Lesson 2: The Area of Right Triangles

Student Outcomes

- Students justify the area formula for a right triangle by viewing the right triangle as part of a rectangle composed of two right triangles.

Lesson Notes

For students to complete the Exploratory Challenge, they will need the attached templates to this lesson, as well as scissors, a ruler, and glue. Students may need more than one copy of each triangle.

Students will use the attached template to develop the formula necessary to calculate the area of a right triangle. The templates will also allow students to visualize why the area of a right triangle is exactly half of the area of a rectangle with the same dimensions. They will calculate the area of two different right triangles to see that the formula works for more than just the first triangle given. Once students develop the formula, they can use substitution and the given dimensions to calculate the area.

Classwork

Discussion (1 minute)

- What are some properties of a right triangle?
  - *Three-sided polygon.*
  - *One interior angle must be exactly* $90°$.

Exploratory Challenge (14 minutes)

MP.1

Students work in groups of 2–3 to discover the formula that can be used to calculate the area of a right triangle. Each group will need the templates attached to this lesson, glue, a ruler, and scissors.

Exploratory Challenge

a. **Use the shapes labeled with an X to predict the formula needed to calculate the area of a right triangle. Explain your prediction.**

Formula for the area of right triangles:

$$A = \frac{1}{2} \times \text{base} \times \text{height or } A = \frac{\text{base} \times \text{height}}{2}$$

Area of the given triangle:

$$A = \frac{1}{2} \times 3 \text{ in.} \times 2 \text{ in.} = 3 \text{ in}^2$$

Scaffolding:

It students are struggling, use some guiding questions:

- What do you know about the area of a rectangle?
- How are the area of a triangle and rectangle related?
- Can you fit the triangle inside the rectangle?

MP.1

b. **Use the shapes labeled with a Y to determine if the formula you discovered in part (a) is correct.**

Does your area formula for triangle Y match the formula you got for triangle X?

Answers will vary; however, the area formulas should be the same if students discovered the correct area formula.

If so, do you believe you have the correct formula needed to calculate the area of a right triangle? Why or why not?

Answers will vary.

If not, which formula do you think is correct? Why?

Answers will vary.

Area of the given triangle:

$A = \frac{1}{2} \times 3 \text{ in.} \times 3 \text{ in.} = 4.5 \text{ in}^2$

Discussion (5 minutes)

- What is the area formula for right triangles?
  - *The area formula of a right triangle is* $A = \frac{1}{2}bh$*, or* $A = \frac{bh}{2}$.
- How do we know this formula is correct?
  - *Each right triangle represents half of a rectangle. The area formula of a rectangle is* $A = bh$*, but since a right triangle only covers half the area of a rectangle, we take the area of the rectangle and multiply it by half, or divide by* 2.
- How can we determine which side of a right triangle is the base and which side is the height?
  - *Similar to a parallelogram, the base and the height of a right triangle are perpendicular to each other, so they form the right angle of the triangle. However, it does not matter which of these two sides are labeled the base and which is labeled the height. The commutative property of multiplication allows us to calculate the formula in any order.*

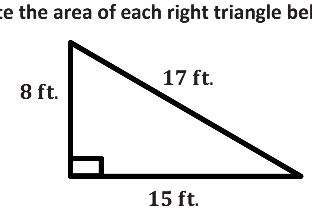
Exercises (15 minutes)

Students complete each exercise independently. Students may use a calculator.

Exercises

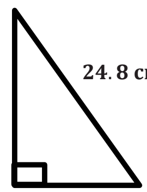
Calculate the area of each right triangle below. Each figure is not drawn to scale.

1.



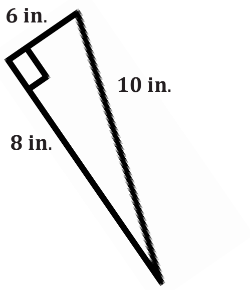
$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(8 \text{ ft.})(15 \text{ ft.})$$
$$= 60 \text{ ft}^2$$

2.



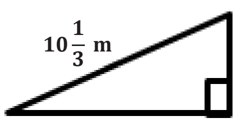
$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(11.4\text{ cm})(17.7\text{ cm})$$
$$= 100.89\text{ cm}^2$$

3.



