



Lesson 9: The Mean Absolute Deviation (MAD)

Student Outcomes

- Students calculate the mean absolute deviation (MAD) for a given data set.
- Students interpret the MAD as the average distance of data values from the mean.

Lesson Notes

Variability was discussed informally in Lesson 8. This lesson focuses on developing a more formal measure of variability in a data distribution called the mean absolute deviation, denoted by MAD. The concept of deviation from the mean should be clear to students by now, since previous lessons used deviations to develop the idea of the mean as a balance point. This lesson challenges students to answer why absolute values of deviations are used in calculating the MAD.

Mean absolute deviation is the measure of variability used in the middle school curriculum. At the high school level, deviations are squared instead of using the absolute value. This leads to other important measures of variability called the variance and the standard deviation.

Classwork

Example 1 (5 minutes): Variability

Example 1: Variability

In Lesson 8, Robert tried to decide to which of two cities he would rather move, based on comparing their mean annual temperatures. Since the mean yearly temperature for New York City and San Francisco turned out to be about the same, he decided instead to compare the cities based on the variability in their monthly temperatures from the overall mean. He looked at the two distributions and decided that the New York City temperatures were more spread out from their mean than were the San Francisco temperatures from their mean.

Read through Example 1 as a class, and recall the main idea of Lesson 8. Then ask students:

- What is variability?
 - *The spread of data in a distribution from some focal point in the distribution (such as the mean).*
- What does a distribution that has no variability look like?
 - *All of the data points are the same.*
- What does a distribution that has a lot of variability look like?
 - *The data points are spread far apart.*

Suggest a visual way to order several data sets from the one with least variability to the one with most variability.

Exercises 1–3 (7–10 minutes)

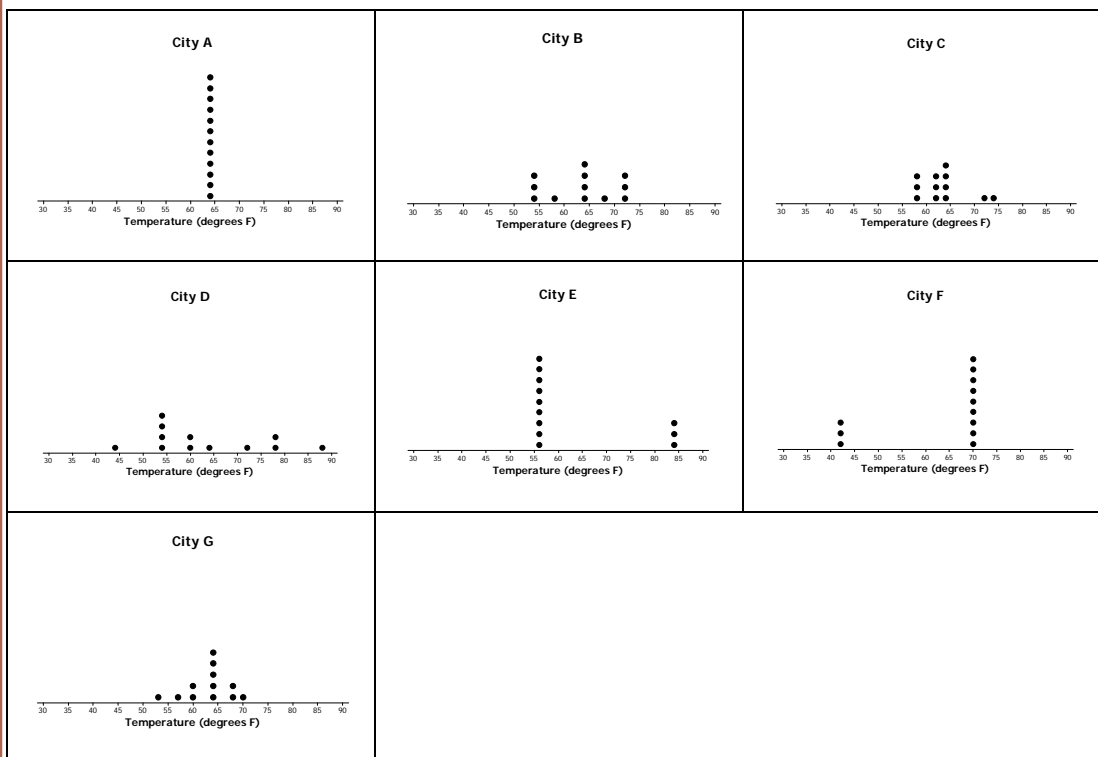
Let students work in small groups on this exercise. Then confirm answers as a class. The discussion of Exercise 3 leads into Example 2.

