions. They can be referred to simply as proportions since students are familiar with this terminology. Expressions of this type will be treated carefully in algebra as they involve a discussion about why the denominator of such expressions cannot be equal to zero. That discussion is not included in this lesson.

Classwork

Concept Development (3 minutes)

- Some linear equations may not look like linear equations upon first glance. A simple example that you should recognize is

$$\frac{x}{5}=\frac{6}{12}.$$

- What do we call this kind of problem, and how do we solve it?
  - *This is a proportion. We can solve this by multiplying both sides of the equation by* 5*. We can also solve it by multiplying each numerator with the other fraction's denominator.*

Students may not think of multiplying each numerator with the other fraction's denominator because multiplying both sides by 5 requires fewer steps and uses the multiplication property of equality that has been used to solve other equations. If necessary, state the theorem and give a brief explanation.

THEOREM. Given rational numbers A, B, C, and D, so that $B \neq 0$ and $D \neq 0$, the property states

$$\text{If } \frac{A}{B}=\frac{C}{D}, \text{ then } AD = BC.$$

- To find the value of x, we can multiply each numerator by the other fraction's denominator.

$$\frac{x}{5}=\frac{6}{12}$$
$$12x = 6(5)$$

- It should be more obvious now that we have a linear equation. We can now solve it as usual using the properties of equality.

$$\begin{aligned} 12x &= 30 \\ x &= \frac{30}{12} \\ x &= \frac{5}{2} \end{aligned}$$

- In this lesson, our work will be similar, but the numerator and/or the denominator of the fractions may contain more than one term. However, the way we solve these kinds of problems remains the same.

Example 1 (5 minutes)

- Given a linear equation in disguise, we will *try* to solve it. To do so, we must first assume that the following equation is true for some number x.

$$\frac{x-1}{2} = \frac{x+\frac{1}{3}}{4}$$

- We want to make this equation look like the linear equations we are used to. For that reason, we will multiply both sides of the equation by 2 and 4, as we normally do with proportions:

$$2\left(x+\frac{1}{3}\right) = 4(x-1).$$

- Is this a linear equation? How do you know?
  - *Yes, this is a linear equation because the expressions on the left and right of the equal sign are linear expressions.*
- Notice that the expressions that contained more than one term were put in parentheses. We do that so we do not make a mistake and forget to use the distributive property.
- Now that we have a linear equation, we will use the distributive property and solve as usual.

$$\begin{aligned} 2\left(x+\frac{1}{3}\right) &= 4(x-1) \\ 2x+\frac{2}{3} &= 4x-4 \\ 2x-2x+\frac{2}{3} &= 4x-2x-4 \\ \frac{2}{3} &= 2x-4 \\ \frac{2}{3}+4 &= 2x-4+4 \\ \frac{14}{3} &= 2x \\ \frac{1}{2}\cdot\frac{14}{3} &= \frac{1}{2}\cdot 2x \\ \frac{7}{3} &= x \end{aligned}$$

- How can we verify that $\frac{7}{3}$ is the solution to the equation?
  - *We can replace x with $\frac{7}{3}$ in the original equation.*

$$\frac{x-1}{2}=\frac{x+\frac{1}{3}}{4}$$
$$\frac{\frac{7}{3}-1}{2}=\frac{\frac{7}{3}+\frac{1}{3}}{4}$$
$$\frac{\frac{4}{3}}{2}=\frac{\frac{8}{3}}{4}$$
$$4\left(\frac{4}{3}\right)=2\left(\frac{8}{3}\right)$$
$$\frac{16}{3}=\frac{16}{3}$$

- Since $\frac{7}{3}$ made the equation true, we know it is a solution to the equation.

