Lesson 19: Surface Area and Volume in the Real World

Student Outcomes

- Students determine the surface area of three-dimensional figures in real-world contexts.
- Students choose appropriate formulas to solve real-life volume and surface area problems.

Classwork

Fluency Exercise (5 minutes): Area of Shapes

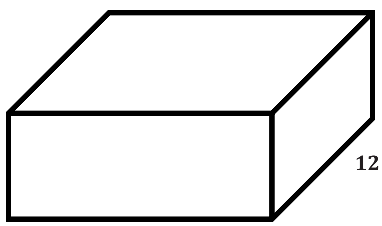
RWBE: Refer to the Rapid White Board Exchange section in the Module Overview for directions to administer an RWBE.

Opening Exercise (4 minutes)

MP.1

Opening Exercise

A box needs to be painted. How many square inches will need to be painted to cover every surface?



$SA = 2(15\text{ in.})(12\text{ in.}) + 2(15\text{ in.})(6\text{ in.}) + 2(12\text{ in.})(6\text{ in.})$

$SA = 360\text{ in}^2 + 180\text{ in}^2 + 144\text{ in}^2$

$SA = 684\text{ in}^2$

A juice box is 4 in. tall, 1 in. wide, and 2 in. long. How much juice fits inside the juice box?

$V = 1\text{ in.} \times 2\text{ in.} \times 4\text{ in.} = 8\text{ in}^3$

How did you decide how to solve each problem?

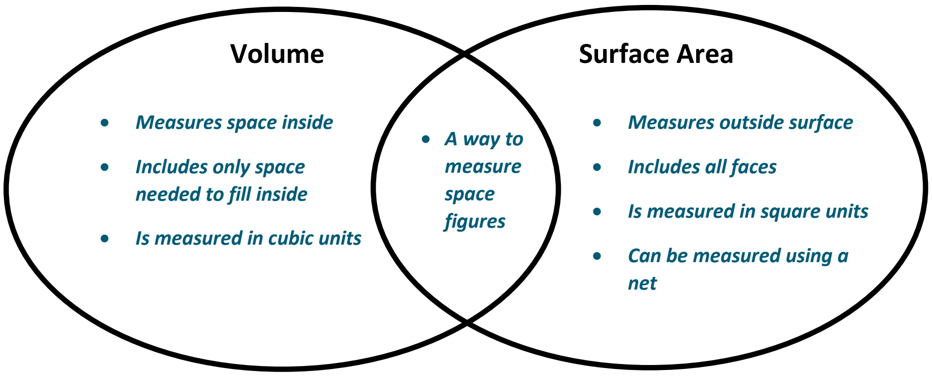
I chose to use surface area to solve the first problem because you would need to know how much area the paint would need to cover. I chose to use volume to solve the second problem because you would need to know how much space is inside the juice box to determine how much juice it can hold.

If students struggle deciding whether to calculate volume or surface area, use the Venn diagram below to help them make the correct decision.

Discussion (5 minutes)

Students need to be able to recognize the difference between volume and surface area. As a class, complete the Venn diagram below so students have a reference when completing the application problems.

Discussion



Example 1 (5 minutes)

Work through the word problem below with students. Students should be leading the discussion in order for them to be prepared to complete the exercises.

Example 1

Vincent put logs in the shape of a rectangular prism. He built this rectangular prism of logs outside his house. However, it is supposed to snow, and Vincent wants to buy a cover so the logs will stay dry. If the pile of logs creates a rectangular prism with these measurements:

33 cm long, 12 cm wide, and 48 cm high,

what is the minimum amount of material needed to make a cover for the wood pile?

- Where do we start?
  - *We need to find the size of the cover for the logs, so we need to calculate the surface area. In order to find the surface area, we need to know the dimensions of the pile of logs.*
- Why do we need to find the surface area and not the volume?
  - *We want to know the size of the cover Vincent wants to buy. If we calculated volume, we would not have the information Vincent needs when he goes shopping for a cover.*
- What are the dimensions of the pile of logs?
  - *The length is* 33 cm*, the width is* 12 cm*, and the height is* 48 cm.

