Lesson 2: Linear and Nonlinear Expressions in x

Student Outcomes

- Students know the properties of linear and nonlinear expressions in x.
- Students transcribe and identify expressions as linear or nonlinear.

Classwork

Discussion (4 minutes)

- A symbolic statement in x with an equal sign is called an equation in x. The equal sign divides the equation into two parts, the left side and the right side. The two sides are called *expressions*.
- For sake of simplicity, we will only discuss expressions in x, but know that we can write expressions in any symbol.
- The following chart contains both linear and nonlinear expressions in x. Sort them into two groups and be prepared to explain what is different about the two groups.

MP.3

$5x + 3$	$-8x + \frac{7}{9} - 3$	$9 - x^2$
$4x^2 - 9$	$0.31x + 7 - 4.2x$	$\left(\frac{x}{2}\right)^3 + 1$
$11(x + 2)$	$-(6 - x) + 15 - 9x$	$7 + x^{-4} + 3x$

Linear expressions are noted in red in the table below.

$5x + 3$	$-8x + \frac{7}{9} - 3$	$9 - x^2$
$4x^2 - 9$	$0.31x + 7 - 4.2x$	$\left(\frac{x}{2}\right)^3 + 1$
$11(x + 2)$	$-(6 - x) + 15 - 9x$	$7 + x^{-4} + 3x$

- Identify which equations you placed in each group. Explain your reasoning for grouping the equations.
  - *Equations that contained an exponent of x other than 1 were put into one group. The other equations were put into another group. That seemed to be the only difference between the types of equations given.*
- Linear expressions in x are special types of expressions. Linear expressions are expressions that are sums of constants and products of a constant and x raised to a power of 0, which simplifies to a value of 1, or a power of 1. Nonlinear expressions are also sums of constants and products of a constant and a power of x. However, nonlinear expressions will have a power of x that is not equal to 1 or 0.
- The reason we want to be able to distinguish linear expressions from nonlinear expressions is because we will soon be solving linear equations. Nonlinear equations will be a set of equations you learn to solve in Algebra I, though we will begin to solve simple nonlinear equations later this year (Module 7). We also want to be able to recognize linear equations in order to predict the shape of their graph, which is a concept we will learn more about later in this module.

Example 1 (3 minutes)

- A linear expression in x is an expression where each term is either a constant or a product of a constant and x. For example, the expression $(57 - x)$ is a linear expression. However, the expression $2x^2 + 9x + 5$ is not a linear expression. Why is $2x^2 + 9x + 5$ not a linear expression in x?
  - *Students should say that $2x^2 + 9x + 5$ is not a linear expression because the terms of linear expressions must either be a constant or the product of a constant and x. The term $2x^2$ does not fit the definition of a linear expression in x.*

Scaffolding:

- Terms are any product of an integer power of x and a constant, or just a constant.
- Constants are fixed numbers.
- When a term is the product of a constant(s) and a power of x, the constant is called a coefficient.

Example 2 (4 minutes)

- Let's examine the expression $4 + 3x^5$ more deeply. To begin, we want to identify the terms of the expression. How many terms are there, and what are they?
  - *There are two terms, 4 and $3x^5$.*
- How many terms are comprised of just constants, and what are they?
  - *There is one constant term, 4.*
- How many terms have coefficients, and what are they?
  - *There is one term with a coefficient, 3.*
- Is $4 + 3x^5$ a linear or nonlinear expression in x? Why or why not?
  - *The expression $4 + 3x^5$ is a nonlinear expression in x because it is the sum of a constant and the product of a constant and positive integer power of $x > 1$.*

Example 3 (4 minutes)

- How many terms does the expression $7x + 9 + 6 + 3x$ have? What are they?
  - *As is, this expression has 4 terms: $7x$, 9, 6, and $3x$.*
- This expression can be transformed using some of our basic properties of numbers. For example, if we apply the commutative property of addition, we can rearrange the terms from $7x + 9 + 6 + 3x$ to $7x + 3x + 9 + 6$. Then, we can apply the associative property of addition:

$$(7x + 3x) + (9 + 6).$$

  Next, we apply the distributive property:

$$(7 + 3)x + (9 + 6).$$

  Finally,

$$10x + 15.$$

- How many terms with coefficients does the expression $10x + 15$ have? What are they?
  - *The expression has one term with a coefficient, $10x$. For this term, the coefficient is 10.*
- Is $10x + 15$ a linear or nonlinear expression in x? Why or why not?
  - *The expression $10x + 15$ is a linear expression in x because it is the sum of constants and products that contain x to the 1^{st} power.*