Students are asked to order the seven data sets from least variability to most variability. Students will no doubt suggest different orderings. Several orderings are reasonable – focus on the students’ explanations for ordering the distributions. What is important is not their suggested orderings but rather their arguments to support their orderings. Also, the goal for this example is for students to realize that they need to have a more formal way of deciding the best ordering. Sabina suggests that a formula is needed, and she proceeds in this lesson to develop one.

MP.3

Exercises 1–3

The following temperature distributions for seven other cities all have a mean temperature of approximately 63 degrees. They do not have the same variability. Consider the following dot plots of the mean yearly temperatures of the seven cities in degrees Fahrenheit.



- Which distribution has the smallest variability of the temperatures from its mean of 63 degrees? Explain your answer.
City A, because all points are the same.
- Which distribution(s) seems to have the most variability of the temperatures from the mean of 63 degrees? Explain your answer.
One or more of the following is acceptable: Cities D, E, and F. They appear to have data points spread furthest from the mean.

3. Order the seven distributions from least variability to most variability. Explain why you listed the distributions in the order that you chose.

Several orderings are reasonable. Focus on students' explanations for choosing the order.

Example 2 (5 minutes): Measuring Variability

Example 2: Measuring Variability

Based on just looking at the distributions, there are different orderings of variability that seem to make some sense. Sabina is interested in developing a formula that will give a number that measures the variability in a data distribution. She would then use the formula for each data set and order the distributions from lowest to highest. She remembers from a previous lesson that a deviation is found by subtracting the mean from a data point. The formula was summarized as: $\text{deviation} = \text{data point} - \text{mean}$. Using deviations to develop a formula measuring variability is a good idea to consider.

No doubt students had different orderings of variability for the seven cities in Exercise 2. Sabina suggests that, in this example, a formula is needed to give a formal ordering. Since variability is being viewed from the mean, it seems reasonable that a formula should be based on how far data points are from the mean. Recall that a deviation results from subtracting the mean from a data point, *or* $\text{deviation} = \text{data point} - \text{mean}$. She concludes that it seems to be a good idea to use deviations in developing a formula for a measure of variability.

Ask students:

- Do you think using deviations is a good basis for a formula to measure variability?
 - *Yes. A deviation measures how far a data point is from the mean of its distribution. That certainly addresses the concept of variability.*
- When are deviations negative?
 - *When they are located to the left of the mean.*
- When are deviations positive?
 - *When they are located to the right of the mean.*

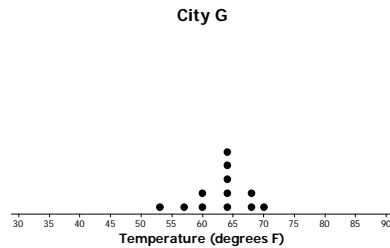
Exercises 4–6 (5 minutes)

Let students work in pairs.

In this exercise, City *G* is used to focus on calculating deviations and verifying that the sum of deviations is equal to zero. This means summing deviations is not a good measure of variability because it always turns out to be zero (by the development of the mean as a balance). A graph is drawn of City *G* to illustrate the values of the deviations.

Exercises 4–6

The dot plot for the temperatures in City G is shown below. Use the dot plot and the mean temperature of 63 degrees to answer the following questions.



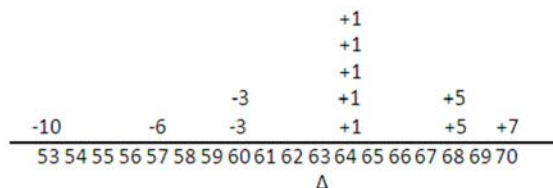
4. Fill in the following table for City G temperature deviations.

Temp	Deviation	Result
53	$53 - 63$	-10
57	$57 - 63$	-6
60	$60 - 63$	-3
60	$60 - 63$	-3
64	$64 - 63$	$+1$
64	$64 - 63$	$+1$
64	$64 - 63$	$+1$
64	$64 - 63$	$+1$
64	$64 - 63$	$+1$
68	$68 - 63$	$+5$
68	$68 - 63$	$+5$
70	$70 - 63$	$+7$
Sum		0

5. Why should the sum of your deviations column be equal to zero? (Hint: Recall the balance interpretation of the mean of a data set.)

The mean is the value that makes the sum of the positive and negative deviations 0. It is the balance point.

6. Another way to graph the deviations is to write them on a number line as follows. What is the sum of the positive deviations (the deviations to the right of the mean)? What is the sum of the negative deviations (the deviations to the left of the mean)? What is the total sum of the deviations?



