|  |
| --- |
|  |

Lesson 13: Angle Sum of a Triangle

**Student Outcomes**

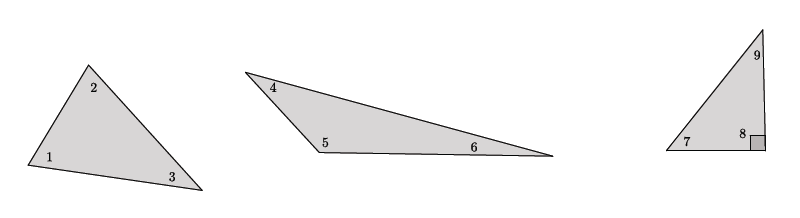
* Students know the angle sum theorem for triangles; the sum of the interior angles of a triangle is always .
* Students present informal arguments to draw conclusions about the angle sum of a triangle.

Classwork

Concept Development (3 minutes)

* The angle sum theorem for triangles states that the sum of the interior angles of a triangle is always   
  ( sum of ).
* It does not matter what kind of triangle it is (i.e., acute, obtuse, right); when you add the measure of the three angles, you always get a sum of .

**Concept Development**

****

**Note that the sum of angles and must equal because of the known right angle in the right triangle.**

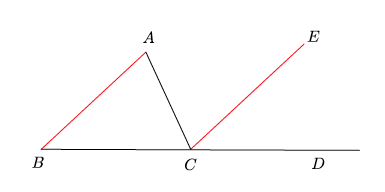
We want to prove that the angle sum of any triangle is . To do so, we will use some facts that we already know about geometry:

* A straight angle is in measure.
* Corresponding angles of parallel lines are equal in measure (corr. , ).
* Alternate interior angles of parallel lines are equal in measure (alt. , ).

**Exploratory Challenge 1 (13 minutes)**

Provide students 10 minutes of work time. Once the 10 minutes have passed, review the solutions with the students before moving on to Exploratory Challenge 2.

Exploratory Challenge 1

Let triangle be given. On the ray from to , take a point so that is between and . Through point , draw a line parallel to , as shown. Extend the parallel lines and *.* Line is the transversal that intersects the parallel lines.

* 1. Name the three interior angles of triangle .

**,,**

* 1. Name the straight angle.

Our goal is to show that the three interior angles of triangle are equal to the angles that make up the straight angle. We already know that a straight angle is in measure. If we can show that the interior angles of the triangle are the same as the angles of the straight angle, then we will have proven that the interior angles of the triangle have a sum of .

* 1. What kinds of angles are and ? What does that mean about their measures?

**MP.3**

and are corresponding angles. Corresponding angles of parallel lines are equal in measure (corr. , ).

* 1. What kinds of angles are and ? What does that mean about their measures?

and are alternate interior angles. Alternate interior angles of parallel lines are equal in measure (alt. , ).

* 1. We know that . Use substitution to show that the three interior angles of the triangle have a sum of .

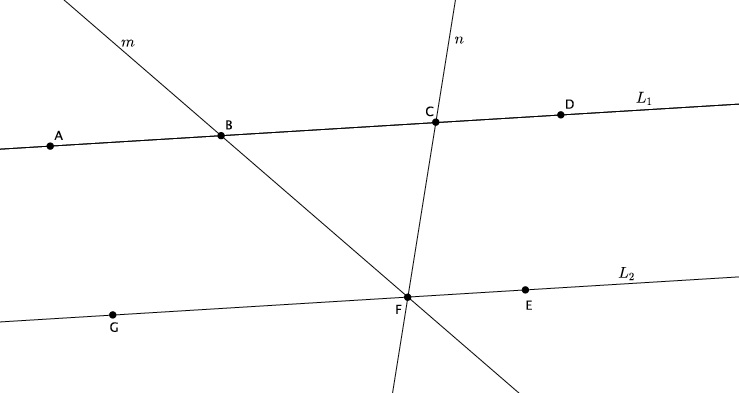
( sum of )**.**

Exploratory Challenge 2 (20 minutes)

Provide students 15 minutes of work time. Once the 15 minutes have passed, review the solutions with the students.

Exploratory Challenge

The figure below shows parallel lines and . Let and be transversals that intersect at points and , respectively, and at point , as shown. Let be a point on to the left of , be a point on to the right of , be a point on to the left of , and be a point on to the right of .



* 1. Name the triangle in the figure.
  2. Name a straight angle that will be useful in proving that the sum of the interior angles of the triangle is .

As before, our goal is to show that the interior angles of the triangle are equal to the straight angle. Use what you learned from Exploratory Challenge 1 to show that interior angles of a triangle have a sum of .

* 1. Write your proof below.

The straight angle is comprised of angles , , and . Alternate interior angles of parallel lines are equal in measure (alt. , ). For that reason, and. Since is a straight angle, it is equal to . Then, . By substitution, . Therefore, the sum of the interior angles of a triangle is (sum of )**.**

**MP.3**

Closing (4 minutes)

Summarize, or have students summarize, the lesson.

