Lesson 13: Angle Sum of a Triangle

Student Outcomes

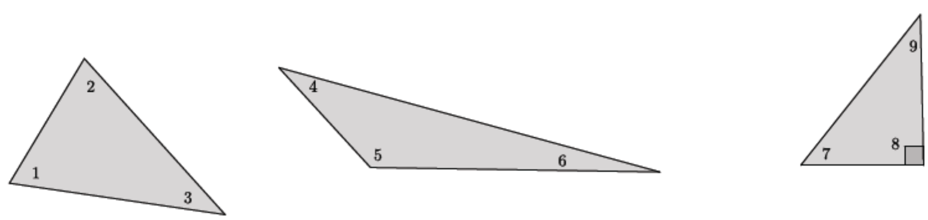
- Students know the angle sum theorem for triangles; the sum of the interior angles of a triangle is always $180°$.
- Students present informal arguments to draw conclusions about the angle sum of a triangle.

Classwork

Concept Development (3 minutes)

- The angle sum theorem for triangles states that the sum of the interior angles of a triangle is always $180°$ ($\angle$ sum of $\triangle$).
- It does not matter what kind of triangle it is (i.e., acute, obtuse, right); when you add the measure of the three angles, you always get a sum of $180°$.

Concept Development



$$\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 5 + \angle 6 = \angle 7 + \angle 8 + \angle 9 = 180$$

Note that the sum of angles 7 and 9 must equal $90°$ because of the known right angle in the right triangle.

We want to prove that the angle sum of any triangle is $180°$. To do so, we will use some facts that we already know about geometry:

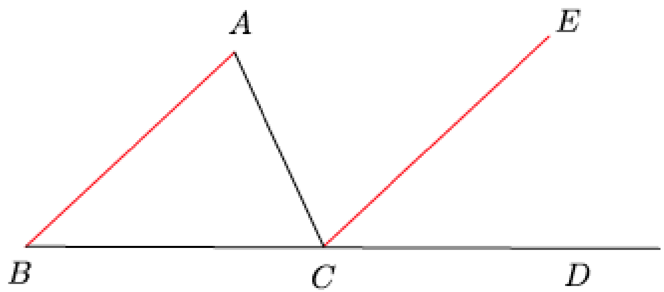
- A straight angle is $180°$ in measure.
- Corresponding angles of parallel lines are equal in measure (corr. $\angle s$, $\overline{AB} \parallel \overline{CD}$).
- Alternate interior angles of parallel lines are equal in measure (alt. $\angle s$, $\overline{AB} \parallel \overline{CD}$).

Exploratory Challenge 1 (13 minutes)

Provide students 10 minutes of work time. Once the 10 minutes have passed, review the solutions with the students before moving on to Exploratory Challenge 2.

Exploratory Challenge 1

Let triangle ABC be given. On the ray from B to C, take a point D so that C is between B and D. Through point C, draw a line parallel to AB, as shown. Extend the parallel lines AB and CE. Line AC is the transversal that intersects the parallel lines.



a. **Name the three interior angles of triangle ABC.**

$\angle ABC$, $\angle BAC$, $\angle BCA$

MP.3

b. **Name the straight angle.**

$\angle BCD$

Our goal is to show that the three interior angles of triangle ABC are equal to the angles that make up the straight angle. We already know that a straight angle is $180°$ in measure. If we can show that the interior angles of the triangle are the same as the angles of the straight angle, then we will have proven that the interior angles of the triangle have a sum of $180°$.

c. **What kinds of angles are $\angle ABC$ and $\angle ECD$? What does that mean about their measures?**

$\angle ABC$ and $\angle ECD$ are corresponding angles. Corresponding angles of parallel lines are equal in measure (corr. $\angle$s, $\overline{AB} \parallel \overline{CE}$).

d. **What kinds of angles are $\angle BAC$ and $\angle ECA$? What does that mean about their measures?**

$\angle BAC$ and $\angle ECA$ are alternate interior angles. Alternate interior angles of parallel lines are equal in measure (alt. $\angle$s, $\overline{AB} \parallel \overline{CE}$).

e. **We know that $\angle BCD = \angle BCA + \angle ECA + \angle ECD = 180°$. Use substitution to show that the three interior angles of the triangle have a sum of $180°$.**