Scaffolding:

- Add to the poster or handout made in the previous lesson showing that *long* represents length, *wide* represents width, and *high* represents height.
- Later, students will have to recognize that *deep* also represents height. Therefore, this vocabulary word should also be added to the poster.

- How do we calculate the surface area to determine the size of the cover?
  - *We can use the surface area formula for a rectangular prism.*

$$SA = 2(33 \text{ cm})(12 \text{ cm}) + 2(33 \text{ cm})(48 \text{ cm}) + 2(12 \text{ cm})(48 \text{ cm})$$
$$SA = 792 \text{ cm}^2 + 3168 \text{ cm}^2 + 1152 \text{ cm}^2$$
$$SA = 5112 \text{ cm}^2$$

- What is different about this problem from other surface area problems of rectangular prisms you have encountered? How does this change the answer?
  - *If Vincent just wants to cover the wood to keep it dry, he does not need to cover the bottom of the pile of logs. Therefore, the cover can be smaller.*
- How can we change our answer to find the exact size of the cover Vincent needs?
  - *We know the area of the bottom of the pile of logs has the dimensions* 33 cm *and* 12 cm. *We can calculate the area and subtract this area from the total surface area.*
  - *The area of the bottom of the pile of firewood is* 396 cm^2*; therefore, the total surface area of the cover would need to be* $5112 \text{ cm}^2 - 396 \text{ cm}^2 = 4716 \text{ cm}^2$.

Exercises 1–6 (17 minutes)

Students complete the volume and surface area problems in small groups.

MP.1

Exercises 1–6

Use your knowledge of volume and surface area to answer each problem.

1. **Quincy Place wants to add a pool to the neighborhood. When determining the budget, Quincy Place determined that it would also be able to install a baby pool that requires less than 15 cubic feet of water. Quincy Place has three different models of a baby pool to choose from.**

   Choice One: 5 feet $\times$ 5 feet $\times$ 1 foot

   Choice Two: 4 feet $\times$ 3 feet $\times$ 1 foot

   Choice Three: 4 feet $\times$ 2 feet $\times$ 2 feet

   Which of these choices is best for the baby pool? Why are the others not good choices?

   Choice One Volume: $5 \text{ ft.} \times 5 \text{ ft.} \times 1 \text{ ft.} = 25$ cubic feet

   Choice Two Volume: $4 \text{ ft.} \times 3 \text{ ft.} \times 1 \text{ ft.} = 12$ cubic feet

   Choice Three Volume: $4 \text{ ft.} \times 2 \text{ ft.} \times 2 \text{ ft.} = 16$ cubic feet

   Choice Two is within the budget because it holds less than 15 cubic feet of water. The other two choices do not work because they require too much water, and Quincy Place will not be able to afford the amount of water it takes to fill the baby pool.

2. **A packaging firm has been hired to create a box for baby blocks. The firm was hired because it could save money by creating a box using the least amount of material. The packaging firm knows that the volume of the box must be 18 cm^3.**
   a. **What are possible dimensions for the box if the volume must be exactly 18 cm^3?**

      Choice 1: $1 \text{ cm} \times 1 \text{ cm} \times 18 \text{ cm}$

      Choice 2: $1 \text{ cm} \times 2 \text{ cm} \times 9 \text{ cm}$

      Choice 3: $1 \text{ cm} \times 3 \text{ cm} \times 6 \text{ cm}$

      Choice 4: $2 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$

MP.1

b. Which set of dimensions should the packaging firm choose in order to use the least amount of material? Explain.

