Lesson 1: The Area of Parallelograms Through Rectangle Facts

Student Outcomes

- Students show the area formula for the region bounded by a parallelogram by composing it into rectangles. They understand that the area of a parallelogram is the area of the region bounded by the parallelogram.

Lesson Notes

In order to participate in the discussions, each student will need the parallelogram templates attached to this lesson, along with the following: scissors, glue, ruler, and paper on which to glue their shapes.

Classwork

Fluency Exercise (5 minutes): Multiplication of Fractions

Sprint: Refer to the Sprints and the Sprint Delivery Script sections in the Module Overview for directions to administer a Sprint.

Opening Exercise (4 minutes)

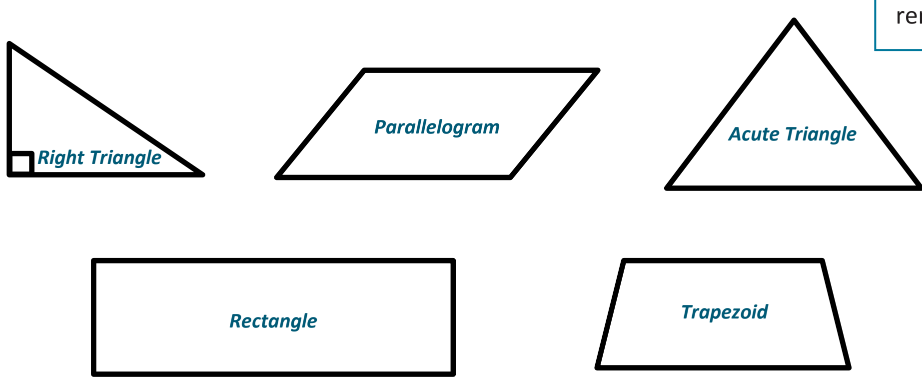
Students name the given shapes.

Scaffolding:

Some students may not know this vocabulary yet, so creating a poster or chart for student desks may help them to remember these terms.

Opening Exercise

Name each shape.



- Identify the shape that is commonly referred to as a parallelogram. How do you know it is a parallelogram?

Note: A rectangle is considered a parallelogram, but is commonly called a rectangle because it is a more specific name.

  - *The shape is a quadrilateral (4-sided) and has two sets of parallel lines.*

- What are some quadrilaterals that you know?
  - *Answers will vary.*
- Today, we are going to find the area of one of these quadrilaterals: the parallelogram. We are going to use our knowledge of the area of rectangles to help us. Who can remind us what we mean by area?
  - *The number of square units that make up the inside of the shape.*

Note: English language learners would benefit from a further discussion of area that relates to things they have personal connections to.

- Talk to your neighbor about how to calculate the area of a rectangle.

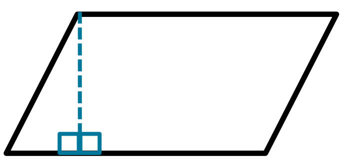
Pick someone who can clearly explain how to find the area of a rectangle.

  - *Count the number of square units inside the shape (if that is given), or multiply the base by the height.*

Discussion (10 minutes)

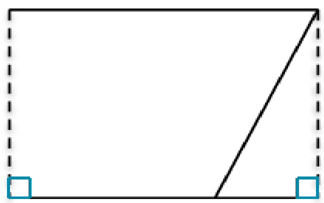
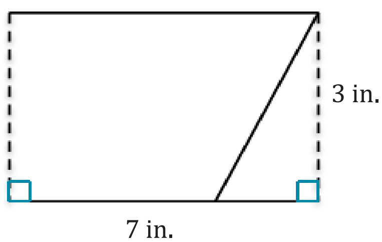
MP.7

Provide each student with the picture of a parallelogram provided as an attachment to this lesson.

- What shape do you have in front of you?
  - *A parallelogram.*
- Work with a partner to make a prediction of how we would calculate the area of the shape.
  - *Answers will vary.*
- Cut out the parallelogram.
- Since we know how to find the area of a rectangle, how can we change the parallelogram into a rectangle?
  - *Cut off a right triangle on one side of the parallelogram, and glue it to the other side.*
- Draw a dotted line, perpendicular to the base, to show the triangle you will cut. Fold your paper along this line.
  - 

  Check to make sure all students have drawn the dotted line in the correct place before instructing them to cut. Explain that the fold on the line shows that the two right angles form a $180°$ angle.
- Could the dotted line be drawn in a different location? If so, where?
  - *The dotted line can be drawn in a different location. It could be drawn on the other side of the parallelogram, displayed below.*