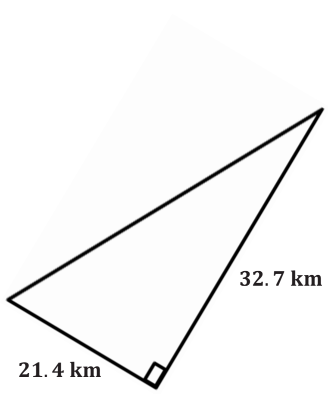
$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(6\text{ in.})(8\text{ in.})$$
$$= 24\text{ in}^2$$

4.



$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}\left(8\frac{2}{3}\text{ m}\right)\left(5\frac{3}{5}\text{ m}\right)$$
$$= \frac{1}{2}\left(\frac{26}{3}\text{ m}\right)\left(\frac{28}{5}\text{ m}\right)$$
$$= \frac{728}{30}\text{ m}^2$$
$$= 24\frac{8}{30}\text{ m}^2 \text{ or } 24\frac{4}{15}\text{ m}^2$$

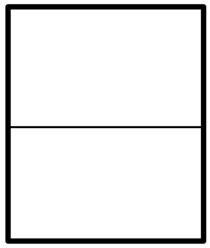
5.



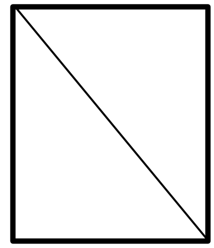
$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(32.7\text{ km})(21.4\text{ km})$$
$$= 349.89\text{ km}^2$$

6. **Mr. Jones told his students they each need a half of a piece of paper. Calvin cut his piece of paper horizontally and Matthew cut his piece of paper diagonally. Which student has the larger area on their half piece of paper? Explain.**

Calvin's Paper



Matthew's Paper



After cutting the paper, both Calvin and Matthew have the same area. Calvin cut his into two rectangles that are each half the area of the original piece of paper. Matthew cut his paper into two equivalent right triangles that are also half the area of the original piece of paper.

7. **Ben requested that the rectangular stage be split into two equal sections for the upcoming school play. The only instruction he gave was that he needed the area of each section to be half of the original size. If Ben wants the stage to be split into two right triangles, did he provide enough information? Why or why not?**

Ben did not provide enough information because the stage may be split horizontally or vertically through the middle of the rectangle. This would result in two equal pieces, but they would not be right triangles.

8. **If the area of a right triangle is 6.22 sq. in. and its base is 3.11 in., write an equation that relates the area to the height, h, and the base. Solve the equation to determine the height.**

$$6.22\ \text{in}^2 = \frac{1}{2}(3.11\ \text{in.})h$$
$$6.22\ \text{in}^2 = (1.555\ \text{in.})h$$
$$6.22\ \text{in}^2 \div 1.555\ \text{in.} = (1.555\ \text{in.})h \div 1.555\ \text{in.}$$
$$4\ \text{in.} = h$$

Therefore, the height of the right triangle is 4 in.

Closing (5 minutes)

- How are the area formulas of rectangles and right triangles related?
  - *When a rectangle and a right triangle have the same dimensions, the area of the right triangle is exactly half of the area of the rectangle. Therefore, the area formulas of rectangles and right triangles are related because the area formula of a rectangle is divided by 2 (or multiplied by a half) in order to translate it to the area formula for a right triangle.*

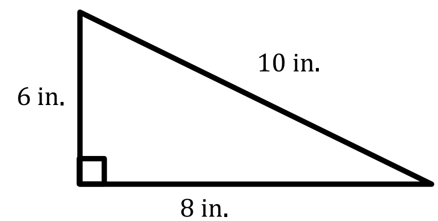
Exit Ticket (5 minutes)