Example 2 (4 minutes)

- Can we solve the following equation? Explain.

$$\frac{\frac{1}{5}-x}{7}=\frac{2x+9}{3}$$

  - *We need to multiply each numerator with the other fraction's denominator.*
- So,

$$\frac{\frac{1}{5}-x}{7}=\frac{2x+9}{3}$$
$$7(2x+9)=3\left(\frac{1}{5}-x\right).$$

- What would be the next step?
  - *Use the distributive property.*

- Now we have

$$\begin{aligned}
7(2x+9) &= 3\left(\frac{1}{5}-x\right)\\
14x+63 &= \frac{3}{5}-3x\\
14x+3x+63 &= \frac{3}{5}-3x+3x\\
17x+63 &= \frac{3}{5}\\
17x+63-63 &= \frac{3}{5}-63\\
17x &= \frac{3}{5}-\frac{315}{5}\\
17x &= -\frac{312}{5}\\
\frac{1}{17}(17x) &= \left(-\frac{312}{5}\right)\frac{1}{17}\\
x &= -\frac{312}{85}.
\end{aligned}$$

- Is this a linear equation? How do you know?
    - *Yes, this is a linear equation because the left and right side are linear expressions.*

Example 3 (5 minutes)

- Can we solve the following equation? If so, go ahead and solve it. If not, explain why not.

Example 3

Can this equation be solved?

$$\frac{6+x}{7x+\frac{2}{3}}=\frac{3}{8}$$

Give students a few minutes to work. Provide support to students as needed.

- *Yes, we can solve the equation because we can multiply each numerator with the other fraction's denominator and then use the distributive property to begin solving it.*

$$\frac{6+x}{7x+\frac{2}{3}}=\frac{3}{8}$$
$$(6+x)8=\left(7x+\frac{2}{3}\right)3$$
$$48+8x=21x+2$$
$$48+8x-8x=21x-8x+2$$
$$48=13x+2$$
$$48-2=13x+2-2$$
$$46=13x$$
$$\frac{46}{13}=x$$

Example 4 (5 minutes)

- Can we solve the following equation? If so, go ahead and solve it. If not, explain why not.

Example 4

Can this equation be solved?

$$\frac{7}{3x+9}=\frac{1}{8}$$

Give students a few minutes to work. Provide support to students as needed.

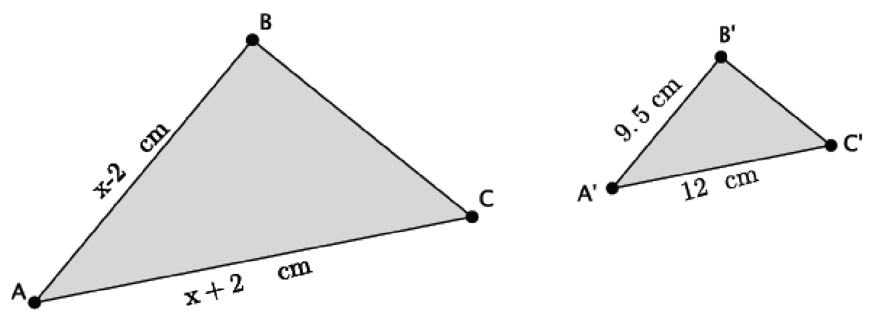
- *Yes, we can solve the equation because we can multiply each numerator with the other fraction's denominator and then use the distributive property to begin solving it.*

$$\frac{7}{3x+9}=\frac{1}{8}$$
$$7(8)=(3x+9)1$$
$$56=3x+9$$
$$56-9=3x+9-9$$
$$47=3x$$
$$\frac{47}{3}=x$$

Example 5 (5 minutes)

Example 5

In the diagram below, $\triangle ABC \sim \triangle A'B'C'$. Using what we know about similar triangles, we can determine the value of x.



- Begin by writing the ratios that represent the corresponding sides.

$$\frac{x-2}{9.5}=\frac{x+2}{12}$$

It is possible to write several different proportions in this case. If time, discuss this fact with students.