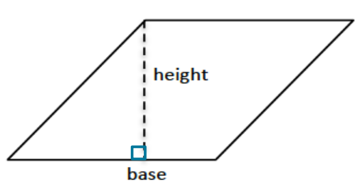
- The base and height of a parallelogram form a right angle.
- Measure, in inches, the base and height of the parallelogram using the correct mathematical tools.
  - *The base is 7 inches, and the height is 3 inches.*

- Cut along the dotted line.
- Glue both parts of the parallelogram onto a piece of paper to make a rectangle.
  - 
- Why is the new shape classified as a rectangle?
  - *The new shape is a rectangle because it is a quadrilateral that has four right angles.*
- Use the correct mathematical tool to measure, in inches, and label each side of the rectangle created from the original parallelogram.
  - 

MP.7

- How do these measurements compare to the base and height of the parallelogram?
  - *They are the same.*
- When we moved the right triangle, did the area inside the shape change? Explain.
  - *The area did not change because both shapes are the same size. The original quadrilateral just looks different.*
- What is the area of the rectangle?
  - 21 *square inches or* 21 *inches squared or* 21 in^2.

Note: English language learners would benefit from a discussion on why all three of these answers represent the same value.

- If the area of the rectangle is 21 square inches, what is the area of the original parallelogram? Why?
  - *The area of the original parallelogram is also* 21 *square inches because both shapes have the same amount of space inside.*
- We know the formula for the area of a rectangle is $\text{Area} = \text{base} \times \text{height}$, or $A = bh$. What is the formula to calculate the area of a parallelogram?
  - *The formula to calculate the area of a parallelogram would be the same as a rectangle,* $A = bh$.
- Examine the given parallelogram, and label the base and height.
  - 

MP.7

- Why is the height the vertical line and not the slanted edge?

Note: English language learners may need a further explanation of the meaning of the slanted edge.

  - *If we look back to the rectangle we created, the base and height of both the rectangle and the original parallelogram are perpendicular to each other. Therefore, the height of a parallelogram is the perpendicular line segment drawn from the top base to the bottom base.*

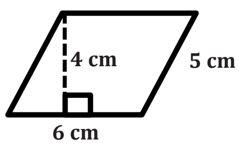
Exercise 1 (5 minutes)

Students work individually to complete the following problems.

Exercises

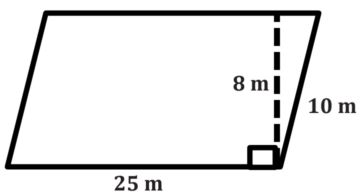
1. Find the area of each parallelogram below. Note that the figures are not drawn to scale.

a.



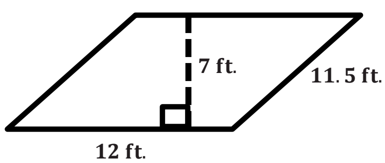
$A = bh$
$= 6\text{ cm}(4\text{ cm})$
$= 24\text{ cm}^2$

b.



$A = bh$
$= 25\text{ m}(8\text{ m})$
$= 200\text{ m}^2$

c.



$A = bh$
$= 12\text{ ft.}(7\text{ ft.})$
$= 84\text{ ft}^2$

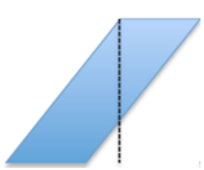
Scaffolding:

English language learners may need some clarification about what it means to not be drawn to scale and why this may be the case.

Discussion (8 minutes)

Give each student a copy of the slanted parallelogram shown below.

- How could we construct a rectangle from this parallelogram?
  - *Answers will vary.*
- Why can't we use the same method we used previously?
  - *The vertical dotted line does not go through the entire parallelogram.*



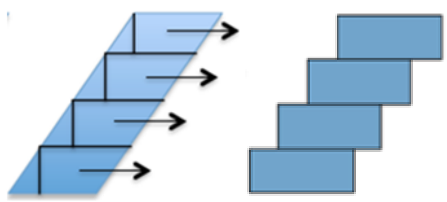
Students may struggle drawing the height because they will not be sure whether part of the height can be outside of the parallelogram.

- Cut out the shape.
- To solve this problem, we are actually going to cut the parallelogram horizontally into four equal pieces. Use the appropriate measurement tool to determine where to make the cuts.

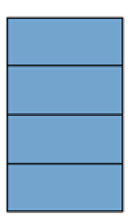
Allow time for students to think about how to approach this problem. If time allows, have students share their thoughts before the teacher demonstrates how to move forward.

Demonstrate these cuts before allowing students to make the cuts.

