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Lesson 2: Multiplication of Numbers in Exponential Form

Student Outcomes

* Students use the definition of exponential notation to make sense of the first law of exponents.
* Students see a rule for simplifying exponential expressions involving division as a consequence of the first law of exponents.
* Students write equivalent numerical and symbolic expressions using the first law of exponents.

Classwork

Discussion (8 minutes)

We have to find out the basic properties of this new concept, “raising a number to a power.” There are three simple ones, and we will discuss them in this and the next lesson.

*Scaffolding:*

* Use concrete numbers for , , and .
* (1) How to multiply different powers of the same number : if , are positive integers, what is ?

Let students explore on their own and then in groups: .

* + *Answer:*

In general, if is any number and are positive integers, then

because

In general, if is any number and are positive integers, then

because

.

Examples 1–2

Work through Examples 1 and 2 in the same manner as just shown (supplement with additional examples if needed).

It is preferable to write the answers as an addition of exponents to emphasize the use of the identity. That step should not be left out. That is, does not have the same instructional value as .

*Scaffolding:*

* Remind students that to remove ambiguity, bases that contain fractions or negative numbers require parentheses.

**Example 1**

**Example 2**

* What is the analog of in the context of repeated addition of a number ?

Allow time for a brief discussion.

MP.2

&

MP.7

* + *If we add copies of and then add to it another copies of , we end up adding copies of By the distributive law:*

*.*

This is further confirmation of what we observed at the beginning of Lesson 1: the exponent in in the context of repeated multiplication corresponds exactly to the in in the context of repeated addition.

Exercises 1–20 (9 minutes)

Students complete Exercises 1–8 independently. Check answers, and then have students complete Exercises 9–20.

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| Exercise 1 | Exercise 5  Let be a number. |
| Exercise 2 | Exercise 6  Let f be a number. |
| Exercise 3 | Exercise 7  Let be a number. |
| Exercise 4 | Exercise 8  Let be a positive integer. If , what is ? |

In Exercises 9–16, students will need to think about how to rewrite some factors so the bases are the same. Specifically, and . Make clear that these expressions can only be simplified when the bases are the same. Also included is a non-example, that cannot be simplified using this identity. Exercises 17–20 are further applications of the identity.

What would happen if there were more terms with the same base? Write an equivalent expression for each problem.

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| Exercise 9 | Exercise 10 |

Can the following expressions be simplified? If so, write an equivalent expression. If not, explain why not.

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| Exercise 11 | Exercise 14 |
| Exercise 12 | Exercise 15 |
| Exercise 13 | Exercise 16    Cannot be simplified. Bases are different and cannot be rewritten in the same base. |

Exercise 17

Let be a number. Simplify the expression of the following number:

Exercise 18

Let and be numbers. Use the distributive law to simplify the expression of the following number:

Exercise 19

Let and be numbers. Use the distributive law to simplify the expression of the following number:

Exercise 20

Let and be numbers. Use the distributive law to simplify the expression of the following number:

Discussion (9 minutes)

Now that we know something about multiplication, we actually know a little about how to divide numbers in exponential notation too. This is not a new law of exponents to be memorized but a (good) consequence of knowing the first law of exponents. Make this clear to students.

* (2) We have just learned how to multiply two different positive integer powers of the same number . It is time to ask how to *divide* different powers of a number . If , are positive integers, what is ?

*Scaffolding:*

* Use concrete numbers for , , and .

Allow time for a brief discussion.

* What is ? (Observe: The power in the numerator is bigger than the power of in the denominator. The general case of arbitrary exponents will be addressed in Lesson 5, so all problems in this lesson will have bigger exponents in the numerator than in the denominator.)
  + *Expect students to write . However, we should nudge them to see how the formula comes into play.*
  + *Answer:*

*by*

*by equivalent fractions*

Observe that the exponent in *is* the difference of and (see the numerator  on the first line).

In general, if is nonzero and , are positive integers, then:

*Note to Teacher:*

* The restriction on and here is to prevent negative exponents from coming up in problems before students learn about them.

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Since , then there is a positive integer , so that . Then, we can rewrite the identity as follows:

by

by equivalent fractions

because implies

Therefore, , if .

In general, if is nonzero and, are positive integers, then

This formula is as far as we can go. We cannot write down in terms of exponents because makes no sense at the moment since we have no meaning for a negative exponent. This explains why the formula above requires . This also motivates our search for a definition of negative exponent, as we shall do in Lesson 5.

* What is the analog of , if in the context of repeated addition of a number ?
  + *Division is to multiplication as subtraction is to addition, so if copies of a number is subtracted from copies of , and, then by the distributive law. (Incidentally, observe once more how the exponent in in the context of repeated multiplication, corresponds exactly to the in in the context of repeated addition.)*

MP.7

Examples 3–4

Work through Examples 3 and 4 in the same manner as shown (supplement with additional examples if needed).