Name ____ Date ____________________

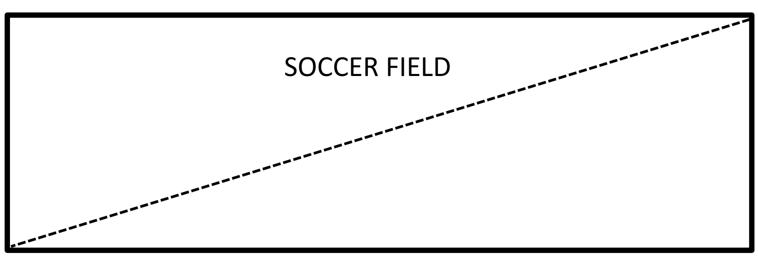
Lesson 2: The Area of Right Triangles

Exit Ticket

1. Calculate the area of the right triangle. Each figure is not drawn to scale.



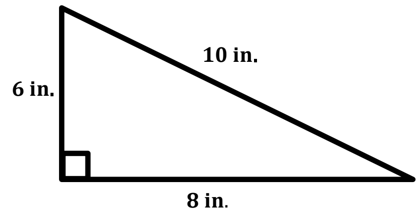
2. Dan and Joe are responsible for cutting the grass on the local high school soccer field. Joe cuts a diagonal line through the field, as shown in the diagram below, and says that each person is responsible for cutting the grass on one side of the line. Dan says that this is not fair because he will have to cut more grass than Joe. Is Dan correct? Why or why not?



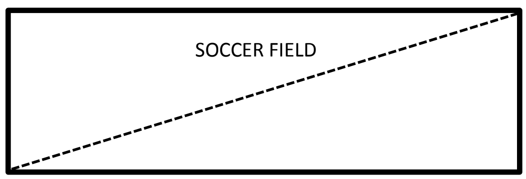
Exit Ticket Sample Solutions

1. Calculate the area of the right triangle. Each figure is not drawn to scale.

$A = \frac{1}{2}bh = \frac{1}{2}(8\text{ in.})(6\text{ in.}) = 24\text{ in}^2$



2. Dan and Joe are responsible for cutting the grass on the local high school soccer field. Joe cuts a diagonal line through the field and says that each person is responsible for cutting the grass on one side of the line. Dan says that this is not fair because he will have to cut more grass than Joe. Is Dan correct? Why or why not?

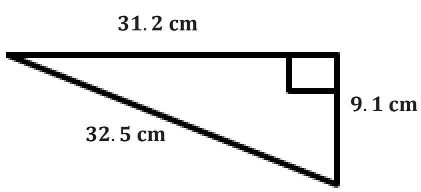


Dan is not correct. The diagonal line Joe cut in the grass would split the field into two right triangles. The area of each triangle is exactly half the area of the entire field because the area formula for a right triangle is $A = \frac{1}{2} \times \text{base} \times \text{height}$.

Problem Set Sample Solutions

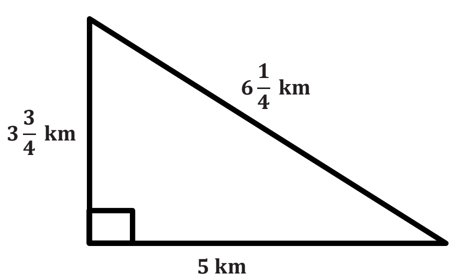
Calculate the area of each right triangle below. Note that the figures are not drawn to scale.

1.



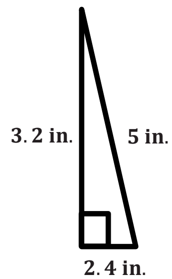
$A = \frac{1}{2}bh = \frac{1}{2}(31.2\text{ cm})(9.1\text{ cm}) = 141.96\text{ cm}^2$

2.



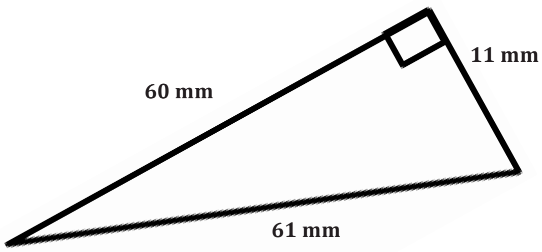
$A = \frac{1}{2}bh = \frac{1}{2}(5\text{ km})\left(3\frac{3}{4}\text{ km}\right) = \frac{1}{2}\left(\frac{5}{1}\text{ km}\right)\left(\frac{15}{4}\text{ km}\right) = \frac{75}{8}\text{ km}^2 = 9\frac{3}{8}\text{ km}^2$

3.



