Lesson 12: Angles Associated with Parallel Lines

Student Outcomes

- Students know that corresponding angles, alternate interior angles, and alternate exterior angles of parallel lines are equal. Students know that when these pairs of angles are equal, then lines are parallel.
- Students know that corresponding angles of parallel lines are equal because of properties related to translation. Students know that alternate interior angles of parallel lines are equal because of properties related to rotation.
- Students present informal arguments to draw conclusions about angles formed when parallel lines are cut by a transversal.

Classwork

Exploratory Challenge 1 (7 minutes)

Students complete the Exploratory Challenge individually or in pairs. Students will investigate the properties of angles formed by two non-parallel lines cut by a transversal.

Exploratory Challenge 1

In the figure below, L_1 is not parallel to L_2, and m is a transversal. Use a protractor to measure angles 1–8. Which, if any, are equal? Explain why. (Use your transparency if needed.)

∠1 = ∠3, ∠2 = ∠4, ∠5 = ∠7, and ∠6 = ∠8. The pairs of angles listed are equal because they are vertical angles. Vertical angles are always equal because a rotation of 180° around the vertex of the angle will map it to its opposite angle.

Discussion (5 minutes)

Discuss what the students noticed about the angles in the first diagram with non-parallel lines. Ask students the following questions during the discussion.

- What did you notice about the pairs of angles in the first diagram when the lines, L_1 and L_2, were not parallel?
  - $\angle 1 = \angle 3, \angle 2 = \angle 4, \angle 5 = \angle 7$, and $\angle 6 = \angle 8$. *Vertical angles were equal in measure.*
- Why are vertical angles equal in measure?
  - *We can rotate the angle around its vertex $180°$, and it maps onto its opposite angle. Since rotations are angle-preserving, it means that the angles are equal in measure.*
- Angles that are on the same side of the transversal in corresponding positions (above each of L_1 and L_2 or below each of L_1 and L_2) are called **corresponding angles**. Name a pair of corresponding angles in the diagram. (Note to teacher: Have students name all pairs of corresponding angles from the diagram, one pair at a time.)
  - *$\angle 1$ and $\angle 5$, $\angle 4$ and $\angle 8$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$*
- When angles are on opposite sides of the transversal and between (inside) the lines L_1 and L_2, they are called **alternate interior angles**. Name a pair of alternate interior angles. (Note to teacher: Have students name both pairs of alternate interior angles from the diagram, one pair at a time.)
  - *$\angle 4$ and $\angle 6$, $\angle 3$ and $\angle 5$*
- When angles are on opposite sides of the transversal and outside of the parallel lines (above L_1 and below L_2), they are called **alternate exterior angles**. Name a pair of alternate exterior angles. (Note to teacher: Have students name both pairs of alternate exterior angles from the diagram, one pair at a time.)
  - *$\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$*

Exploratory Challenge 2 (9 minutes)

Students complete the Exploratory Challenge individually or in pairs. Students will investigate the properties of angles formed by two parallel lines cut by a transversal.

Exploratory Challenge 2

In the figure below, $L_1 \parallel L_2$, and m is a transversal. Use a protractor to measure angles 1–8. List the angles that are equal in measure.

$\angle 1 = \angle 3 = \angle 5 = \angle 7$ *and* $\angle 2 = \angle 4 = \angle 6 = \angle 8$

a. **What did you notice about the measures of ∠1 and ∠5? Why do you think this is so? (Use your transparency if needed.)**

∠1 and ∠5 are equal in measure. We can translate ∠1 along a vector on line m so that the vertex of ∠1 maps onto the vertex of ∠5. Translations are angle-preserving, so the two angles will coincide.

b. **What did you notice about the measures of ∠3 and ∠7? Why do you think this is so? (Use your transparency if needed.) Are there any other pairs of angles with this same relationship? If so, list them.**

