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Lesson 6: Rotations of 180 Degrees

Student Outcomes

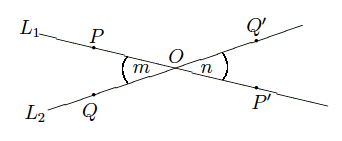
* Students learn that a rotation of degrees moves a point on the coordinate plane to .
* Students learn that a rotation of degrees around a point, not on the line, produces a line parallel to the given line.

Classwork

**Example 1 (5 minutes)**

* Rotations of degrees are special. Recall, a rotation of degrees around is a rigid motion so that if is any point in the plane, , , and are collinear (i.e., they lie on the same line).
* Rotations of degrees occur in many situations. For example, the frequently cited fact that vertical angles [vert. ] at the intersection of two lines are equal, follows immediately from the fact that -degree rotations are angle-preserving. More precisely, let two lines and intersect at , as shown:

Example 1

The picture below shows what happens when there is a rotation of around center .

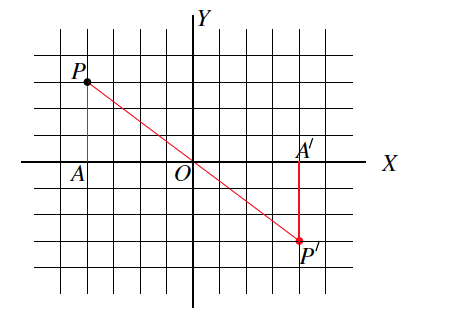
* We want to show that the vertical angles [vert. ], and , are equal (i.e., ). If we let be the -degree rotation around , then maps to More precisely, if and are points on and , respectively (as shown above), let and . Then, maps () to (), and since is angle-preserving, we have .

**Example 2 (5 minutes)**

* Let’s look at a -degree rotation, around the origin of a coordinate system. If a point has coordinates , it is generally said that is the point with coordinates .
* Suppose the point has coordinates ; we will show that the coordinates of are .

Example 2

The picture below shows what happens when there is a rotation of around center , the origin of the coordinate plane.



* Let Let the *vertical line* (i.e., the line parallel to the *-*axis) through meet the -axis at a point . Because the coordinates of are , the point has coordinates by the way coordinates are defined. In particular, is of distance from , and since is length-preserving, the point is also of distance from . However, is a -degree rotation around , so also lies on the -axis but on the opposite side of the -axis from . Therefore, the coordinates of are Now, is a right angle and—since maps it to and also preserves degrees—we see that is also a right angle. This means that is the point of intersection of the vertical line through and the -axis. Since we already know that has coordinates of , then the -coordinate of is , by definition.
* Similarly, the -coordinate of being implies that the -coordinate of is . Altogether, we have proved that the -degree rotation of a point of coordinates is a point with coordinates

The reasoning is perfectly general: The same logic shows that the -degree rotation around the origin of a point of coordinates is the point with coordinates , as desired.

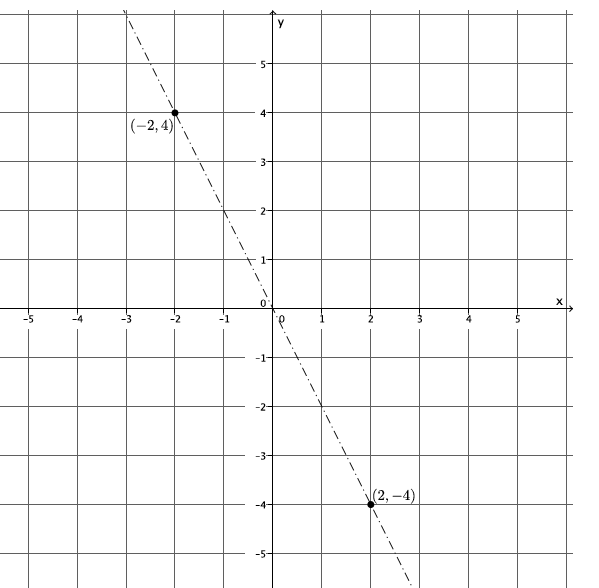
Exercises 1–9 (16 minutes)

Students complete Exercises 1–2independently. Check solutions. Then, let students work in pairs on Exercises 3–4. Students complete Exercises 5–9independently in preparation for the example that follows.