- Now that we have the ratios, solve for x and find the lengths of AB and AC.

$$\frac{x-2}{9.5}=\frac{x+2}{12}$$
$$(x-2)12=9.5(x+2)$$
$$12x-24=9.5x+19$$
$$12x-24+24=9.5x+19+24$$
$$12x=9.5x+43$$
$$12x-9.5x=9.5x-9.5x+43$$
$$2.5x=43$$
$$x=\frac{43}{2.5}$$
$$x=17.2$$

The length of $|AB|=15.2$ cm, and the length of $|AC|=19.2$ cm.

Exercises 1–4 (8 minutes)

Students complete Exercises 1–4 independently.

Exercises

Solve the following equations of rational expressions, if possible.

1. $\dfrac{2x+1}{9} = \dfrac{1-x}{6}$

$$\frac{2x+1}{9} = \frac{1-x}{6}$$
$$9(1-x) = (2x+1)6$$
$$9 - 9x = 12x + 6$$
$$9 - 9x + 9x = 12x + 9x + 6$$
$$9 = 21x + 6$$
$$9 - 6 = 21x + 6 - 6$$
$$3 = 21x$$
$$\frac{3}{21} = \frac{21}{21}x$$
$$\frac{1}{7} = x$$

2. $\dfrac{5+2x}{3x-1} = \dfrac{6}{7}$

$$\frac{5+2x}{3x-1} = \frac{6}{7}$$
$$(5+2x)7 = (3x-1)6$$
$$35 + 14x = 18x - 6$$
$$35 - 35 + 14x = 18x - 6 - 35$$
$$14x = 18x - 41$$
$$14x - 18x = 18x - 18x - 41$$
$$-4x = -41$$
$$\frac{-4}{-4}x = \frac{-41}{-4}$$
$$x = \frac{41}{4}$$

3. $\dfrac{x+9}{12} = \dfrac{-2x - \frac{1}{2}}{3}$

$$\frac{x+9}{12} = \frac{-2x - \frac{1}{2}}{3}$$
$$12\left(-2x - \frac{1}{2}\right) = (x+9)3$$
$$-24x - 6 = 3x + 27$$
$$-24x + 24x - 6 = 3x + 24x + 27$$
$$-6 = 27x + 27$$
$$-6 - 27 = 27x + 27 - 27$$
$$-33 = 27x$$
$$\frac{-33}{27} = \frac{27}{27}x$$
$$-\frac{11}{9} = x$$

4. $\dfrac{8}{3-4x} = \dfrac{5}{2x+\frac{1}{4}}$

$$\frac{8}{3-4x} = \frac{5}{2x+\frac{1}{4}}$$
$$8\left(2x+\frac{1}{4}\right) = (3-4x)5$$
$$16x+2 = 15-20x$$
$$16x+2-2 = 15-2-20x$$
$$16x = 13-20x$$
$$16x+20x = 13-20x+20x$$
$$36x = 13$$
$$\frac{36}{36}x = \frac{13}{36}$$
$$x = \frac{13}{36}$$

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know that proportions that have more than one term in the numerator and/or denominator can be solved the same way we normally solve a proportion.
- When multiplying a fraction with more than one term in the numerator and/or denominator by a number, we should put the expressions with more than one term in parentheses so that we are less likely to forget to use the distributive property.

Lesson Summary

Some proportions are linear equations in disguise and are solved the same way we normally solve proportions.

When multiplying a fraction with more than one term in the numerator and/or denominator by a number, put the expressions with more than one term in parentheses so that you remember to use the distributive property when transforming the equation. For example:

$$\frac{x+4}{2x-5} = \frac{3}{5}$$
$$5(x+4) = 3(2x-5).$$

The equation $5(x+4) = 3(2x-5)$ is now clearly a linear equation and can be solved using the properties of equality.

Exit Ticket (5 minutes)

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Exit Ticket

Solve the following equations for x.