∠3 and ∠7 are equal in measure. We can translate ∠3 along a vector on line m so that the vertex of ∠3 maps onto the vertex of ∠7. Translations are angle-preserving, so the two angles will coincide. Other pairs of angles with this same relationship are ∠4 and ∠8, and ∠2 and ∠6.

c. **What did you notice about the measures of ∠4 and ∠6? Why do you think this is so? (Use your transparency if needed.) Is there another pair of angles with this same relationship?**

The measures of ∠4 and ∠6 are equal. A rotation of $180°$ around a center would map ∠4 to ∠6. Rotations are angle-preserving, so we know that ∠4 and ∠6 are equal. ∠3 and ∠5 have the same relationship.

Discussion (15 minutes)

Discuss what the students noticed about the angles in the second diagram with parallel lines. Ask students the following questions during the discussion.

- We are going to discuss what you observed when you measured and compared angles in Exploratory Challenge 2. What we will do is make some informal arguments to prove some of the things you noticed. Each time you answer "why," "how do you know," or "explain," you are making an informal argument.

- Were the vertical angles in Exploratory Challenge 2 equal like they were in Exploratory Challenge 1? Why?

  □ *Yes, for the same reason they were equal in the first diagram, rotation.*

- What other angles were equal in the second diagram when the lines L_1 and L_2 were parallel?

  □ *∠1 = ∠3 = ∠5 = ∠7 and ∠2 = ∠4 = ∠6 = ∠8*

- Let's look at just ∠1 and ∠5. What kind of angles are these, and how do you know?

  □ *They are corresponding angles because they are on the same side of the transversal and in corresponding positions (i.e., above each of L_1 and L_2 or below each of L_1 and L_2).*

- We have already said that these two angles are equal in measure. Who can explain why this is so?

  □ *Translation along a vector in line m will map ∠1 onto ∠5. Translations preserve degrees of angles, so the two angles are equal in measure.*

Note to teacher: Insert and label a point at the vertex of each angle, points A and B, for example. Draw the vector between the two points. Trace one of the angles on a transparency, and demonstrate to students that a translation along a vector of exactly length AB would map one angle onto the other. Further, if there is a translation along vector $\overrightarrow{AB}$, then $Translation(L_1) = L_2$. The vector $\overrightarrow{AB}$ on line m is also translated, but remains on line m. Therefore, all of the angles formed by the intersection of L_1 and m are translated along $\overrightarrow{AB}$ according to the translation. That is why the corresponding angles are equal in measure, i.e., $Translation(∠1) = ∠5$.

- What did you notice about ∠3 and ∠7?

  □ *These two angles were also equal in measure for the same reason as ∠1 and ∠5.*

- What other pairs of corresponding angles are in the diagram?

  □ *∠4 and ∠8, and ∠2 and∠6*

- In Exploratory Challenge 1, the pairs of corresponding angles we named were not equal in measure. Given the information provided about each diagram, can you think of why this is so?
    - *In the first diagram, the lines L_1 and L_2 were not parallel. A translation of one of the angles would not map onto the other angle.*
- Are ∠4 and ∠6 corresponding angles? If not, why not?
    - *No, they are not on the same side of the transversal in corresponding locations.*
- What kind of angles are ∠4 and ∠6? How do you know?
    - *∠4 and ∠6 are alternate interior angles because they are on opposite sides of the transversal and inside the lines L_1 and L_2.*
- We have already said that ∠4 and ∠6 are equal in measure. Why do you think this is so?
    - *You can use rotation to map one of the angles onto the other. Rotations are angle-preserving.*

Note to teacher: Mark the midpoint, point M for example, of the segment between the vertices of the two angles. Trace one of the angles on a transparency and demonstrate to students that a 180° rotation around the point M would map one of the angles onto the other. Further, we know that a 180° rotation of a line around a point not on the line maps to a parallel line. If we let there be the rotation of 180° around point M, then $Rotation(L_1) = L_2$. The rotation of point M maps to itself under rotation, and the line containing M, line m, will also map to itself under rotation. For that reason, the angles formed at the intersection of L_1 and line m, under rotation, will map to angles at the intersection of L_2 and line m, but on the other side of the transversal, specifically, $Rotation(∠4) = ∠6$.