Choice 1: $SA = 2(1\text{ cm})(1\text{ cm}) + 2(1\text{ cm})(18\text{ cm}) + 2(1\text{ cm})(18\text{ cm}) = 74\text{ cm}^2$

Choice 2: $SA = 2(1\text{ cm})(2\text{ cm}) + 2(1\text{ cm})(9\text{ cm}) + 2(2\text{ cm})(9\text{ cm}) = 58\text{ cm}^2$

Choice 3: $SA = 2(1\text{ cm})(3\text{ cm}) + 2(1\text{ cm})(6\text{ cm}) + 2(3\text{ cm})(6\text{ cm}) = 54\text{ cm}^2$

Choice 4: $SA = 2(2\text{ cm})(3\text{ cm}) + 2(2\text{ cm})(3\text{ cm}) + 2(3\text{ cm})(3\text{ cm}) = 42\text{ cm}^2$

The packaging firm should choose Choice 4 because it requires the least amount of material. In order to find the amount of material needed to create a box, the packaging firm would have to calculate the surface area of each box. The box with the smallest surface area requires the least amount of material.

3. A gift has the dimensions of $50\text{ cm} \times 35\text{ cm} \times 5\text{ cm}$. You have wrapping paper with dimensions of $75\text{ cm} \times 60\text{ cm}$. Do you have enough wrapping paper to wrap the gift? Why or why not?

Surface Area of the Present: $SA = 2(50\text{ cm})(35\text{ cm}) + 2(50\text{ cm})(5\text{ cm}) + 2(35\text{ cm})(5\text{ cm}) =$
$3500\text{ cm}^2 + 500\text{ cm}^2 + 350\text{ cm}^2 = 4350\text{ cm}^2$

Area of Wrapping Paper: $A = 75\text{ cm} \times 60\text{ cm} = 4,500\text{ cm}^2$

I do have enough paper to wrap the present because the present requires $4,350$ square centimeters *of paper, and I have* $4,500$ square centimeters *of wrapping paper.*

4. Tony bought a flat rate box from the post office to send a gift to his mother for Mother's Day. The dimensions of the medium size box are 14 inches $\times$ 12 inches $\times$ 3.5 inches. What is the volume of the largest gift he can send to his mother?

Volume of the Box: $14\text{ in.} \times 12\text{ in.} \times 3.5\text{ in.} = 588\text{ in}^3$

Tony would have 588 cubic inches *of space to fill with a gift for his mother.*

5. A cereal company wants to change the shape of its cereal box in order to attract the attention of shoppers. The original cereal box has dimensions of 8 inches $\times$ 3 inches $\times$ 11 inches. The new box the cereal company is thinking of would have dimensions of 10 inches $\times$ 10 inches $\times$ 3 inches.

a. Which box holds more cereal?

Volume of Original Box: $V = 8\text{ in.} \times 3\text{ in.} \times 11\text{ in.} = 264\text{ in}^3$

Volume of New Box: $V = 10\text{ in.} \times 10\text{ in.} \times 3\text{ in.} = 300\text{ in}^3$

The new box holds more cereal because it has a larger volume.

b. Which box requires more material to make?

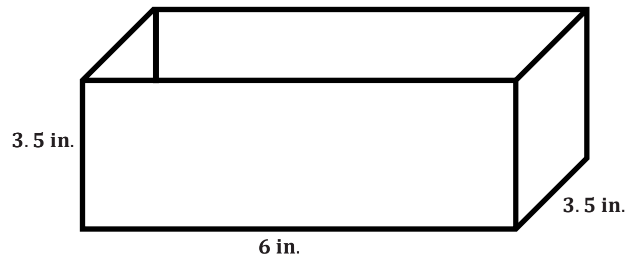
Surface Area of Original Box: $SA = 2(8\text{ in.})(3\text{ in.}) + 2(8\text{ in.})(11\text{ in.}) + 2(3\text{ in.})(11\text{ in.}) =$
$48\text{ in}^2 + 176\text{ in}^2 + 66\text{ in}^2 = 290\text{ in}^2$

Surface Area of New Box: $SA = 2(10\text{ in.})(10\text{ in.}) + 2(10\text{ in.})(3\text{ in.}) + 2(10\text{ in.})(3\text{ in.}) =$
$200\text{ in}^2 + 60\text{ in}^2 + 60\text{ in}^2 = 320\text{ in}^2$

The new box requires more material than the original box because the new box has a larger surface area.

MP.1

6. Cinema theaters created a new popcorn box in the shape of a rectangular prism. The new popcorn box has a length of 6 inches, a width of 3.5 inches, and a height of 3.5 inches but does not include a lid.