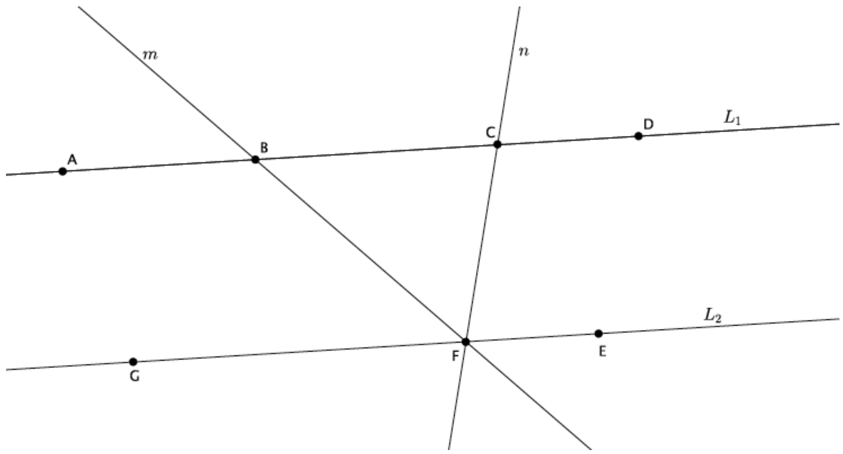
$\angle BCD = \angle BCA + \angle BAC + \angle ABC = 180°$ ($\angle$ sum of $\triangle$).

Exploratory Challenge 2 (20 minutes)

Provide students 15 minutes of work time. Once the 15 minutes have passed, review the solutions with the students.

Exploratory Challenge 2

The figure below shows parallel lines L_1 and L_2. Let m and n be transversals that intersect L_1 at points B and C, respectively, and L_2 at point F, as shown. Let A be a point on L_1 to the left of B, D be a point on L_1 to the right of C, G be a point on L_2 to the left of F, and E be a point on L_2 to the right of F.



a. **Name the triangle in the figure.**

$\triangle BCF$

b. **Name a straight angle that will be useful in proving that the sum of the interior angles of the triangle is $180°$.**

$\angle GFE$

As before, our goal is to show that the interior angles of the triangle are equal to the straight angle. Use what you learned from Exploratory Challenge 1 to show that interior angles of a triangle have a sum of $180°$.

c. **Write your proof below.**

MP.3

The straight angle $\angle GFE$ is comprised of angles $\angle GFB$, $\angle BFC$, and $\angle EFC$. Alternate interior angles of parallel lines are equal in measure (alt. $\angle$s, $\overline{AD} \parallel \overline{CE}$). For that reason, $\angle BCF = \angle EFC$ and $\angle CBF = \angle GFB$. Since $\angle GFE$ is a straight angle, it is equal to $180°$. Then, $\angle GFE = \angle GFB + \angle BFC + \angle EFC = 180°$. By substitution, $\angle GFE = \angle CBF + \angle BFC + \angle BCF = 180°$. Therefore, the sum of the interior angles of a triangle is $180°$ ($\angle$ sum of $\triangle$).

Closing (4 minutes)

Summarize, or have students summarize, the lesson.

- All triangles have a sum of interior angles equal to $180°$.
- We can prove that a triangle has a sum of interior angles equal to that of a straight angle using what we know about alternate interior angles and corresponding angles of parallel lines.

Lesson Summary

All triangles have a sum of interior angles equal to $180°$.

The proof that a triangle has a sum of interior angles equal to $180°$ is dependent upon the knowledge of straight angles and angle relationships of parallel lines cut by a transversal.

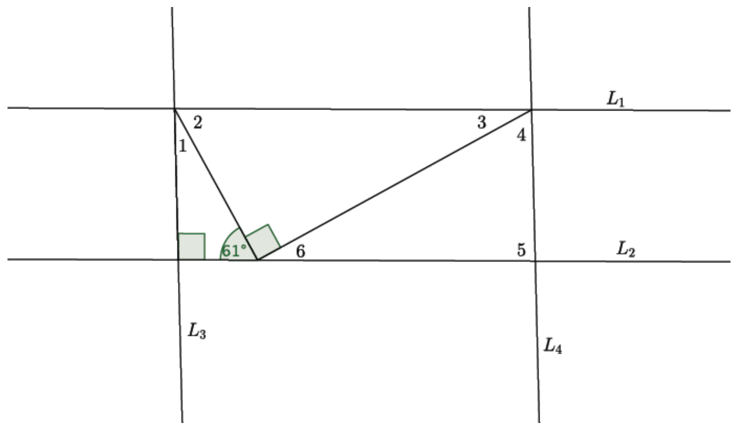
Exit Ticket (5 minutes)