- Name another pair of alternate interior angles.
    - *∠3 and ∠5*
- In Exploratory Challenge 1, the pairs of alternate interior angles we named were not equal in measure. Given the information provided about each diagram, can you think of why this is so?
    - *In the first diagram, the lines L_1 and L_2 were not parallel. A rotation around a point would not map one angle onto the other angle.*
- Are ∠1 and ∠7 corresponding angles? If not, why not?
    - *No, they are not on the same side of the transversal, so they cannot be corresponding angles.*
- Are ∠1 and ∠7 alternate interior angles? If not, why not?
    - *No, they are not inside of the lines L_1 and L_2, so they cannot be interior angles.*
- What kind of angles are ∠1 and ∠7?
    - *They are alternate exterior angles because they are on opposite sides of the transversal and above L_1 and below L_2.*
- Name another pair of alternate exterior angles.
    - *∠2 and ∠8*
- These pairs of alternate exterior angles were not equal in measure in Exploratory Challenge 1. Given the information provided about each diagram, can you think of why this is so?
    - *In the first diagram, the lines L_1 and L_2 were not parallel. A rotation of one of the angles would not map onto the other angle.*
- If you know that pairs of corresponding angles, alternate interior angles, and alternate exterior angles are congruent, what do you think is true about the lines?
    - *The lines are parallel when pairs of corresponding angles, alternate interior angles, and alternate exterior angles are congruent.*

State the following theorem and its converse:

- <u>Theorem</u>: When parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent, the pairs of alternate interior angles are congruent, and the pairs of alternate exterior angles are congruent.
- The <u>converse</u> of the theorem states that if you know that corresponding angles are congruent, then you can be sure that the lines cut by a transversal are parallel. How could you phrase the converse of the theorem with respect to other types of angles we have learned?
    - *When alternate interior angles are congruent, then the lines cut by a transversal are parallel.*
    - *When alternate exterior angles are congruent, then the lines cut by a transversal are parallel.*

Closing (4 minutes)

Provide students two minutes to fill in the example portion of the lesson summary in the student materials.

Summarize, or have students summarize, the lesson.

- When a pair of parallel lines are cut by a transversal, then any corresponding angles, any alternate interior angles, and any alternate exterior angles are equal in measure.
- The reason that specific pairs of angles are equal is because of the properties we learned about the basic rigid motions, specifically that they are angle-preserving.
- When a pair of non-parallel lines are cut by a transversal, any corresponding angles, any alternate interior angles, and any alternate exterior angles are not equal.

Lesson Summary

Angles that are on the same side of the transversal in corresponding positions (above each of L_1 and L_2 or below each of L_1 and L_2) are called <u>corresponding angles</u>. For example, $\angle 2$ and $\angle 4$ are corresponding angles.

When angles are on opposite sides of the transversal and between (inside) the lines L_1 and L_2, they are called <u>alternate interior angles</u>. For example, $\angle 3$ and $\angle 7$ are alternate interior angles.

When angles are on opposite sides of the transversal and outside of the parallel lines (above L_1 and below L_2), they are called <u>alternate exterior angles</u>. For example, $\angle 1$ and $\angle 5$ are alternate exterior angles.