> *Scaffolding:*
> English language learners may not be familiar with the term *lid*. Provide an illustration or demonstration.

a. How much material is needed to create the box?

Surface Area of the Box: $SA = 2(6 \text{ in.})(3.5 \text{ in.}) + 2(6 \text{ in.})(3.5 \text{ in.}) + 2(3.5 \text{ in.})(3.5 \text{ in.}) =$

$$42 \text{ in}^2 + 42 \text{ in}^2 + 24.5 \text{ in}^2 = 108.5 \text{ in}^2$$

The box does not have a lid, so we have to subtract the area of the lid from the surface area.

Area of Lid: $6 \text{ in.} \times 3.5 \text{ in.} = 21 \text{ in}^2$

Total Surface Area: $108.5 \text{ in}^2 - 21 \text{ in}^2 = 87.5 \text{ in}^2$

87.5 square inches *of material is needed to create the new popcorn box.*

b. How much popcorn does the box hold?

Volume of the Box: $V = 6 \text{ in.} \times 3.5 \text{ in.} \times 3.5 \text{ in.} = 73.5 \text{ in}^3$

Closing (4 minutes)

- Is it possible for two containers having the same volume to have different surface areas? Explain.
  - *Yes, it is possible to have two containers to have the same volume but different surface areas. This was the case in Exercise 2. All four boxes would hold the same amount of baby blocks (same volume), but required a different amount of material (surface area) to create the box.*
- If you want to create an open box with dimensions 3 inches $\times$ 4 inches $\times$ 5 inches, which face should be the base if you want to minimize the amount of material you use?
  - *The face with dimensions* 4 inches $\times$ 5 inches *should be the base because that face would have the largest area.*

If students have a hard time understanding an open box, use a shoe box to demonstrate the difference between a closed box and an open box.

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 19: Surface Area and Volume in the Real World

Exit Ticket

Solve the word problem below.

Kelly has a rectangular fish aquarium with an open top that measures 18 inches long, 8 inches wide, and 12 inches tall.

a. What is the maximum amount of water in cubic inches the aquarium can hold?

b. If Kelly wanted to put a protective covering on the four glass walls of the aquarium, how big does the cover have to be?

Exit Ticket Sample Solutions

Solve the word problem below.

Kelly has a rectangular fish aquarium that measures 18 inches long, 8 inches wide, and 12 inches tall.

a. What is the maximum amount of water the aquarium can hold?

Volume of the Aquarium: $V = 18 \text{ in.} \times 8 \text{ in.} \times 12 \text{ in.} = 1728 \text{ in}^3$

The maximum amount of water the aquarium can hold is $1,728$ *cubic inches.*

b. If Kelly wanted to put a protective covering on the four glass walls of the aquarium, how big does the cover have to be?

Surface Area of the Aquarium: $SA = 2(18 \text{ in.})(8 \text{ in.}) + 2(18 \text{ in.})(12 \text{ in.}) + 2(8 \text{ in.})(12 \text{ in.}) = 288 \text{ in}^2 + 432 \text{ in}^2 + 192 \text{ in}^2 = 912 \text{ in}^2$

We only need to cover the four glass walls, so we can subtract the area of both the top and bottom of the aquarium.

Area of Top: $A = 18 \text{ in.} \times 8 \text{ in.} = 144 \text{ in}^2$

Area of Bottom: $A = 18 \text{ in.} \times 8 \text{ in.} = 144 \text{ in}^2$

Surface Area of the Four Walls: $SA = 912 \text{ in}^2 - 144 \text{ in}^2 - 144 \text{ in}^2 = 624 \text{ in}^2$

Kelly would need 624 in^2 *to cover the four walls of the aquarium.*

Problem Set Sample Solutions

Solve each problem below.

1. Dante built a wooden, cubic toy box for his son. Each side of the box measures 2 feet.

a. How many square feet of wood did he use to build the box?

Surface Area of the Box: $SA = 6(2 \text{ ft})^2 = 6(4 \text{ ft}^2) = 24 \text{ ft}^2$

Dante would need 24 *square feet of wood to build the box.*

b. How many cubic feet of toys will the box hold?