Name ____ Date ____________________

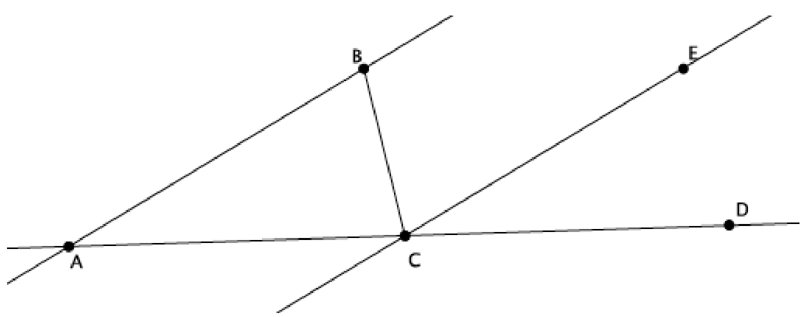
Lesson 13: Angle Sum of a Triangle

Exit Ticket

1. If $L_1 \parallel L_2$, and $L_3 \parallel L_4$, what is the measure of $\angle 1$? Explain how you arrived at your answer.

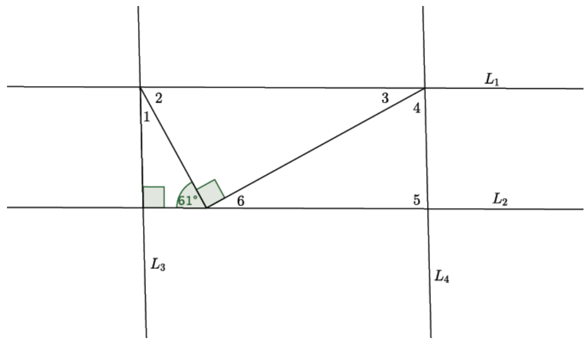


2. Given Line AB is parallel to Line CE, present an informal argument to prove that the interior angles of triangle ABC have a sum of $180°$.



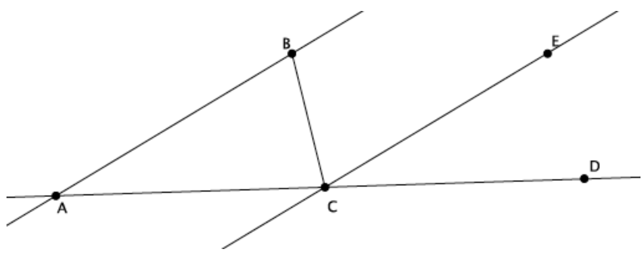
Exit Ticket Sample Solutions

1. **If $L_1 \parallel L_2$, and $L_3 \parallel L_4$, what is the measure of $\angle 1$? Explain how you arrived at your answer.**



The measure of angle 1 is $29°$. I know that the angle sum of triangles is $180°$. I already know that two of the angles of the triangle are $90°$ and $61°$.

2. **Given Line AB is parallel to Line CE, present an informal argument to prove that the interior angles of triangle ABC have a sum of $180°$.**

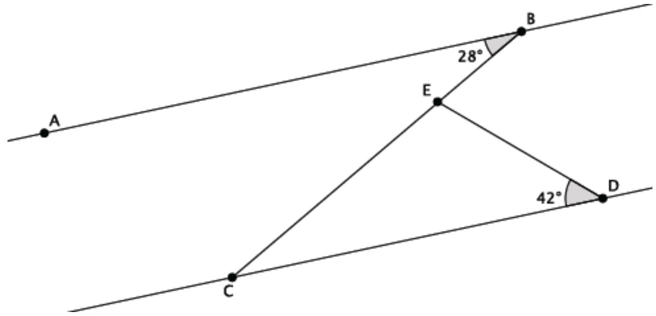


Since AB is parallel to CE, then the corresponding angles $\angle BAC$ and $\angle ECD$ are equal in measure. Similarly, angles $\angle ABC$ and $\angle ECB$ are equal in measure because they are alternate interior angles. Since $\angle ACD$ is a straight angle, i.e., equal to $180°$ in measure, substitution shows that triangle ABC has a sum of $180°$. Specifically, the straight angle is made up of angles $\angle ACB$, $\angle ECB$, and $\angle ECD$. $\angle ACB$ is one of the interior angles of the triangle and one of the angles of the straight angle. We know that angle $\angle ABC$ has the same measure as angle $\angle ECB$ and that angle $\angle BAC$ has the same measure as $\angle ECD$. Therefore, the sum of the interior angles will be the same as the angles of the straight angle, which is $180°$.