- We have four parallelograms. How can we use them to calculate the area of the original parallelogram?
  - *Turn each of the parallelograms into rectangles.*
- How can we make these parallelograms into rectangles?
  - *Cut a right triangle off of every parallelogram, and move the right triangle to the other side of the parallelogram.*

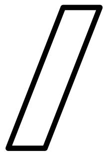


- How can we show that the original parallelogram forms a rectangle?
  - *If we push all the rectangles together, they will form one rectangle.*

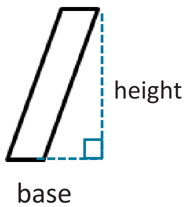


- Therefore, it does not matter how tilted a parallelogram is. The formula to calculate the area will always be the same as the area formula of a rectangle.

- Draw and label the height of the parallelogram below.



  - *Students should connect two dotted lines as below:*

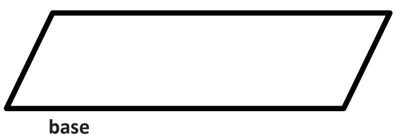


Exercise 2 (5 minutes)

Students complete the exercises individually.

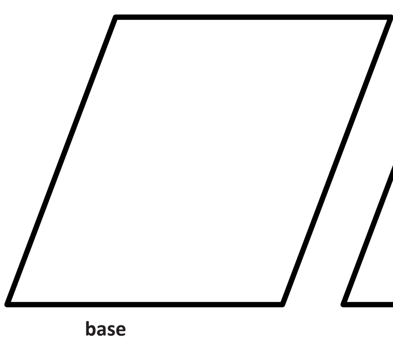
2. Draw and label the height of each parallelogram. Use the correct mathematical tool to measure (in inches) the base and height, and calculate the area of each parallelogram.

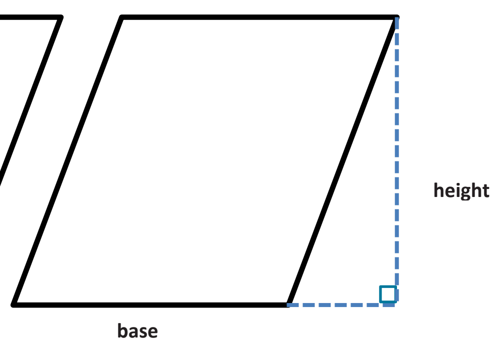
a.



$A = bh = (0.5\text{ in.})(2\text{ in.}) = 1\text{ in}^2$

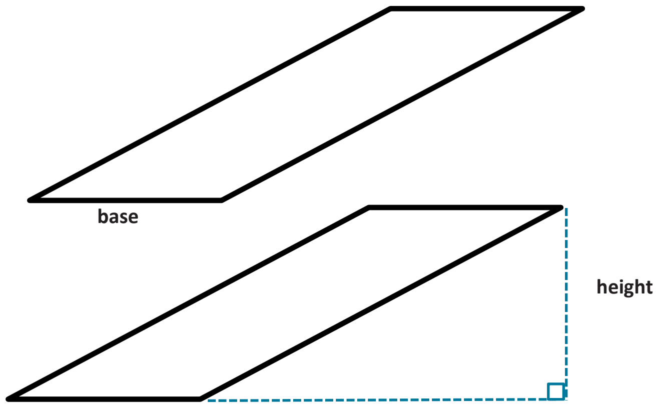
b.





$A = bh = (1.5\text{ in.})(2\text{ in.}) = 3\text{ in}^2$

c.



$A = bh = (1 \text{ in.})(1 \text{ in.}) = 1 \text{ in}^2$

3. If the area of a parallelogram is $\frac{35}{42}$ cm^2 and the height is $\frac{1}{7}$ cm, write an equation that relates the height, base, and area of the parallelogram. Solve the equation.

$$\frac{35}{42} \text{ cm}^2 = b\left(\frac{1}{7} \text{ cm}\right)$$
$$\frac{35}{42} \text{ cm}^2 \div \frac{1}{7} \text{ cm} = b\left(\frac{1}{7} \text{ cm}\right) \div \frac{1}{7} \text{cm}$$
$$\frac{35}{6} \text{cm} = b$$
$$5\frac{5}{6} \text{ cm} = b$$

Scaffolding:

English language learners may benefit from a sentence starter such as "The formulas are the same because …"

Closing (3 minutes)

- Why are the area formulas for rectangles and parallelograms the same?
  - *The area formulas for rectangles and parallelograms are the same because a parallelogram can be changed to a rectangle. By cutting a right triangle from one side of the parallelogram and connecting it to the other side of the parallelogram, a rectangle is formed.*

Lesson Summary

The formula to calculate the area of a parallelogram is $A = bh$, where b represents the base and h represents the height of the parallelogram.