Volume of the Box: $V = 2 \text{ ft.} \times 2 \text{ ft.} \times 2 \text{ ft.} = 8 \text{ ft}^3$

The toy box would hold 8 *cubic feet of toys.*

2. A company that manufactures gift boxes wants to know how many different sized boxes having a volume of 50 cubic centimeters it can make if the dimensions must be whole centimeters.

a. List all the possible whole number dimensions for the box.

Choice One: $1 \text{ cm} \times 1 \text{ cm} \times 50 \text{ cm}$

Choice Two: $1 \text{ cm} \times 2 \text{ cm} \times 25 \text{ cm}$

Choice Three: $1 \text{ cm} \times 5 \text{ cm} \times 10 \text{ cm}$

Choice Four: $2 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm}$

b. **Which possibility requires the least amount of material to make?**

Choice One: $SA = 2(1\text{ cm})(1\text{ cm}) + 2(1\text{ cm})(50\text{ cm}) + 2(1\text{ cm})(50\text{ cm}) = 2\text{ cm}^2 + 100\text{ cm}^2 + 100\text{ cm}^2 = 202\text{ cm}^2$

Choice Two: $SA = 2(1\text{ cm})(2\text{ cm}) + 2(1\text{ cm})(25\text{ cm}) + 2(2\text{ cm})(25\text{ cm}) = 4\text{ cm}^2 + 50\text{ cm}^2 + 100\text{ cm}^2 = 154\text{ cm}^2$

Choice Three: $SA = 2(1\text{ cm})(5\text{ cm}) + 2(1\text{ cm})(10\text{ cm}) + 2(5\text{ cm})(10\text{ cm}) = 10\text{ cm}^2 + 20\text{ cm}^2 + 100\text{ cm}^2 = 130\text{ cm}^2$

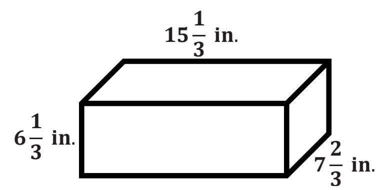
Choice Four: $SA = 2(2\text{ cm})(5\text{ cm}) + 2(2\text{ cm})(5\text{ cm}) + 2(5\text{ cm})(5\text{ cm}) = 20\text{ cm}^2 + 20\text{ cm}^2 + 50\text{ cm}^2 = 90\text{ cm}^2$

Choice Four requires the least amount of material because it has the smallest surface area.

c. **Which box would you recommend the company use? Why?**

I would recommend the company use the box with dimensions of $2\text{ cm} \times 5\text{ cm} \times 5\text{ cm}$ *(Choice Four) because it requires the least amount of material to make; so, it would cost the company the least amount of money to make.*

3. **A rectangular box of rice is shown below. How many cubic inches of rice can fit inside?**



Volume of the Rice Box: $V = 15\frac{1}{3}\text{ in.} \times 7\frac{2}{3}\text{ in.} \times 6\frac{1}{3}\text{ in.} = \frac{20102}{27}\text{ in}^3 = 744\frac{14}{27}\text{ in}^3$

4. **The Mars Cereal Company has two different cereal boxes for Mars Cereal. The large box is 8 inches wide, 11 inches high, and 3 inches deep. The small box is 6 inches wide, 10 inches high, and 2.5 inches deep.**

a. **How much more cardboard is needed to make the large box than the small box?**

Surface Area of the Large Box: $SA = 2(8\text{ in.})(11\text{ in.}) + 2(8\text{ in.})(3\text{ in.}) + 2(11\text{ in.})(3\text{ in.}) = 176\text{ in}^2 + 48\text{ in}^2 + 66\text{ in}^2 = 290\text{ in}^2$

Surface Area of the Small Box: $SA = 2(6\text{ in.})(10\text{ in.}) + 2(6\text{ in.})(2.5\text{ in.}) + 2(10\text{ in.})(2.5\text{ in.}) = 120\text{ in}^2 + 30\text{ in}^2 + 50\text{ in}^2 = 200\text{ in}^2$