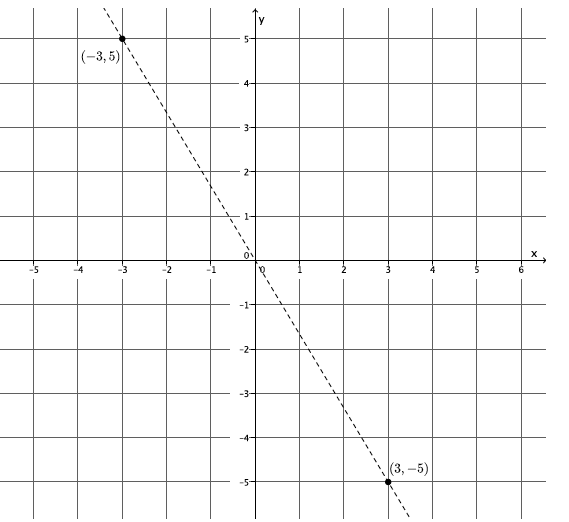
Exercises 1–9

1. Using your transparency, rotate the plane degrees, about the origin. Let this rotation be. What are the coordinates of ?

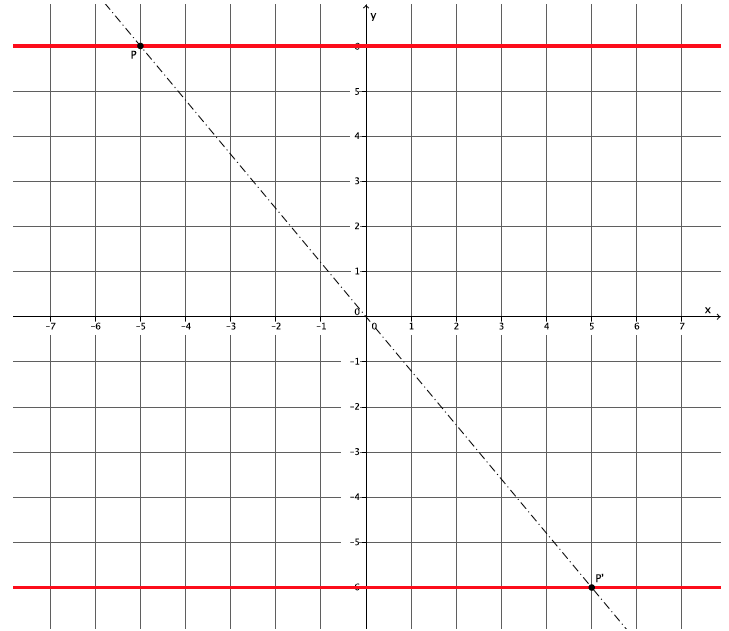
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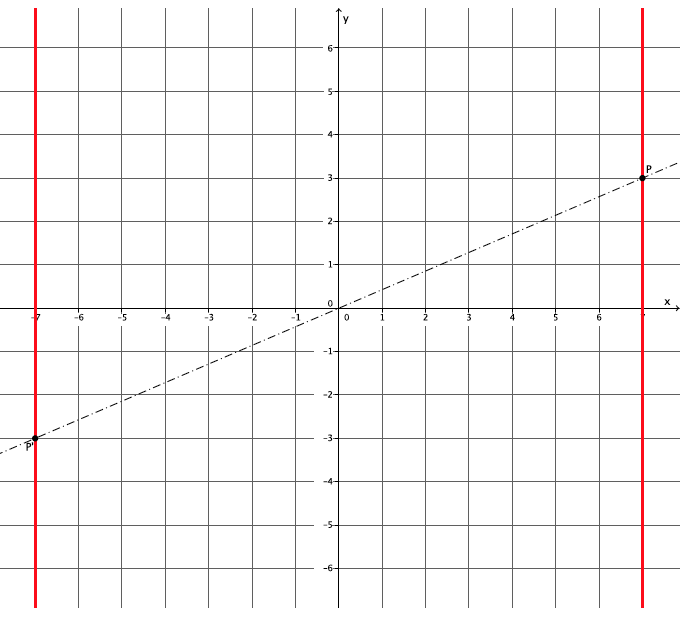