Example 4 (2 minutes)

- How many terms does the expression $5 + 9x \cdot 7 + 2x^9$ have? What are they?
  - *The expression has three terms:* 5, $9x \cdot 7$, *and* $2x^9$.
- How many terms with coefficients does the expression $5 + 9x \cdot 7 + 2x^9$ have? What are they?
  - *The expression has two terms with coefficients:* $63x$ *and* $2x^9$. *The coefficients are* 63 *and* 2.
- Is $5 + 9x \cdot 7 + 2x^9$ a linear or nonlinear expression in x? Why or why not?
  - *The expression* $5 + 9x \cdot 7 + 2x^9$ *is a nonlinear expression in* x *because it is the sum of constants and products that contain* x *raised to a power that is greater than* 1.

Example 5 (2 minutes)

- Is $94 + x + 4x^{-6} - 2$ a linear or nonlinear expression in x? Why or why not?
  - *Students may first say that it is neither a linear nor a nonlinear expression in* x *because of the* -2. *Remind them that subtraction can be rewritten as a sum, i.e.,* $+(-2)$; *therefore, this expression does fit the definition of nonlinear.*

Example 6 (2 minutes)

- Is the expression $x^1 + 9x - 4$ a linear expression in x?
  - *Yes,* $x^1 + 9x - 4$ *is a linear expression in* x *because* x^1 *is the same as* x.
- What powers of x are acceptable in the definition of a linear expression in x?
  - *Only the power of* 1 *is acceptable because* x^1*is, by definition, just* x.

Exercises 1–12 (14 minutes)

Students complete Exercises 1–12 independently.

Exercises

Write each of the following statements in Exercises 1–12 as a mathematical expression. State whether or not the expression is linear or nonlinear. If it is nonlinear, then explain why.

1. **The sum of a number and four times the number.**

   Let x be a number; then, $x + 4x$ is a linear expression.

2. **The product of five and a number.**

   Let x be a number; then, $5x$ is a linear expression.

3. **Multiply six and the reciprocal of the quotient of a number and seven.**

   Let x be a number; then, $6 \cdot \frac{7}{x}$ is a nonlinear expression. The expression is nonlinear because the number $\frac{7}{x} = 7 \cdot \frac{1}{x} = 7 \cdot x^{-1}$. The exponent of the x is the reason it is not a linear expression.

4. Twice a number subtracted from four times a number, added to 15.

   Let x be a number; then, $15 + (4x - 2x)$ is a linear expression.

5. The square of the sum of six and a number.

   Let x be a number; then, $(x + 6)^2$ is a nonlinear expression. When you multiply $(x + 6)^2$, you get $x^2 + 12x + 36$. The x^2 is the reason it is not a linear expression.

6. The cube of a positive number divided by the square of the same positive number.

   Let x be a number; then, $\frac{x^3}{x^2}$ is a nonlinear expression. However, if you simplify the expression to x, then it is linear.

7. The sum of four consecutive numbers.

   Let x be the first number; then, $x + (x + 1) + (x + 2) + (x + 3)$ is a linear expression.

8. Four subtracted from the reciprocal of a number.

   Let x be a number; then, $\frac{1}{x} - 4$ is a nonlinear expression. The term $\frac{1}{x}$ is the same as x^{-1}, which is why this expression is not linear. It is possible that a student may let x be the reciprocal of a number, $\frac{1}{x}$, which would make the expression linear.

9. Half of the product of a number multiplied by itself three times.

   Let x be a number; then, $\frac{1}{2} \cdot x \cdot x \cdot x$ is a nonlinear expression. The term $\frac{1}{2} \cdot x \cdot x \cdot x$ is the same as $\frac{1}{2}x^3$, which is why this expression is not linear.

10. The sum that shows how many pages Maria read if she read 45 pages of a book yesterday and $\frac{2}{3}$ of the remaining pages today.

    Let x be the number of remaining pages of the book; then, $45 + \frac{2}{3}x$ is a linear expression.

11. An admission fee of $\$10$ plus an additional $\$2$ per game.

    Let x be the number of games; then, $10 + 2x$ is a linear expression.

12. Five more than four times a number and then twice that sum.

    Let x be the number; then, $2(4x + 5)$ is a linear expression.

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We have definitions for linear and nonlinear expressions.
- We know how to use the definitions to identify expressions as linear or nonlinear.
- We can write expressions that are linear and nonlinear.