Difference: $290\text{ in}^2 - 200\text{ in}^2 = 90\text{ in}^2$

The large box requires 90 square inches *more material than the small box.*

b. **How much more cereal does the large box hold than the small box?**

Volume of the Large Box: $V = 8\text{ in.} \times 11\text{ in.} \times 3\text{ in.} = 264\text{ in}^3$

Volume of the Small Box: $V = 6\text{ in.} \times 10\text{ in.} \times 2.5\text{ in.} = 150\text{ in}^3$

Difference: $264\text{ in}^3 - 150\text{ in}^3 = 114\text{ in}^3$

The large box holds 114 cubic inches *more cereal than the small box.*

5. A swimming pool is 8 meters long, 6 meters wide, and 2 meters deep. The water-resistant paint needed for the pool costs $\$6$ per square meter. How much will it cost to paint the pool?

   a. How many faces of the pool do you have to paint?

   You will have to point 5 faces.

   b. How much paint (in square meters) do you need to paint the pool?

   $SA = 2(8\text{ m} \times 6\text{ m}) + 2(8\text{ m} \times 2\text{ m}) + 2(6\text{ m} \times 2\text{ m}) = 96\text{ m}^2 + 32\text{ m}^2 + 24\text{ m}^2 = 152\text{ m}^2$

   Area of Top of Pool: $8\text{ m} \times 6\text{ m} = 48\text{ m}^2$

   Total Paint Needed: $152\text{ m}^2 - 48\text{ m}^2 = 104\text{ m}^2$

   c. How much will it cost to paint the pool?

   $104\text{ m}^2 \times \$6 = \$624$

   It will cost $\$624$ to paint the pool.

6. Sam is in charge of filling a rectangular hole with cement. The hole is 9 feet long, 3 feet wide, and 2 feet deep. How much cement will Sam need?

   $V = 9\text{ ft.} \times 3\text{ ft.} \times 2\text{ ft.} = 54\text{ ft}^3$

   Sam will need 54 cubic feet of cement to fill the hole.

7. The volume of Box D subtracted from the volume of Box C is 23.14 cubic centimeters. Box D has a volume of 10.115 cubic centimeters.

   a. Let C be the volume of Box C in cubic centimeters. Write an equation that could be used to determine the volume of Box C.

   $C - 10.115\text{ cm}^3 = 23.14\text{ cm}^3$

   b. Solve the equation to determine the volume of Box C.

   $C - 10.115\text{ cm}^3 + 10.115\text{ cm}^3 = 23.14\text{ cm}^3 + 10.115\text{ cm}^3$

   $C = 33.255\text{ cm}^3$

   c. The volume of Box C is one-tenth the volume of another box, Box E. Let E represent the volume of Box E in cubic centimeters. Write an equation that could be used to determine the volume of Box E, using the result from part (b).

   $33.255\text{ cm}^3 = \frac{1}{10}E$

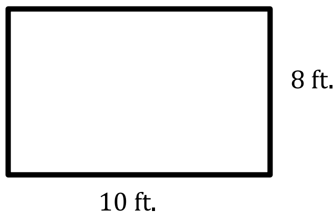
   d. Solve the equation to determine the volume of Box E.

   $33.255\text{ cm}^3 \div \frac{1}{10} = \frac{1}{10}E \div \frac{1}{10}$

   $332.55\text{ cm}^3 = E$

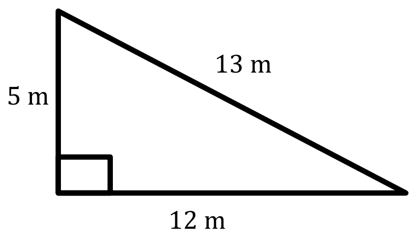
Area of Shapes

1.



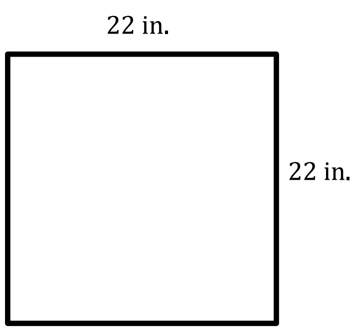
$A = 80\ \text{ft}^2$

2.