Problem Set Sample Solutions

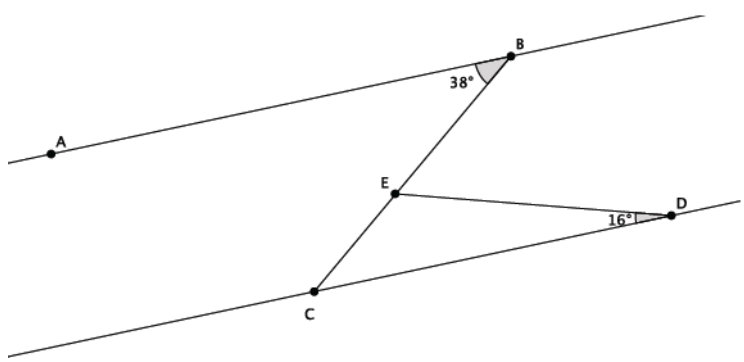
Students practice presenting informal arguments about the sum of the angles of a triangle using the theorem to find the measures of missing angles.

1. **In the diagram below, line AB is parallel to line CD, i.e., $L_{AB} \parallel L_{CD}$. The measure of angle $\angle ABC = 28°$, and the measure of angle $\angle EDC = 42°$. Find the measure of angle $\angle CED$. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle.**



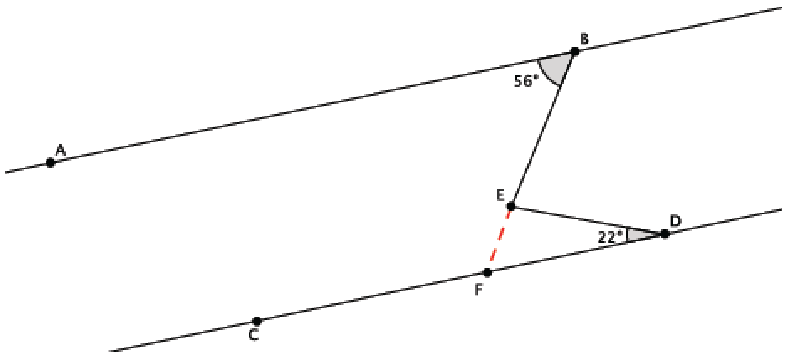
The measure of angle $\angle CED = 110°$. This is the correct measure for the angle because $\angle ABC$ and $\angle DCE$ are alternate interior angles of parallel lines. That means that the angles are congruent and have the same measure. Since the angle sum of a triangle is $180°$, then the measure of $\angle CED = 180° - (28° + 42°) = 110°$.

2. **In the diagram below, line AB is parallel to line CD, i.e., $L_{AB} \parallel L_{CD}$. The measure of angle $\angle ABE = 38°$, and the measure of angle $\angle EDC = 16°$. Find the measure of angle $\angle BED$. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle. (Hint: Find the measure of angle $\angle CED$ first, and then use that measure to find the measure of angle $\angle BED$.)**



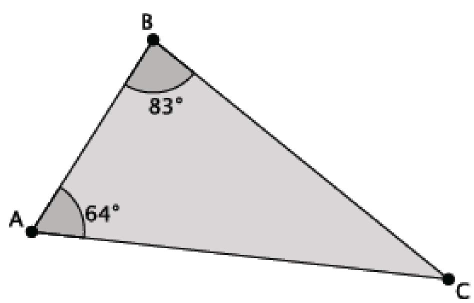
The measure of angle $\angle BED = 54°$. This is the correct measure for the angle because $\angle ABC$ and $\angle DCE$ are alternate interior angles of parallel lines. That means that the angles are congruent and have the same measure. Since the angle sum of a triangle is $180°$, then the measure of $\angle CED = 180° - (38° + 16°) = 126°$. The straight angle $\angle BEC$ is made up of $\angle CED$ and $\angle BED$. Since we know straight angles are $180°$ in measure, and angle $\angle CED = 126°$, then $\angle BED = 54°$.

3. In the diagram below, line AB is parallel to line CD, i.e., $L_{AB} \parallel L_{CD}$. The measure of angle $\angle ABE = 56°$, and the measure of angle $\angle EDC = 22°$. Find the measure of angle $\angle BED$. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle. (Hint: Extend the segment BE so that it intersects line CD.)