1. Let be the rotation of the plane by degrees, about the origin. Without using your transparency, find .

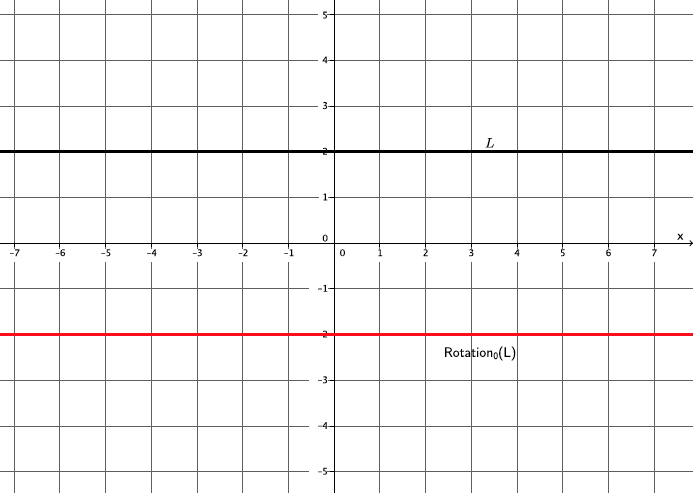
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1. Let be the rotation of degrees around the origin. Let be the line passing through parallel to the *-*axis. Find . Use your transparency if needed.

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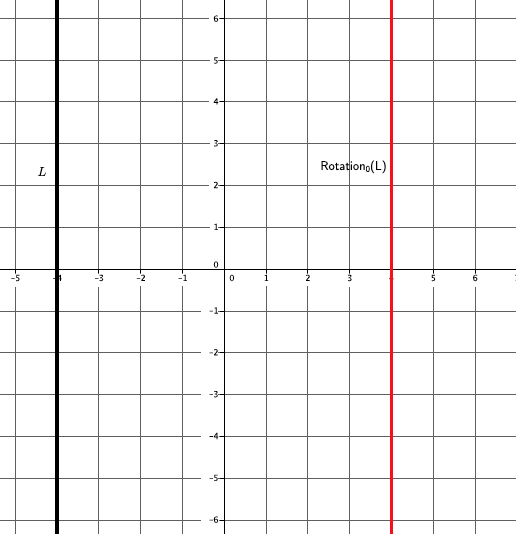
1. Let be the rotation of degrees around the origin. Let be the line passing through parallel to the -axis. Find . Use your transparency if needed.
2. Let be the rotation of degrees around the origin. Let be the line passing through parallel to the *-*axis. Is parallel to ?

Yes, .

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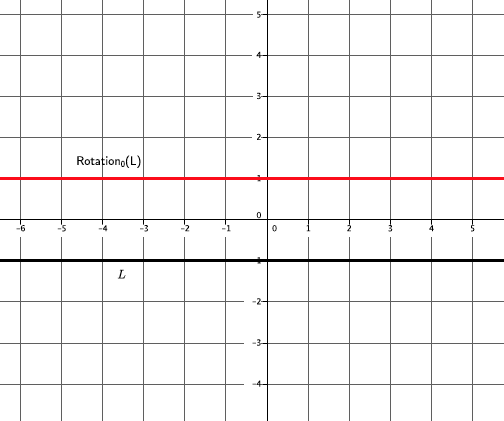
1. Let be the rotation of degrees around the origin. Let be the line passing through parallel to the *-*axis. Is parallel to ?