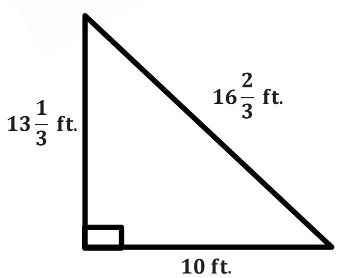
$A = \frac{1}{2}bh = \frac{1}{2}(2.4\text{ in.})(3.2\text{ in.}) = 3.84\text{ in}^2$

4.



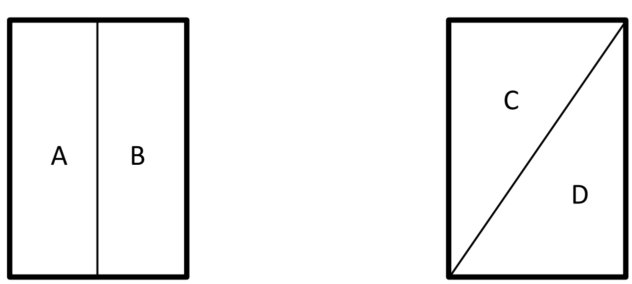
$A = \frac{1}{2}bh = \frac{1}{2}(11\text{ mm})(60\text{ mm}) = 330\text{ mm}^2$

5.



$A = \frac{1}{2}bh = \frac{1}{2}\left(13\frac{1}{3}\text{ ft.}\right)(10\text{ ft.}) = \frac{1}{2}\left(\frac{40}{3}\text{ ft.}\right)\left(\frac{10}{1}\text{ ft.}\right) = \frac{400}{6}\text{ ft}^2 = 66\frac{2}{3}\text{ ft}^2$

6. **Elania has two congruent rugs at her house. She cut one vertically down the middle, and she cut diagonally through the other one.**



After making the cuts, which rug (labeled A, B, C, or D) has the larger area? Explain.

All of the rugs are the same size after making the cuts. The vertical line goes down the center of the rectangle, making two congruent parts. The diagonal line also splits the rectangle in two congruent parts because the area of a right triangle is exactly half the area of the rectangle.

7. **Give the dimensions of a right triangle and a parallelogram with the same area. Explain how you know.**

Answers will vary.

8. **If the area of a right triangle is $\frac{9}{16}$ sq. ft. and the height is $\frac{3}{4}$ ft., write an equation that relates the area to the base, *b*, and the height. Solve the equation to determine the base.**

$$\frac{9}{16}\ \text{ft}^2 = \frac{1}{2}b\left(\frac{3}{4}\ \text{ft.}\right)$$

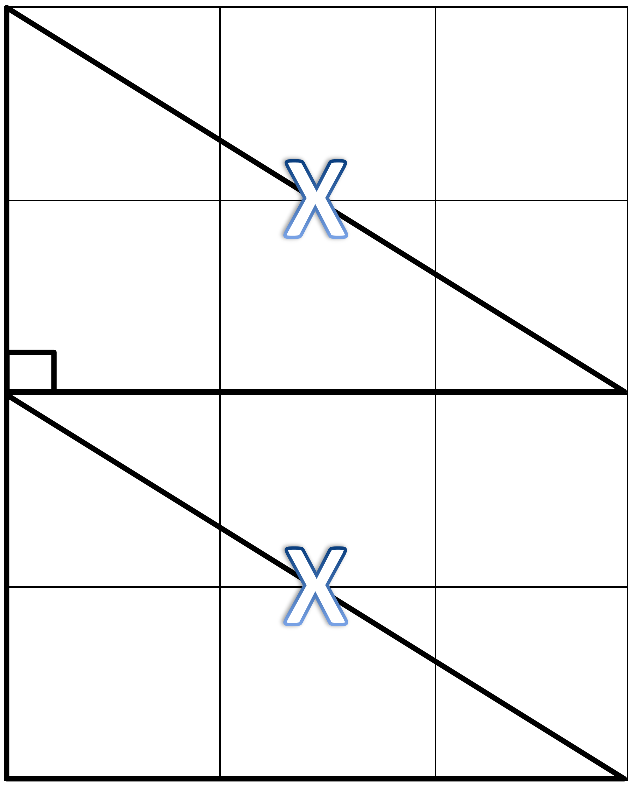
$$\frac{9}{16}\ \text{ft}^2 = \left(\frac{3}{8}\ \text{ft.}\right)b$$