Sum of the positive deviations = +22

Sum of the negative deviations = -22

Sum of all of the deviations = 0

Example 3 (5–7 minutes): Finding the Mean Absolute Deviation (MAD)**Example 3: Finding the Mean Absolute Deviation (MAD)**

By the balance interpretation of the mean, the sum of the deviations for any data set will always be zero. Sabina is disappointed that her idea of developing a measure of variability using deviations isn't working. She still likes the concept of using deviations to measure variability, but the problem is that the sum of the positive deviations is cancelling out the sum of the negative deviations. What would you suggest she do to keep the deviations as the basis for a formula but to avoid the deviations cancelling out each other?

MP.1

This example asks students how they could still use deviations in developing a measure of variability but correct the difficulty of having the negative deviations offset the positive deviations when the deviations are summed. The operation of absolute value should come to mind because it is part of what students have previously studied in mathematics. The example leads them through the calculation of the deviations and then to taking the mean of the absolute deviations.

Ask students:

- If we were to treat the negative deviations as distances, what mathematical operation would do that?
 - *Finding the absolute value of a negative deviation makes it a positive distance. Emphasize that the concept of deviation has been maintained.*
- If we use the absolute value of all the deviations, what will happen to the sum?
 - *It will not be zero.*

Define the mean absolute deviation as the sum of the absolute values of the deviations divided by the number of deviations. (See Exercise 7, part (c).) Teachers may wish to work through Exercise 7, parts (a)–(c) as a class to develop this concept.

Exercises 7–8 (10–12 minutes)

Let students continue to work in pairs or small groups. As previously indicated, teachers may decide to work through Exercise 7, parts (a)–(c) as a class.

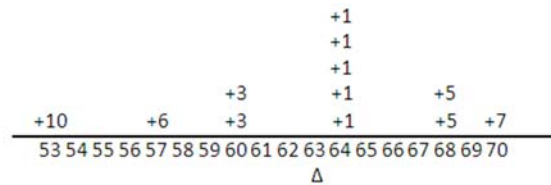
Exercises 7–8

7. One suggestion to possibly help Sabina is to take the absolute value of the deviations.

a. Fill in the following table.

Temp	Deviation	Result	Abs
53	$53 - 63$	-10	$+10$
57	$57 - 63$	-6	$+6$
60	$60 - 63$	-3	$+3$
60	$60 - 63$	-3	$+3$
64	$64 - 63$	$+1$	$+1$
64	$64 - 63$	$+1$	$+1$
64	$64 - 63$	$+1$	$+1$
64	$64 - 63$	$+1$	$+1$
64	$64 - 63$	$+1$	$+1$
68	$68 - 63$	$+5$	$+5$
68	$68 - 63$	$+5$	$+5$
70	$70 - 63$	$+7$	$+7$

- b. From the following graph, what is the sum of the absolute deviations?



The sum of the absolute deviations is +44.

- c. Sabina suggests that the mean of the absolute deviations could be a measure of the variability in a data set. Its value is the average distance that all the data values are from the mean temperature. It is called the Mean Absolute Deviation and is denoted by the letters, MAD. Find the MAD for this data set of City *G* temperatures. Round to the nearest tenth.

The mean absolute deviation is $\frac{44}{12}$ or 3.7 degrees to the nearest tenth of a degree.

- d. Find the MAD for each of the temperature distributions in all seven cities, and use the values to order the distributions from least variability to most variability. Recall that the mean for each data set is 63 degrees. Does the list that you made in Exercise 2 by just looking at the distributions match this list made by ordering MAD values?

**If time is a factor in completing this lesson, assign a city to individual students. After each student has calculated the mean deviation, organize results for the whole class. Direct students to calculate the MAD to the nearest tenth of a degree.*

MAD values:

City *A* = 0

City *B* = 5.3

City *C* = 5.3

City *D* = 10.5

E = 10.5

F = 10.5

G = 3.7

The order from least to greatest is:

A, G, and B, C (tied), and

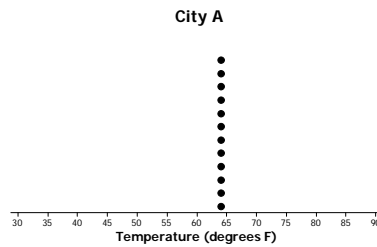
D, E, and F (all tied).

- e. Which of the following is a correct interpretation of the MAD?