1. $\dfrac{5x-8}{3}=\dfrac{11x-9}{5}$

2. $\dfrac{x+11}{7}=\dfrac{2x+1}{-8}$

3. $\dfrac{-x-2}{-4}=\dfrac{3x+6}{2}$

Exit Ticket Sample Solutions

Solve the following equations for x.

1. $\dfrac{5x-8}{3}=\dfrac{11x-9}{5}$

$$\frac{5x-8}{3}=\frac{11x-9}{5}$$
$$5(5x-8)=3(11x-9)$$
$$25x-40=33x-27$$
$$25x-25x-40=33x-25x-27$$
$$-40=8x-27$$
$$-40+27=8x-27+27$$
$$-13=8x$$
$$-\frac{13}{8}=x$$

2. $\dfrac{x+11}{7}=\dfrac{2x+1}{-8}$

$$\frac{x+11}{7}=\frac{2x+1}{-8}$$
$$7(2x+1)=-8(x+11)$$
$$14x+7=-8x-88$$
$$14x+7-7=-8x-88-7$$
$$14x=-8x-95$$
$$14x+8x=-8x+8x-95$$
$$22x=-95$$
$$x=-\frac{95}{22}$$

3. $\dfrac{-x-2}{-4}=\dfrac{3x+6}{2}$

$$\frac{-x-2}{-4}=\frac{3x+6}{2}$$
$$-4(3x+6)=2(-x-2)$$
$$-12x-24=-2x-4$$
$$-12x-24+24=-2x-4+24$$
$$-12x=-2x+20$$
$$-12x+2x=-2x+2x+20$$
$$-10x=20$$
$$x=-2$$

Problem Set Sample Solutions

Students practice solving equations with rational expressions, if a solution is possible.

Solve the following equations of rational expressions, if possible. If the equation cannot be solved, explain why.

1. $\frac{5}{6x-2}=\frac{-1}{x+1}$

$$\begin{aligned}\frac{5}{6x-2}&=\frac{-1}{x+1}\\ 5(x+1)&=-1(6x-2)\\ 5x+5&=-6x+2\\ 5x+5-5&=-6x+2-5\\ 5x&=-6x-3\\ 5x+6x&=-6x+6x-3\\ 11x&=-3\\ x&=-\frac{3}{11}\end{aligned}$$

2. $\frac{4-x}{8}=\frac{7x-1}{3}$

$$\begin{aligned}\frac{4-x}{8}&=\frac{7x-1}{3}\\ 8(7x-1)&=(4-x)3\\ 56x-8&=12-3x\\ 56x-8+8&=12+8-3x\\ 56x&=20-3x\\ 56x+3x&=20-3x+3x\\ 59x&=20\\ \frac{59}{59}x&=\frac{20}{59}\\ x&=\frac{20}{59}\end{aligned}$$

3. $\frac{3x}{x+2}=\frac{5}{9}$

$$\begin{aligned}\frac{3x}{x+2}&=\frac{5}{9}\\ 9(3x)&=(x+2)5\\ 27x&=5x+10\\ 27x-5x&=5x-5x+10\\ 22x&=10\\ \frac{22}{22}x&=\frac{10}{22}\\ x&=\frac{5}{11}\end{aligned}$$

4. $\dfrac{\frac{1}{2}x+6}{3} = \dfrac{x-3}{2}$

$$\begin{aligned}
\frac{\frac{1}{2}x+6}{3} &= \frac{x-3}{2}\\
3(x-3) &= 2\left(\frac{1}{2}x+6\right)\\
3x-9 &= x+12\\
3x-9+9 &= x+12+9\\
3x &= x+21\\
3x-x &= x-x+21\\
2x &= 21\\
x &= \frac{21}{2}
\end{aligned}$$

5. $\dfrac{7-2x}{6} = \dfrac{x-5}{1}$

$$\begin{aligned}
\frac{7-2x}{6} &= \frac{x-5}{1}\\
6(x-5) &= (7-2x)1\\
6x-30 &= 7-2x\\
6x-30+30 &= 7+30-2x\\
6x &= 37-2x\\
6x+2x &= 37-2x+2x\\
8x &= 37\\
\frac{8}{8}x &= \frac{37}{8}\\
x &= \frac{37}{8}
\end{aligned}$$