Yes, .



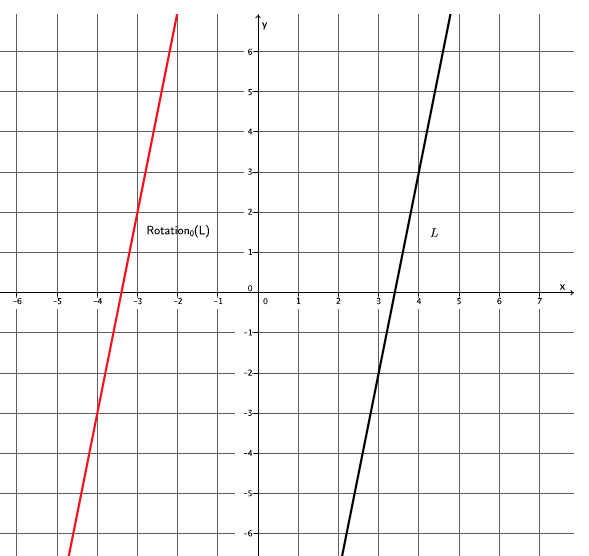
1. Let be the rotation of degrees around the origin. Let be the line passing through parallel to the *-*axis. Is parallel to ?

Yes, .



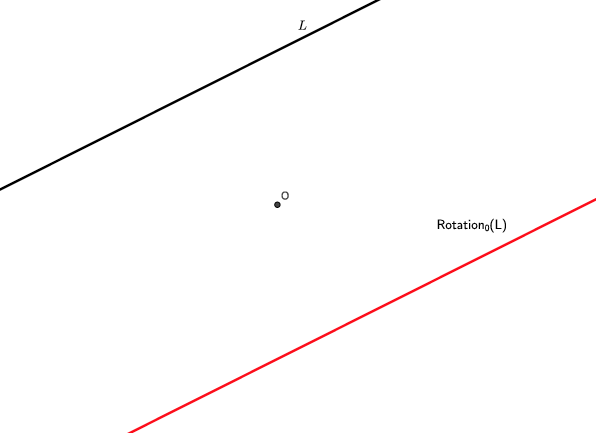
1. Let be the rotation of degrees around the origin. Is parallel to ? Use your transparency if needed.

Yes, .

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1. Let be the rotation of degrees around the origin. Is parallel to ? Use your transparency if needed.

Yes, .



**Example 3 (5 minutes)**

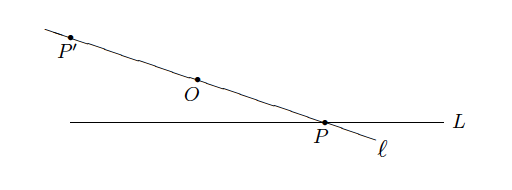
MP.2

*Scaffolding:*

After completing Exercises 5–9, students should be convinced that the theorem is true. Make it clear that their observations can be proven (by contradiction) if we assume something different will happen (e.g., the lines will intersect).

**Theorem**. Let be a point not lying on a given line . Then, the -degree rotation around maps to a line parallel to.

**Proof:**  Let be the -degree rotation around and let be a point on . As usual, denote by . Since is a -degree rotation, ,, lie on the same line (denoted by ).



We want to investigate whether lies on or not. Keep in mind that we want to show that the -degree rotation maps to a line parallel to . If the point lies on , then at some point, the line and intersect, meaning they are not parallel. If we can eliminate the possibility that lies on , then we have to conclude that does not lie on (rotations of degrees make points that are collinear). If lies on , then is a line that joins two points, and *,* on . However is already a line that joins and , so and must be the same line (i.e., ). This is trouble because we know lies on , so implies that lies on . Look at the hypothesis of the theorem: “Let be a point not lying on a given line.” We have a contradiction. So, the possibility that lies on is nonexistent. As we said, this means that does not lie on .