* All triangles have a sum of interior angles equal to .
* We can prove that a triangle has a sum of interior angles equal to that of a straight angle using what we know about alternate interior angles and corresponding angles of parallel lines.

Lesson Summary

**All triangles have a sum of interior angles equal to .**

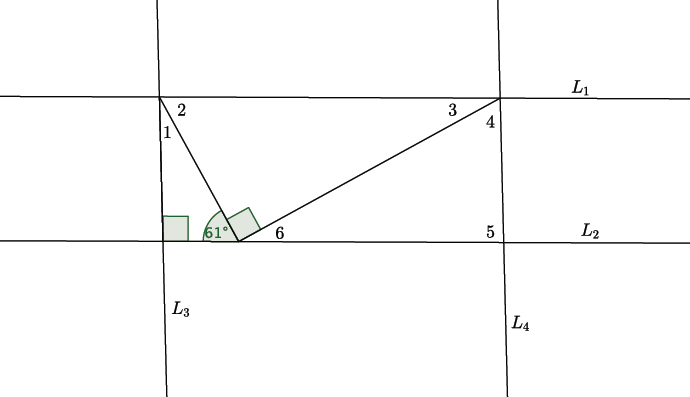
**The proof that a triangle has a sum of interior angles equal to is dependent upon the knowledge of straight angles and angle relationships of parallel lines cut by a transversal.**

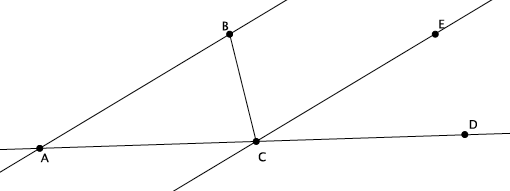
Exit Ticket (5 minutes)

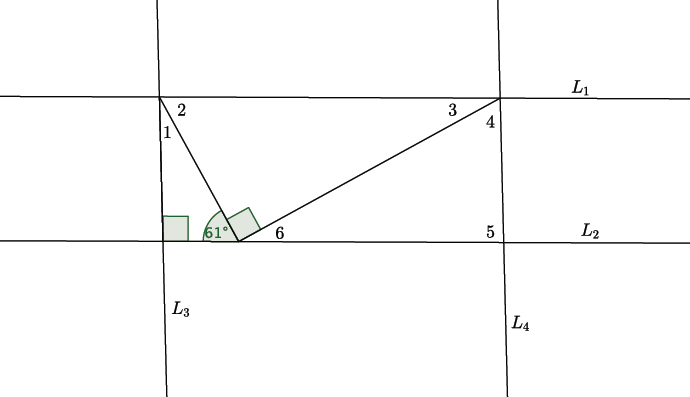
Name Date

Lesson 13: Angle Sum of a Triangle

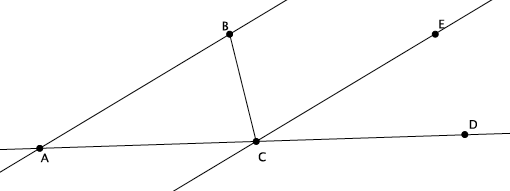
Exit Ticket

1. If , and , what is the measure of ? Explain how you arrived at your answer.
2. Given Line is parallel to Line , present an informal argument to prove that the interior angles of triangle have a sum of .

Exit Ticket Sample Solutions

1. If , and , what is the measure of ? Explain how you arrived at your answer.

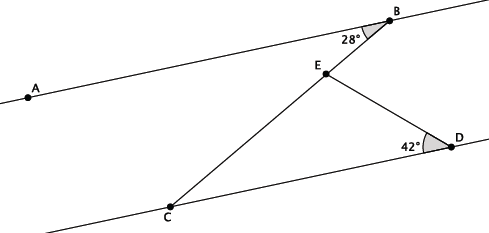
The measure of angle is . I know that the angle sum of triangles is . I already know that two of the angles of the triangle are and .

1. Given Line is parallel to Line *,* present an informal argument to prove that the interior angles of triangle have a sum of .

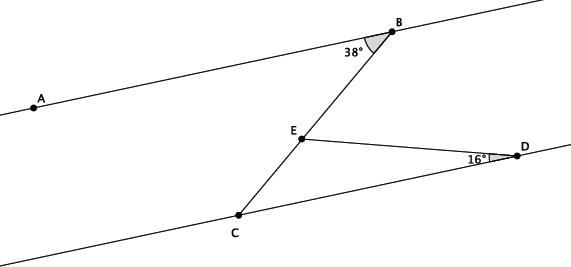
Since is parallel to , then the corresponding angles and are equal in measure. Similarly, angles and are equal in measure because they are alternate interior angles. Since is a straight angle, i.e., equal to in measure, substitution shows that triangle has a sum of . Specifically, the straight angle is made up of angles , , and . is one of the interior angles of the triangle and one of the angles of the straight angle. We know that angle has the same measure as angle and that angle has the same measure as . Therefore, the sum of the interior angles will be the same as the angles of the straight angle, which is .