- The monthly temperatures in City *G* are spread 3.7 degrees from the approximate mean of 63 degrees.
- The monthly temperatures in City *G* are, on average, 3.7 degrees from the approximate mean temperature of 63 degrees.
- The monthly temperatures in City *G* differ from the approximate mean temperature of 63 degrees by 3.7 degrees.

Answer is (ii).

8. The dot plot for City A temperatures follows.



- a. How much variability is there in City A's temperatures? Why?

No variability. The deviations are all 0.

- b. Does the MAD agree with your answer in part (a)?

Yes. The mean absolute deviation is 0.

Lesson Summary

In this lesson, a formula was developed that measures the amount of variability in a data distribution.

- The absolute deviation of a data point is how far away that data point is from the mean.
- The Mean Absolute Deviation (MAD) is computed by finding the mean of the absolute deviations in the distribution.
- The value of MAD is the average distance that all the data values are from the mean.
- A small MAD indicates that the distribution has very little variability.
- A large MAD indicates that the data points are spread far away from the mean.

Exit Ticket (5 minutes)

Name _____

Date_____

Lesson 9: The Mean Absolute Deviation (MAD)

Exit Ticket

1. The Mean Absolute Deviation (MAD) is a measure of variability for a data set. What does a data distribution look like if its MAD equals zero? Explain.
2. Is it possible to have a negative value for the MAD of a data set?
3. Suppose that seven students have the following number of pets: 1, 1, 1, 2, 4, 4, 8.
 - a. The mean number of pets for these seven students is three pets. Use the following table to find the MAD number of pets for this distribution of number of pets.

Student	# of Pets	Deviations	Absolute Deviations
1	1		
2	1		
3	1		
4	2		
5	4		
6	4		
7	8		
Sum			

- b. Explain in words what the MAD means for this data set.

Exit Ticket Sample Solutions

1. The Mean Absolute Deviation (MAD) is a measure of variability for a data set. What does a data distribution look like if its MAD equals zero? Explain.

If the MAD is zero, then all of the deviations are zero. For example, City A had a dot plot with all the same temperatures. They were all the same, so there was no variability, since the MAD measures average temperature from the mean. And it is zero, because all the deviations are zero.

2. Is it possible to have a negative value for the MAD of a data set?

Because a MAD is the average of the absolute values of the deviations, it is always a positive value.

3. Suppose that seven students have the following number of pets: 1, 1, 1, 2, 4, 4, 8.

- a. The mean number of pets for these seven students is three pets. Use the following table to find the MAD number of pets for this distribution of number of pets.

The MAD number of pets is $+\frac{14}{7} = 2$ pets.

Student	# of Pets	Deviations	Absolute Deviations
1	1	$1 - 3 = -2$	$+2$
2	1	$1 - 3 = -2$	$+2$
3	1	$1 - 3 = -2$	$+2$
4	2	$2 - 3 = -1$	$+1$
5	4	$4 - 3 = +1$	$+1$
6	4	$4 - 3 = +1$	$+1$
7	8	$8 - 3 = +5$	$+5$
Sum		0	$+14$

- b. Explain in words what the MAD means for this data set.

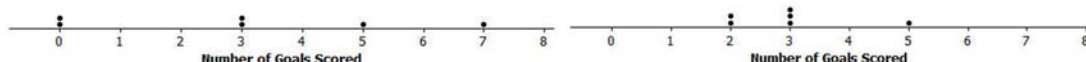
On average, these students' number of pets differ by 2 pets from their mean of 3 pets.

Problem Set Sample Solutions

1. Suppose the dot plot on the left shows the number of goals a boys' soccer team has scored in six games so far this season, and the dot plot on the right shows the number of goals a girls' soccer team has scored in six games so far this season. The mean for both of these teams is 3.

Dot Plot of Number of Goals Scored by Boys' Team

Dot Plot of Number of Goals Scored by Girls' Team FINAL -



- a. Before doing any calculations, which dot plot has the larger MAD? Explain how you know.

The graph of the Boys' team is more spread out and has the larger deviations from the mean.

- b. Use the following tables to find the MAD number of goals for each distribution. Round your calculations to the nearest hundredth.

Boys' Team		
#Goals	Deviations	Absolute Deviations
0	-3	+3
0	-3	+3
3	$3 - 3$	0
3	$3 - 3 = 0$	0
5	$5 - 3 = 2$	+2
7	$7 - 3 = 4$	+4
Sum		+12

Girls' Team		
#Goals	Deviations	Absolute Deviations
2	$2 - 3 = -1$	+1
2	$2 - 3 = -1$	+1
3	$3 - 3 = 0$	0
3	$3 - 3 = 0$	0
3	$3 - 3 = 0$	0
5	$5 - 3 = 2$	+2
Sum		+4

- c. Based on the computed MAD values, for which distribution is the mean a better indication of a typical value? Explain your answer.