It is preferable to write the answers as a subtraction of exponents to emphasize the use of the identity.

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| **Example 3** | **Example 4** |

Exercises 21–32 (10 minutes)

Students complete Exercises 21–24 independently. Check answers, and then have students complete Exercises 25–32 in pairs or small groups.

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| Exercise 21 | Exercise 23 |
| Exercise 22 | Exercise 24 |

Exercise 25

Let , be nonzero numbers. What is the following number?

Exercise 26

Let be a nonzero number. What is the following number?

Can the following expressions be simplified? If yes, write an equivalent expression for each problem. If not, explain why not.

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| Exercise 27 | Exercise 29 |
| Exercise 28 | Exercise 30 |

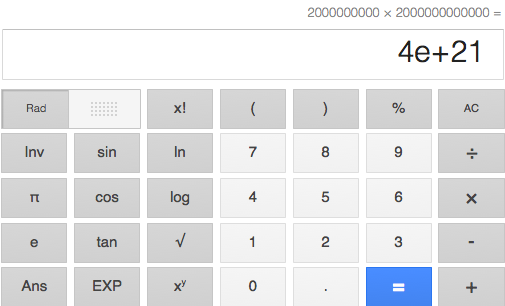
Exercise 31

Let be a number. Simplify the expression of each of the following numbers:



Exercise 32

Anne used an online calculator to multiply . The answer showed up on the calculator as , as shown below. Is the answer on the calculator correct? How do you know?

.

The answer must mean followed by zeroes. That means that the answer on the calculator is correct.

This problem is hinting at scientific notation; i.e., . Accept any reasonable explanation of the answer.

Closing (3 minutes)

Summarize, or have students summarize, the lesson.

* State the two identities and how to write equivalent expressions for each.

Optional Fluency Exercise (2 minutes)

This exercise is not an expectation of the standard, but may prepare students for work with squared numbers in Module 2 with respect to the Pythagorean Theorem. For that reason this is an optional fluency exercise.

Have students chorally respond to numbers squared and cubed that you provide. For example, you say “ squared” and students respond, “.” Next, “ squared” and students respond “.” Have students respond to all squares, in order, up to . When squares are finished, start with “ cubed” and students respond “.” Next, “ cubed” and students respond “.” Have students respond to all cubes, in order, up to . If time allows, you can have students respond to random squares and cubes.

Exit Ticket (2 minutes)

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 2: Multiplication of Numbers in Exponential Form

Exit Ticket

Simplify each of the following numerical expressions as much as possible:

1. Let and be positive integers.
2. Let and be positive integers and.

Exit Ticket Sample Solutions

*Note to Teacher:* Accept both forms of the answer; in other words, the answer that shows the exponents as a sum or difference and the answer where the numbers were actually added or subtracted.

Simplify each of the following numerical expressions as much as possible:

1. Let and be positive integers.

1. Let and be positive integers and.

Problem Set Sample Solutions

To ensure success, students need to complete at least bounces –with support in class.

Students may benefit from a simple drawing of the scenario. It will help them see why the factor of is necessary when calculating the distance traveled for each bounce. Make sure to leave the total distance traveled in the format shown so that students can see the pattern that is developing. Simplifying at any step will make it extremely difficult to write the general statement for number of bounces.

1. A certain ball is dropped from a height of feet. It always bounces up to feet. Suppose the ball is dropped from feet and is caught exactly when it touches the ground after the th bounce. What is the total distance traveled by the ball? Express your answer in exponential notation.

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| Bounce | Computation of Distance Traveled in Previous Bounce | Total Distance Traveled (in feet) |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 30 |  |  |
|  |  |  |

1. If the same ball is dropped from feet and is caught exactly at the highest point after the th bounce, what is the total distance traveled by the ball? Use what you learned from the last problem.

Based on the last problem we know that each bounce causes the ball to travel feet. If the ball is caught at the highest point of the bounce, then the distance traveled on that last bounce is just because it does not make the return trip to the ground. Therefore, the total distance traveled by the ball in this situation is

1. Let and be numbers and , and let and be positive integers. Simplify each of the following expressions as much as possible:

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1. Let the dimensions of a rectangle be ft. by ft. Determine the area of the rectangle. No need to expand all the powers.

Area

sq. ft.

1. A rectangular area of land is being sold off in smaller pieces. The total area of the land is square miles. The pieces being sold are square miles in size. How many smaller pieces of land can be sold at the stated size? Compute the actual number of pieces.

pieces of land can be sold.