6. $\dfrac{2x+5}{2} = \dfrac{3x-2}{6}$

$$\begin{aligned}
\frac{2x+5}{2} &= \frac{3x-2}{6}\\
2(3x-2) &= 6(2x+5)\\
6x-4 &= 12x+30\\
6x-4+4 &= 12x+30+4\\
6x &= 12x+34\\
6x-12x &= 12x-12x+34\\
-6x &= 34\\
x &= -\frac{34}{6}\\
x &= -\frac{17}{3}
\end{aligned}$$

7. $\dfrac{6x+1}{3} = \dfrac{9-x}{7}$

$$\frac{6x+1}{3} = \frac{9-x}{7}$$
$$(6x+1)7 = 3(9-x)$$
$$42x + 7 = 27 - 3x$$
$$42x + 7 - 7 = 27 - 7 - 3x$$
$$42x = 20 - 3x$$
$$42x + 3x = 20 - 3x + 3x$$
$$45x = 20$$
$$\frac{45}{45}x = \frac{20}{45}$$
$$x = \frac{4}{9}$$

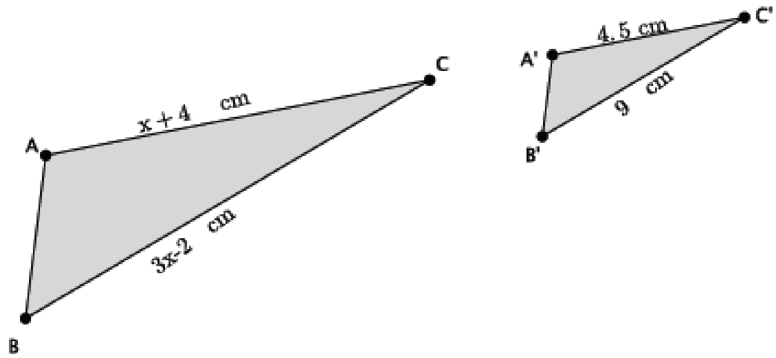
8. $\dfrac{\frac{1}{3}x - 8}{12} = \dfrac{-2-x}{15}$

$$\frac{\frac{1}{3}x - 8}{12} = \frac{-2-x}{15}$$
$$12(-2-x) = \left(\frac{1}{3}x - 8\right)15$$
$$-24 - 12x = 5x - 120$$
$$-24 - 12x + 12x = 5x + 12x - 120$$
$$-24 = 17x - 120$$
$$-24 + 120 = 17x - 120 + 120$$
$$96 = 17x$$
$$\frac{96}{17} = \frac{17}{17}x$$
$$\frac{96}{17} = x$$

9. $\dfrac{3-x}{1-x} = \dfrac{3}{2}$

$$\frac{3-x}{1-x} = \frac{3}{2}$$
$$(3-x)2 = (1-x)3$$
$$6 - 2x = 3 - 3x$$
$$6 - 2x + 2x = 3 - 3x + 2x$$
$$6 = 3 - x$$
$$6 - 3 = 3 - 3 - x$$
$$3 = -x$$
$$-3 = x$$

10. In the diagram below, $\triangle ABC \sim \triangle A'B'C'$. Determine the lengths of AC and BC.



$$\frac{x+4}{4.5} = \frac{3x-2}{9}$$
$$9(x+4) = 4.5(3x-2)$$
$$9x + 36 = 13.5x - 9$$
$$9x + 36 + 9 = 13.5x - 9 + 9$$
$$9x + 45 = 13.5x$$
$$9x - 9x + 45 = 13.5x - 9x$$
$$45 = 4.5x$$
$$10 = x$$

The length of $|AC| = 14$ cm, and the length of $|BC| = 28$ cm.