$$\frac{9}{16}\ \text{ft}^2 \div \frac{3}{8}\ \text{ft.} = \left(\frac{3}{8}\ \text{ft.}\right)b \div \frac{3}{8}\ \text{ft.}$$

$$\frac{3}{2}\ \text{ft.} = b$$

$$1\frac{1}{2}\ \text{ft.} = b$$

Therefore, the base of the right triangle is $1\frac{1}{2}$ ft.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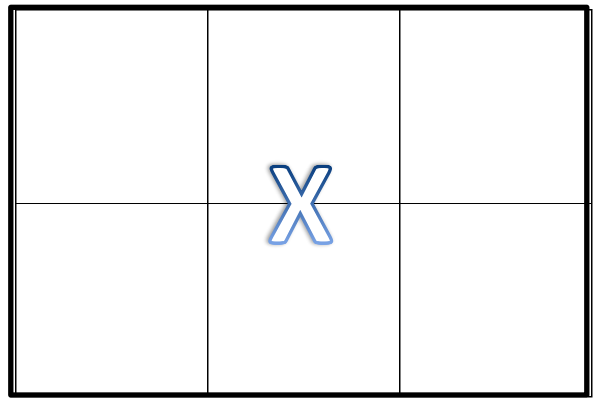
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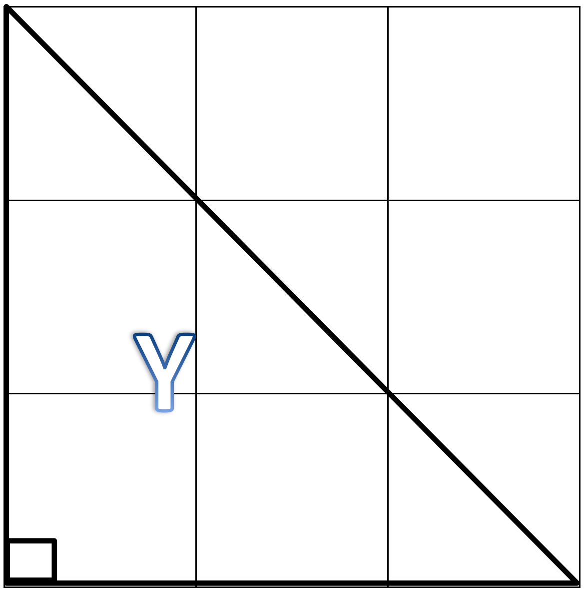
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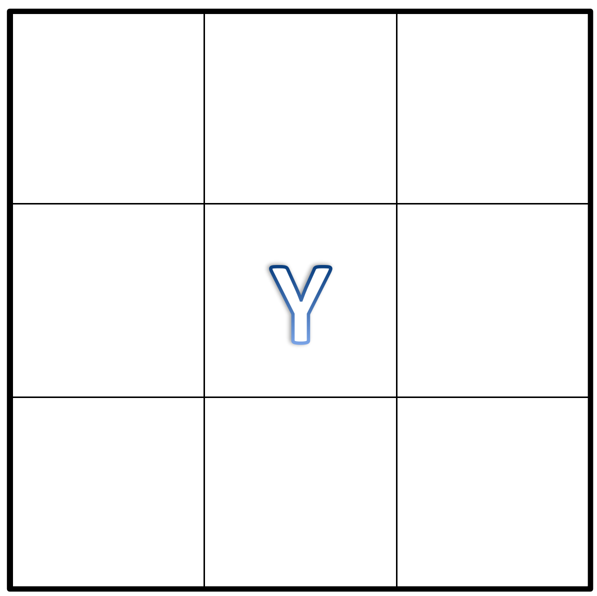
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