What we have proved is that no matter which point we take from , we know does not lie on . But consists of *all* the points of the form where lies on , so what we have proved is that no point of lies on . In other words, and have no point in common (i.e., ). The theorem is proved.

Closing (5 minutes)

Summarize, or have students summarize, the lesson.

* Rotations of degrees are special:
  + A point, , that is rotated degrees around a center , produces a point so that , , are collinear.
  + When we rotate around the origin of a coordinate system, we see that the point with coordinates is moved to the point .
* We now know that when a line is rotated degrees around a point not on the line, it maps to a line parallel to the given line.

Lesson Summary

* **A rotation of degrees around is the rigid motion so that if is any point in the plane, , , and are *collinear* (i.e., lie on the same line).**
* **Given a -degree rotation, around the origin of a coordinate system, and a point with coordinates , it is generally said that is the point with coordinates .**

Theorem: Let be a point not lying on a given line . Then, the -degree rotation around maps to a line parallel to .

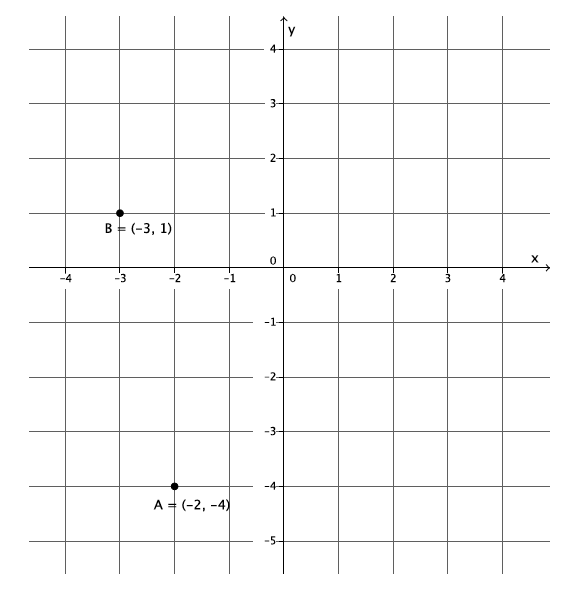
Exit Ticket (5 minutes)

Name Date

Lesson 6: Rotations of 180 Degrees

Exit Ticket

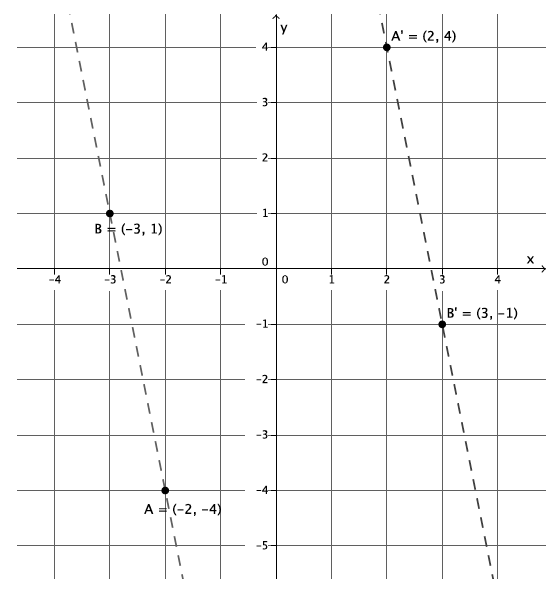
Let there be arotation of degrees about the origin. Point has coordinates and point has coordinates , as shown below.

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1. What are the coordinates of ? Mark that point on the graph so that . What are the coordinates of ? Mark that point on the graph so that .
2. What can you say about the points , , and ? What can you say about the points ,, and ?
3. Connect point to point to make the line . Connect point to point to make the line . What is the relationship between and ?

Exit Ticket Sample Solutions

Let there be a rotation of degrees about the origin. Point has coordinates , and point has coordinates , as shown below.