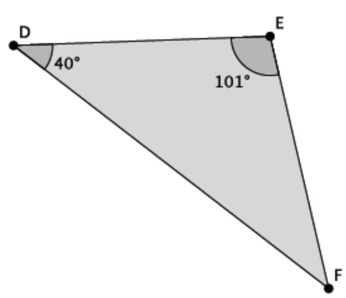
The measure of angle $\angle BED = 78°$. This is the correct measure for the angle because $\angle ABE$ and $\angle DFE$ are alternate interior angles of parallel lines. That means that the angles are congruent and have the same measure. Since the angle sum of a triangle is $180°$, then the measure of $\angle FED = 180° - (56° + 22°) = 102°$. The straight angle $\angle BEF$ is made up of $\angle FED$ and $\angle BED$. Since straight angles are $180°$ in measure, and angle $\angle FED = 102°$, then $\angle BED = 78°$.

4. What is the measure of $\angle ACB$?



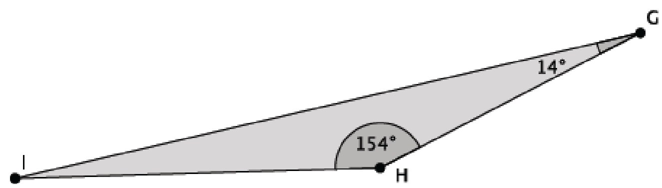
The measure of $\angle ACB$ is $180° - (83° + 64°) = 33°$.

5. What is the measure of $\angle EFD$?



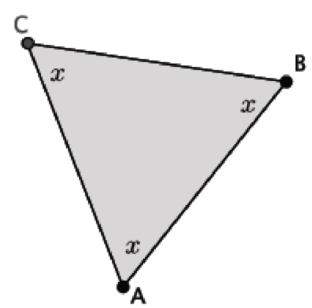
The measure of $\angle EFD$ is $180° - (101° + 40°) = 39°$.

6. What is the measure of $\angle HIG$?



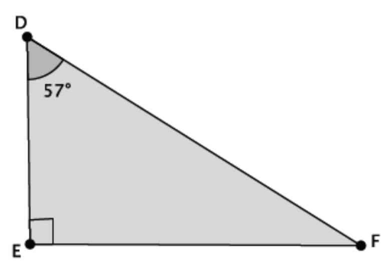
The measure of $\angle HIG$ is $180° - (154° + 14°) = 12°$.

7. What is the measure of $\angle ABC$?



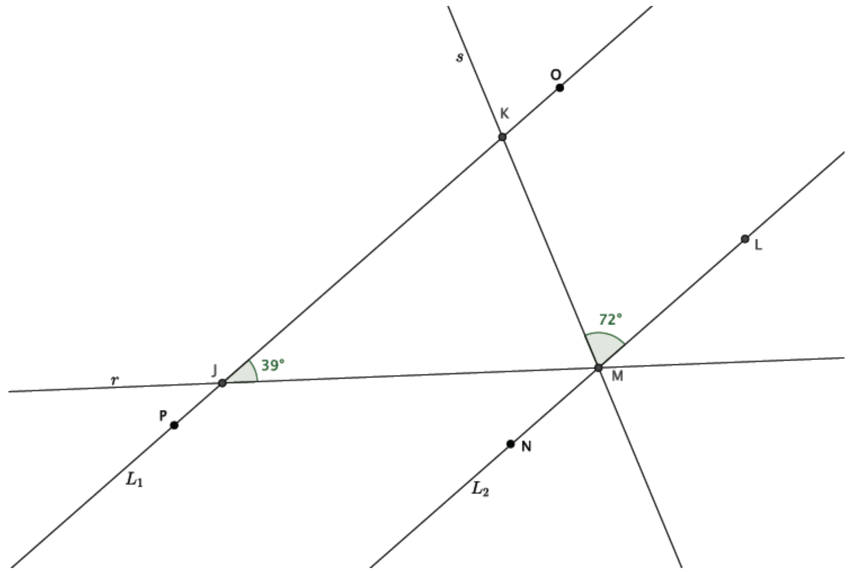
The measure of $\angle ABC$ is $60°$ because $60 + 60 + 60 = 180$.

8. Triangle DEF is a right triangle. What is the measure of $\angle EFD$?



The measure of $\angle EFD$ is $90° - 57° = 33°$.

9. In the diagram below, lines L_1 and L_2 are parallel. Transversals r and s intersect both lines at the points shown below. Determine the measure of $\angle JMK$. Explain how you know you are correct.



The lines L_1 and L_2 are parallel, which means that the alternate interior angles formed by the transversals are equal. Specifically, $\angle LMK = \angle JKM = 72°$. Since triangle ΔJKM has a sum of interior angles equal to $180°$, then $\angle KJM + \angle JMK + \angle JKM = 180°$. By substitution, we have $39 + \angle JMK + 72 = 180$; therefore, $\angle JMK = 69°$.