When parallel lines are cut by a transversal, any corresponding angles, any alternate interior angles, and any alternate exterior angles are equal in measure. If the lines are not parallel, then the angles are not equal in measure.
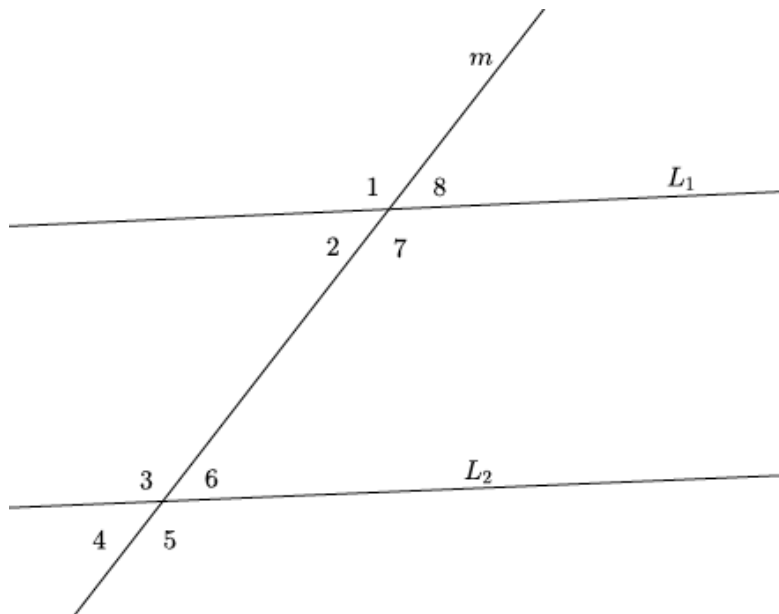

Exit Ticket (5 minutes)

Name _____   Date _____

Lesson 12: Angles Associated with Parallel Lines

Exit Ticket

Use the diagram to answer Questions 1 and 2. In the diagram, lines L_1 and L_2 are intersected by transversal m, forming angles 1–8, as shown.
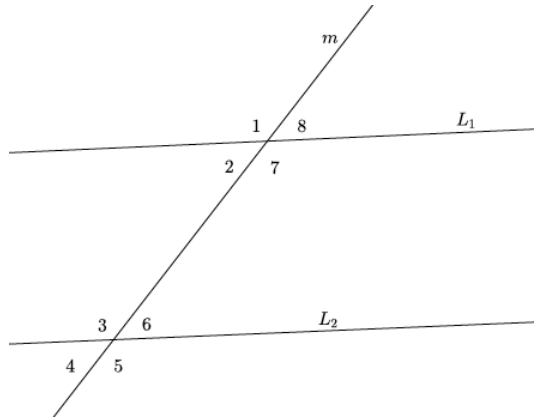

1. If $L_1 \parallel L_2$, what do know about $\angle 2$ and $\angle 6$? Use informal arguments to support your claim.

2. If $L_1 \parallel L_2$, what do know about $\angle 1$ and $\angle 3$? Use informal arguments to support your claim.

Exit Ticket Sample Solutions

Use the diagram to answer Questions 1 and 2. In the diagram, lines L_1 and L_2 are intersected by transversal m, forming angles 1–8, as shown.

1. **If $L_1 \parallel L_2$, what do know about $\angle 2$ and $\angle 6$. Use informal arguments to support your claim.**

   They are alternate interior angles because they are on opposite sides of the transversal and inside of lines L_1 and L_2. Also, the angles are equal in measure because the lines L_1 and L_2 are parallel. If we rotated angle 2 around the midpoint of the segment between the parallel lines, then it would map onto angle 6.

2. **If $L_1 \parallel L_2$, what do know about $\angle 1$ and $\angle 3$? Use informal arguments to support your claim.**

   They are corresponding angles because they are on the same side of the transversal and above each of lines L_1 and L_2. Also, the angles are equal in measure because the lines L_1 and L_2 are parallel. If we translated angle 1 along a vector (the same length as the segment between the parallel lines), then it would map onto angle 3.

Problem Set Sample Solutions

Students practice identifying corresponding, alternate interior, and alternate exterior angles from a diagram.