The height of a parallelogram is the line segment perpendicular to the base. The height is usually drawn from a vertex that is opposite the base.

Exit Ticket (5 minutes)

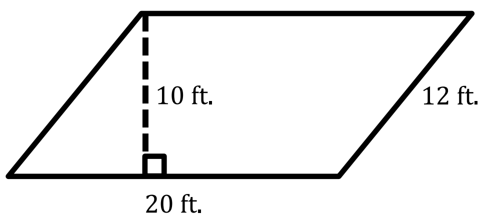
Name ____ Date ____________________

Lesson 1: The Area of Parallelograms Through Rectangle Facts

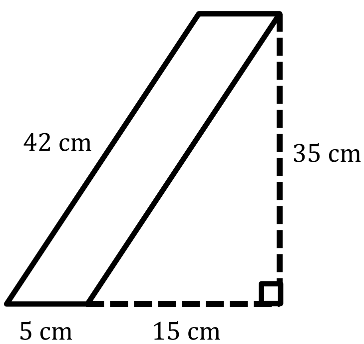
Exit Ticket

Calculate the area of each parallelogram. Note that the figures are not drawn to scale.

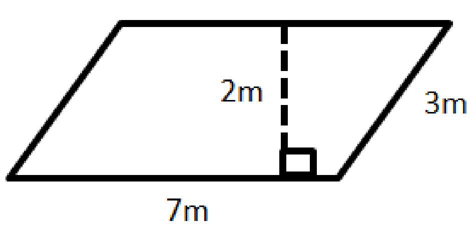
1.



2.



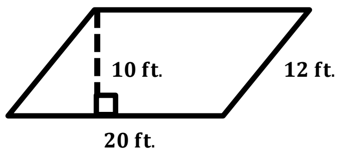
3.



Exit Ticket Sample Solutions

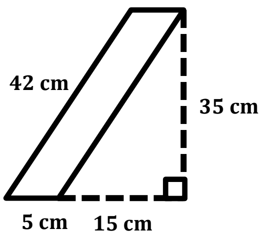
Calculate the area of each parallelogram. Note that the figures are not drawn to scale.

1.



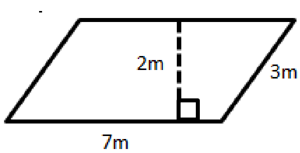
$A = bh = 20 \text{ ft.} (10 \text{ ft.}) = 200 \text{ ft}^2$

2.



$A = bh = 5 \text{ cm} (35 \text{ cm}) = 175 \text{ cm}^2$

3.

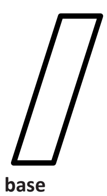


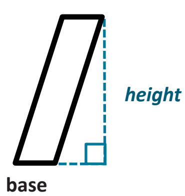
$A = bh = 7 \text{ m}(2 \text{ m}) = 14 \text{ m}^2$

Problem Set Sample Solutions

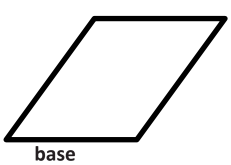
Draw and label the height of each parallelogram.

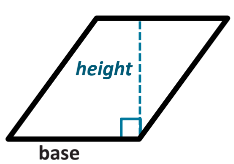
1.





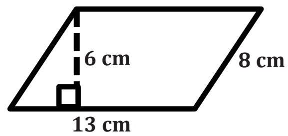
2.





Calculate the area of each parallelogram. The figures are not drawn to scale.

3.

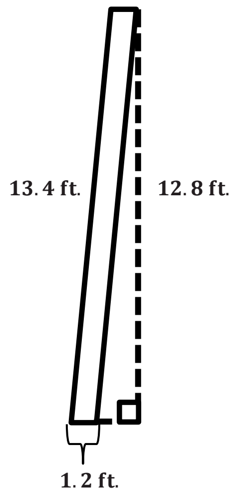


$A = bh$

$= 13\text{ cm}(6\text{ cm})$

$= 78\text{ cm}^2$

4.

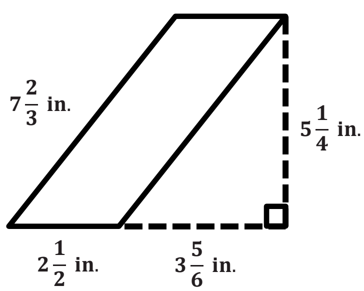


$A = bh$

$= 1.2\text{ ft.}(12.8\text{ ft.})$

$= 15.36\text{ ft}^2$

5.