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1. What are the coordinates of ? Mark that point on the graph so that . What are the coordinates of ? Mark that point on the graph so that .

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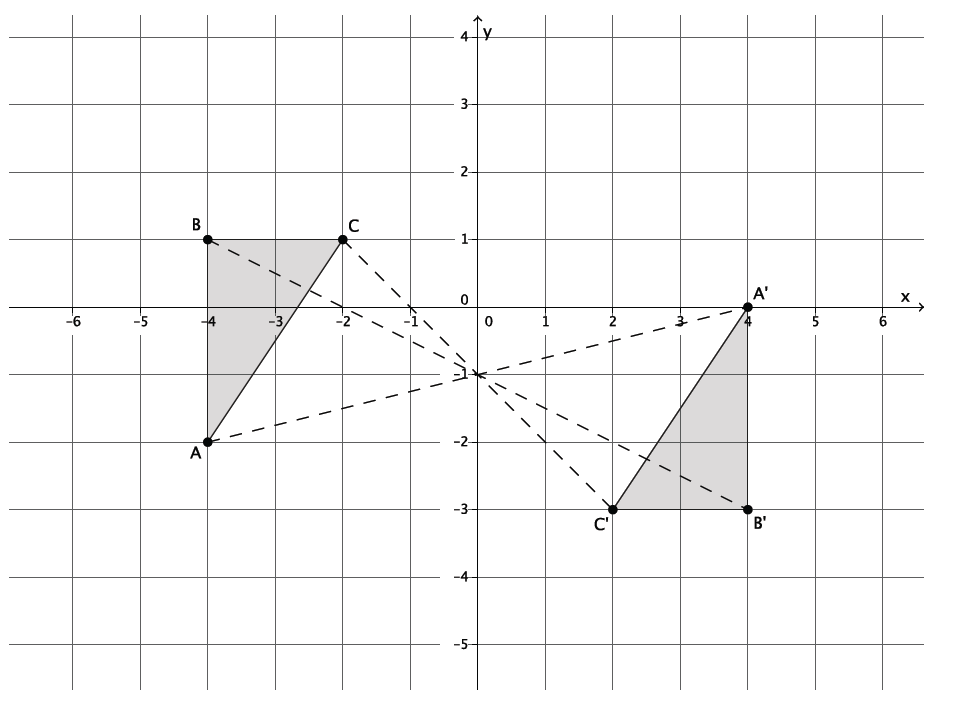
1. What can you say about the points ,, and ? What can you say about the points , , and ?

The points ,, and are collinear. The points ,, and are collinear.

1. Connect point to point to make the line . Connect point to point to make the line . What is the relationship between and ?

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Problem Set Sample Solutions

Use the following diagram for Problems 1–5. Use your transparency as needed.

1. Looking only at segment , is it possible that a rotation would map onto ? Why or why not?

It is possible because the segments are parallel.

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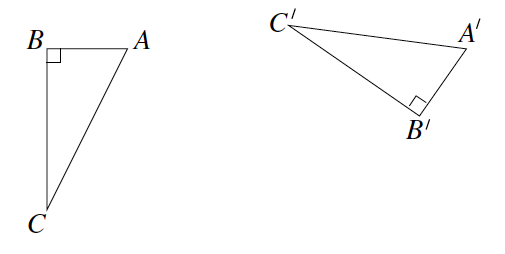
1. Connect point to point , point to point , and point to point . What do you notice? What do you think that point is?

All of the lines intersect at one point. The point is the center of rotation, I checked by using my transparency.

1. Would a rotation map triangle onto triangle ? If so, define the rotation (i.e., degree and center). If not, explain why not.

Let there be a rotation around point . Then, .

1. The picture below shows right triangles  and , where the right angles are at and *.* Given that , and , and that is not parallel to , is there a rotation that would map onto ? Explain.



No, because a rotation of a segment will map to a segment that is parallel to the given one. It is given that is not parallel to ; therefore, a rotation of will not map onto .