> **Lesson Summary**
>
> **<u>Linear expressions</u> are sums of constants and products of constants and x raised to a power of 0 or 1. For example, $4+3x$, $7x+x-15$, and $\frac{1}{2}x+7-2$ are all linear expressions in x.**
>
> **<u>Nonlinear expressions</u> are also sums of constants and products of constants and x raised to a power that is not 0 or 1. For example, $2x^2-9$, $-6x^{-3}+8+x$, and $\frac{1}{x}+8$ are all nonlinear expressions in x.**

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 2: Linear and Nonlinear Expressions in x

Exit Ticket

Write each of the following statements as a mathematic expression. State whether the expression is a linear or nonlinear expression in x.

1. Seven subtracted from five times a number, and then the difference added to nine times a number.

2. Three times a number subtracted from the product of fifteen and the reciprocal of a number.

3. Half of the sum of two and a number multiplied by itself three times.

Exit Ticket Sample Solutions

Write each of the following statements as a mathematic expression. State whether the expression is a linear or nonlinear expression in x.

1. **Seven subtracted from five times a number, and then the difference added to nine times a number.**

   Let x be a number; then, $(5x - 7) + 9x$. The expression is a linear expression.

2. **Three times a number subtracted from the product of fifteen and the reciprocal of a number.**

   Let x be a number; then, $15 \cdot \frac{1}{x} - 3x$. The expression is a nonlinear expression.

3. **Half of the sum of two and a number multiplied by itself three times.**

   Let x be a number; then, $\frac{1}{2}(2 + x^3)$. The expression is a nonlinear expression.

Problem Set Sample Solutions

Students practice writing expressions and identifying them as linear or nonlinear.

Write each of the following statements as a mathematic expression. State whether the expression is linear or nonlinear. If it is nonlinear, then explain why.

1. **A number decreased by three squared.**

   Let x be a number; then, $x - 3^2$ is a linear expression.

2. **The quotient of two and a number, subtracted from seventeen.**

   Let x be a number; then, $17 - \frac{2}{x}$ is a nonlinear expression. The term $\frac{2}{x}$ is the same as $2 \cdot \frac{1}{x}$ and $\frac{1}{x} = x^{-1}$, which is why it is not linear.

3. **The sum of thirteen and twice a number.**

   Let x be a number; then, $13 + 2x$ is a linear expression.

4. **5.2 more than the product of seven and a number.**

   Let x be a number; then, $5.2 + 7x$ is a linear expression.

5. **The sum that represents the number of tickets sold if 35 tickets were sold Monday, half of the remaining tickets were sold on Tuesday, and 14 tickets were sold on Wednesday.**

   Let x be the remaining number of tickets; then, $35 + \frac{1}{2}x + 14$ is a linear expression.

6. The product of 19 and a number, subtracted from the reciprocal of the number cubed.

   Let x be a number; then, $\frac{1}{x^3} - 19x$ is a nonlinear expression. The term $\frac{1}{x^3}$ is the same as x^{-3}, which is why it is not linear.

7. The product of 15 and a number, and then the product multiplied by itself four times.

   Let x be a number; then, $(15x)^4$ is a nonlinear expression. The expression can be written as $15^4 \cdot x^4$. The exponent of 4 with a base of x is the reason it is not linear.

8. A number increased by five and then divided by two.

   Let x be a number; then, $\frac{x+5}{2}$ is a linear expression.

9. Eight times the result of subtracting three from a number.

   Let x be a number; then, $8(x-3)$ is a linear expression.

10. The sum of twice a number and four times a number subtracted from the number squared.

    Let x be a number; then, $x^2 - (2x + 4x)$ is a nonlinear expression. The term x^2 is the reason it is not linear.

11. One-third of the result of three times a number that is increased by 12.

    Let x be a number; then, $\frac{1}{3}(3x + 12)$ is a linear expression.

12. Five times the sum of one-half and a number.

    Let x be a number; then, $5\left(\frac{1}{2} + x\right)$ is a linear expression.

13. Three-fourths of a number multiplied by seven.

    Let x be a number; then, $\frac{3}{4}x \cdot 7$ is a linear expression.

14. The sum of a number and negative three, multiplied by the number.

    Let x be a number; then, $\left(x + (-3)\right)x$ is a nonlinear expression because $\left(x + (-3)\right)x = x^2 - 3x$ after using the distributive property. It is nonlinear because the power of x in the term x^2 is greater than 1.

15. The square of the difference between a number and 10.

    Let x be a number; then, $(x - 10)^2$ is a nonlinear expression because $(x - 10)^2 = x^2 - 20x + 100$. The term x^2 is a positive power of $x > 1$; therefore, this is a not a linear expression.