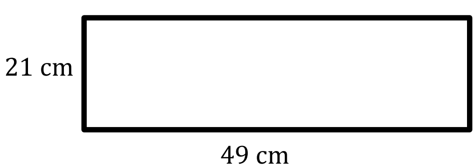
$A = 30\ \text{m}^2$

3.



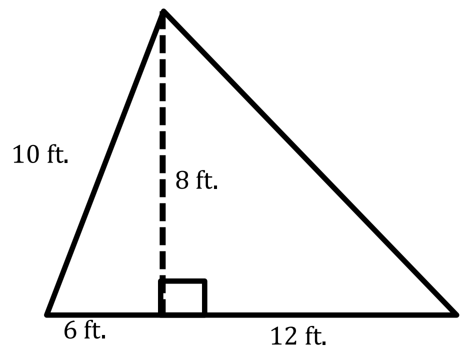
$A = 484\ \text{in}^2$

4.



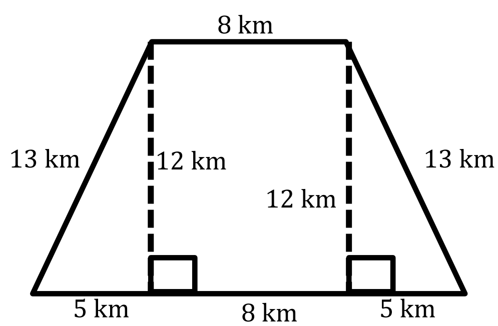
$A = 1,029\ \text{cm}^2$

5.



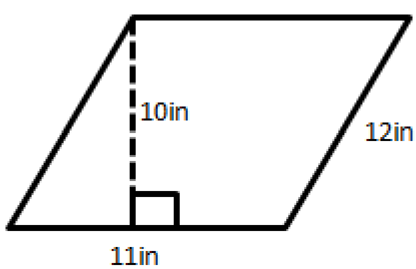
$A = 72 \text{ ft}^2$

6.



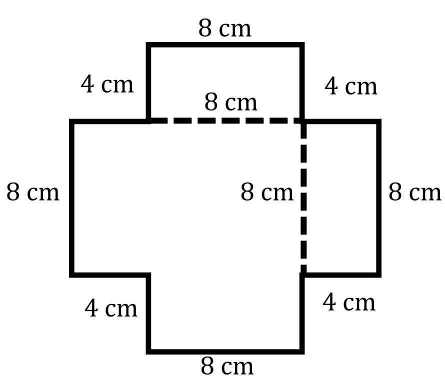
$A = 156 \text{ km}^2$

7.



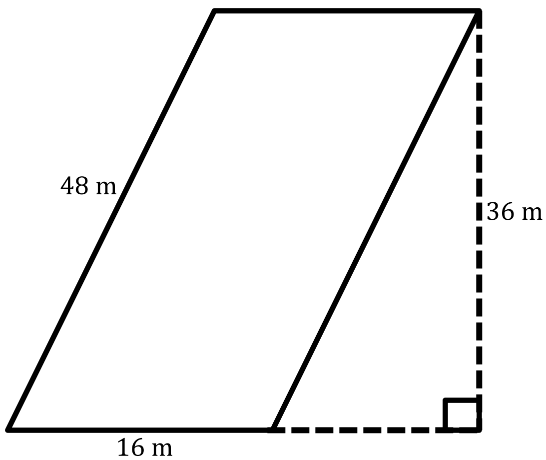
$A = 110 \text{ in}^2$

8.



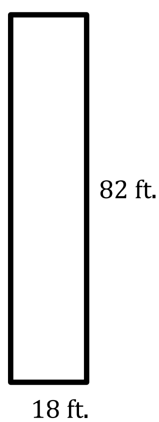
$A = 192 \text{ cm}^2$

9.



$A = 576\text{ m}^2$

10.



$A = 1,476\text{ ft}^2$