The Girls' team, because the measure of variability given by the MAD is lower (0.67 goals) than the Boys' MAD (2 goals). Visually, the data in the Girls' dot plot are more compact around the mean than they are in the Boys' dot plot.

2. Recall Robert's problem of deciding whether to move to New York City or to San Francisco. The table of temperatures (in degrees Fahrenheit) and deviations for the New York City distribution is as follows:

NYC	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temp	39	42	50	61	71	81	85	84	76	65	55	47
Deviation	-24	-21	-13	-2	8	18	22	21	13	2	-8	-16

- a. The dot plot below is written with the deviations above each of the monthly temperatures. What is the sum of all of the deviations? Are you surprised? Explain.

The sum of the negative deviations is: $-24 + -21 + -16 + -13 + -8 + -2 = -84$. The sum of the positive deviations is: $2 + 8 + 13 + 18 + 21 + 22 = +84$. The sum of all of the deviations is $-84 + 84 = 0$. Students should not be surprised, since the sum of the deviations of any data set around its mean is 0.

-24	-21	-16	-13	-8	-2	2	8	13	18	21	22	
39	42	47	50	55	61	65	71	76	81	84	85	

- b. The absolute deviations for the monthly temperatures are shown below. Use this information to calculate the MAD. Explain the MAD in words for this problem.

The sum of the absolute deviations is $2(84) = 168$ degrees. The average of the absolute deviations, MAD, is $\frac{168}{12} = 14$ degrees. On average, the monthly temperatures in New York City differ from the mean of 63 degrees by 14 degrees.

+24	+21	+16	+13	+8	+2	2	8	13	18	21	22	
39	42	47	50	55	61	65	71	76	81	84	85	

- c. Complete the following table and then use the values to calculate the MAD for the San Francisco data distribution.

First of all, note that the sum of the negative deviations is -21 , and the sum of the positive deviations is $+21$, as it should be. The sum of the absolute deviations is $2(21) = 42$. The MAD is the mean of the absolute deviations, which equals $\frac{42}{12} = 3.5$ degrees.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temp	57	60	62	63	64	67	67	68	70	69	63	58
Deviations	-7	-4	-2	-1	0	+3	+3	+4	+6	+5	-1	-6
Absolute Deviations	+7	+4	+2	+1	0	+3	+3	+4	+6	+5	+1	+6

- d. Comparing the MAD values for New York City and San Francisco, which city would Robert choose to move to if he is interested in having a lot of variability in monthly temperatures? Explain using the MAD.

New York City has a MAD of 14 degrees, as compared to 3.5 degrees in San Francisco. Robert should choose New York City if he wants to have more variability in monthly temperatures.

3. Consider the following data of the number of green jellybeans in seven bags sampled from five different candy manufacturers (Awesome, Delight, Finest, Sweeties, YumYum). Note that the mean in each distribution is 42 green jellybeans.

	Bag 1	Bag 2	Bag 3	Bag 4	Bag 5	Bag 6	Bag 7
Awesome	40	40	41	42	42	43	46
Delight	22	31	36	42	48	53	62
Finest	26	36	40	43	47	50	52
Sweeties	36	39	42	42	42	44	49
YumYum	33	36	42	42	45	48	48

- a. Complete the following table of the deviations of the number of green jellybeans from the mean number of green jellybeans in the seven bags.

	Bag 1	Bag 2	Bag 3	Bag 4	Bag 5	Bag 6	Bag 7
Awesome	-2	-2	-1	0	0	+1	+4
Delight	-20	-11	-6	0	+6	+11	+20
Finest	-16	-6	-2	+1	+5	+8	+10
Sweeties	-6	-3	0	0	0	+2	+7
YumYum	-9	-6	0	0	+3	+6	+6

- b. Based on what you learned about MAD, which manufacturer do you think will have the lowest MAD? Calculate the MAD for the manufacturer you selected.

Use the MAD for each manufacturer to evaluate students' responses.

	Bag 1	Bag 2	Bag 3	Bag 4	Bag 5	Bag 6	Bag 7	SUM	MAD
Awesome	+2	+2	+1	0	0	+1	+4	+10	1.4
Delight	+20	+11	+6	0	+6	+11	+20	+74	10.6
Finest	+16	+6	+2	+1	+5	+8	+10	+48	6.9
Sweeties	+6	+3	0	0	0	+2	+7	+18	2.6
YumYum	+9	+6	0	0	+3	+6	+6	+30	4.3