Use the diagram below to do Problems 1–6.
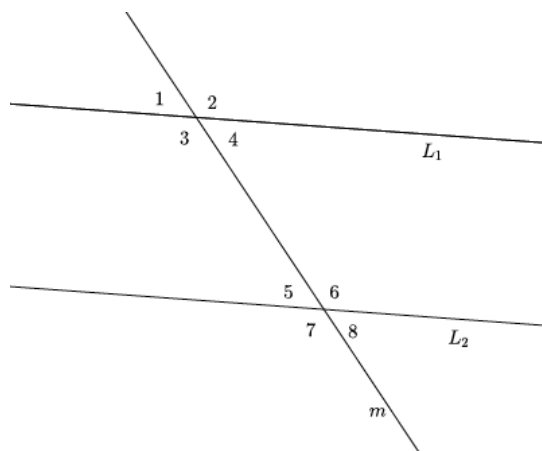
1. **Identify all pairs of corresponding angles. Are the pairs of corresponding angles equal in measure? How do you know?**

    ∠1 and ∠5, ∠4 and ∠8, ∠2 and ∠6, ∠3 and ∠7

    There is no information provided about the lines in the diagram being parallel. For that reason, we do not know if the pairs of corresponding angles are equal. If we knew the lines were parallel, we could use translation to map one angle onto another.

2. **Identify all pairs of alternate interior angles. Are the pairs of alternate interior angles equal in measure? How do you know?**

    ∠4 and ∠5, ∠3 and ∠6

    There is no information provided about the lines in the diagram being parallel. For that reason, we do not know if the pairs of alternate interior angles are equal in measure. If the lines were parallel, we could use rotation to show that the pairs of angles would map onto one another proving they are equal in measure.

3. **Use an informal argument to describe why ∠1 and ∠8 are equal in measure if $L_1 \parallel L_2$.**

    The reason that angle 1 and angle 8 are equal in measure when the lines are parallel is because you can rotate around the midpoint of the segment between the parallel lines. A rotation would then map angle 1 onto angle 8, showing that they are congruent and equal in measure.

4. **Assuming $L_1 \parallel L_2$ if the measure of ∠4 is 73°, what is the measure of ∠8? How do you know?**

    The measure of ∠8 is 73°. This must be true because they are corresponding angles of parallel lines.

5. **Assuming $L_1 \parallel L_2$, if the measure of ∠3 is 107° degrees, what is the measure of ∠6? How do you know?**

    The measure of ∠6 is 107°. This must be true because they are alternate interior angles of parallel lines.

6. **Assuming $L_1 \parallel L_2$, if the measure of ∠2 is 107°, what is the measure of ∠7? How do you know?**

    The measure of ∠7 is 107°. This must be true because they are alternate exterior angles of parallel lines.

7. **Would your answers to Problems 4–6 be the same if you had not been informed that $L_1 \parallel L_2$? Why, or why not?**

    No. The fact that the lines are parallel is the reason we can state that specific pairs of angles are equal. We can use basic rigid motions to prove that angles associated with parallel lines have the property of being equal when they are corresponding, alternate interior, or alternate exterior angles. If the lines are not parallel, then we could still classify the angles, but we would not know anything about their measures.

8. **Use an informal argument to describe why ∠1 and ∠5 are equal in measure if $L_1 \parallel L_2$.**

    The reason that angle 1 and angle 5 are equal in measure when the lines are parallel is because you can translate along a vector equal in length of the segment between the parallel lines; then, angle 1 would map onto angle 5.

9. **Use an informal argument to describe why ∠4 and ∠5 are equal in measure if $L_1 \parallel L_2$.**

    The reason that angle 4 and angle 5 are equal in measure when the lines are parallel is because when you rotate angle 4 around the midpoint of the segment between the parallel lines, angle 4 will map onto angle 5.

10. **Assume that L_1 is not parallel to L_2. Explain why ∠3 ≠ ∠7.**

    If the lines are not parallel, then all we know about angle 3 and angle 7 is that they are corresponding angles. If the lines are parallel, we could use translation to map one angle onto the other to show that they are equal in measure. However, we are to assume that the lines are not parallel, which means that their corresponding angles will not be equal in measure.