Problem Set Sample Solutions

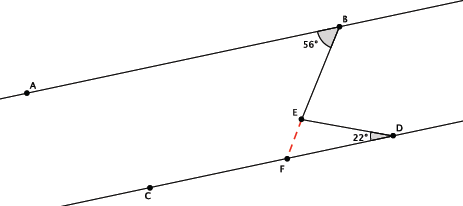
Students practice presenting informal arguments about the sum of the angles of a triangle using the theorem to find the measures of missing angles.

1. In the diagram below, line is parallel to line , i.e., . The measure of angle , and the measure of angle . Find the measure of angle . Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle.

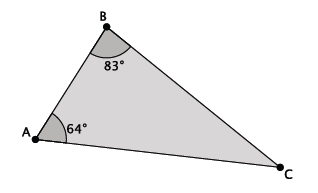
The measure of angle . This is the correct measure for the angle because and are alternate interior angles of parallel lines. That means that the angles are congruent and have the same measure. Since the angle sum of a triangle is , then the measure of .

1. In the diagram below, line is parallel to line , i.e., . The measure of angle , and the measure of angle . Find the measure of angle . Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle. (Hint: Find the measure of angle first, and then use that measure to find the measure of angle .)

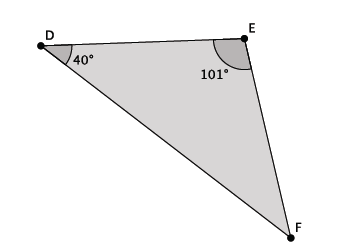
The measure of angle . This is the correct measure for the angle because and are alternate interior angles of parallel lines. That means that the angles are congruent and have the same measure. Since the angle sum of a triangle is , then the measure of . The straight angle is made up of and . Since we know straight angles are in measure, and angle , then .

1. In the diagram below, line is parallel to line , i.e., . The measure of angle , and the measure of angle . Find the measure of angle . Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle. (Hint: Extend the segment so that it intersects line *.*)

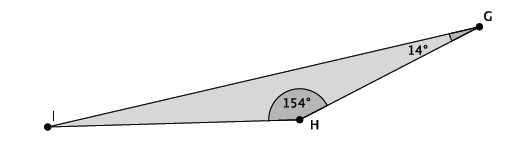
The measure of angle . This is the correct measure for the angle because and are alternate interior angles of parallel lines. That means that the angles are congruent and have the same measure. Since the angle sum of a triangle is , then the measure of . The straight angle is made up of and . Since straight angles are in measure, and angle , then .

1. What is the measure of ?

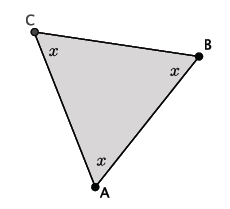
The measure of is .

1. What is the measure of ?

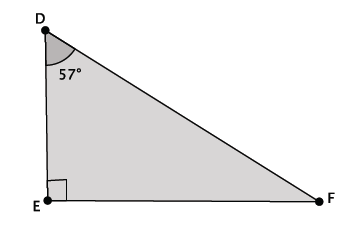
The measure of is .

1. What is the measure of ?

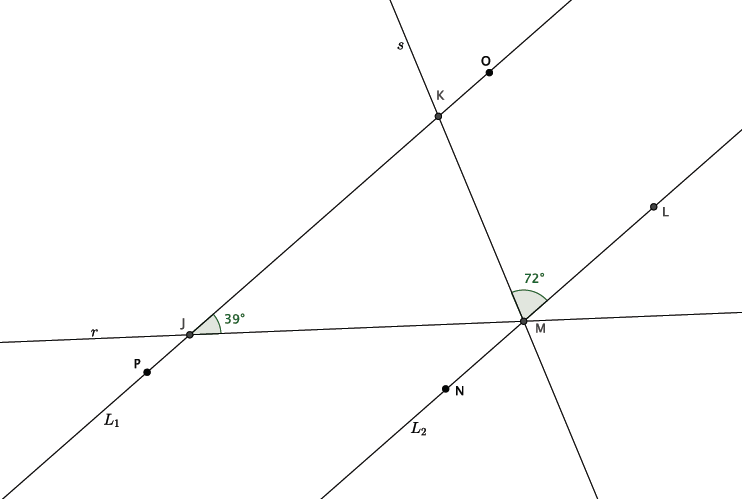
The measure of is .

1. What is the measure of ?

The measure of is because .

1. Triangle is a right triangle. What is the measure of ?

The measure of is .

1. In the diagram below, lines and are parallel. Transversals and intersect both lines at the points shown below. Determine the measure of . Explain how you know you are correct.

The lines and are parallel, which means that the alternate interior angles formed by the transversals are equal. Specifically, . Since triangle has a sum of interior angles equal to , then . By substitution, we have ; therefore, .