$A = bh$

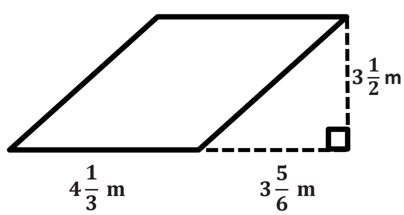
$= 2\frac{1}{2}\text{ in.}\left(5\frac{1}{4}\text{ in.}\right)$

$= \frac{5}{2}\text{ in.}\left(\frac{21}{4}\text{ in.}\right)$

$= \frac{105}{8}\text{ in}^2$

$= 13\frac{1}{8}\text{ in}^2$

6.



$A = bh$

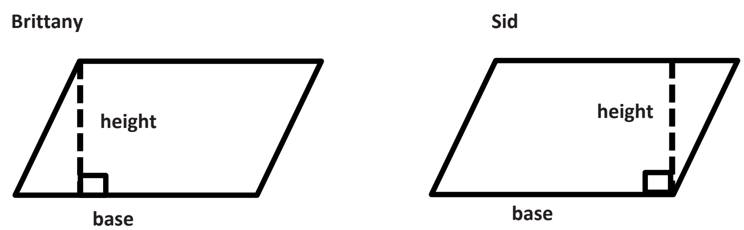
$= 4\frac{1}{3}\text{ m}\left(3\frac{1}{2}\text{ m}\right)$

$= \frac{13}{3}\text{ m}\left(\frac{7}{2}\text{ m}\right)$

$= \frac{91}{6}\text{ m}^2$

$= 15\frac{1}{6}\text{ m}^2$

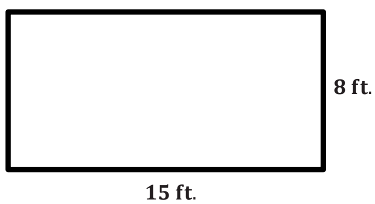
7. **Brittany and Sid were both asked to draw the height of a parallelogram. Their answers are below.**

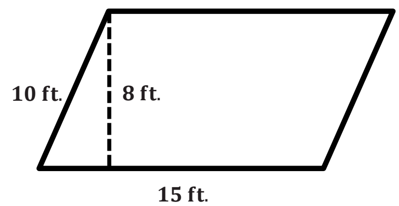


Are both Brittany and Sid correct? If not, who is correct? Explain your answer.

Both Brittany and Sid are correct because both of their heights represent a line segment that is perpendicular to the base and whose endpoint is on the opposite side of the parallelogram.

8. **Do the rectangle and parallelogram below have the same area? Explain why or why not.**





Yes, the rectangle and parallelogram have the same area because if we cut off the right triangle on the left side of the parallelogram, we can move it over to the right side and make the parallelogram into a rectangle. At this time, both rectangles would have the same dimensions; therefore, their areas would be the same.

9. **A parallelogram has an area of 20.3 sq. cm and a base of 2.5 cm. Write an equation that relates the area to the base and height, h. Solve the equation to determine the length of the height.**

$$20.3\text{ cm}^2 = 2.5\text{ cm}(h)$$
$$20.3\text{ cm}^2 \div 2.5\text{ cm} = 2.5\text{ cm}(h) \div 2.5\text{ cm}$$
$$8.12\text{ cm} = h$$

Therefore, the height of the parallelogram is 8.12 cm.

Number Correct: ______

Multiplication of Fractions—Round 1

Directions: Determine the product of the fractions.

1.	$\frac{1}{2} \times \frac{3}{4}$		16.	$\frac{8}{9} \times \frac{3}{4}$	
2.	$\frac{5}{6} \times \frac{5}{7}$		17.	$\frac{3}{4} \times \frac{4}{7}$	
3.	$\frac{3}{4} \times \frac{7}{8}$		18.	$\frac{1}{4} \times \frac{8}{9}$	
4.	$\frac{4}{5} \times \frac{8}{9}$		19.	$\frac{3}{5} \times \frac{10}{11}$	
5.	$\frac{1}{4} \times \frac{3}{7}$		20.	$\frac{8}{13} \times \frac{7}{24}$	
6.	$\frac{5}{7} \times \frac{4}{9}$		21.	$2\frac{1}{2} \times 3\frac{3}{4}$	
7.	$\frac{3}{5} \times \frac{1}{8}$		22.	$1\frac{4}{5} \times 6\frac{1}{3}$	
8.	$\frac{2}{9} \times \frac{7}{9}$		23.	$8\frac{2}{7} \times 4\frac{5}{6}$	
9.	$\frac{1}{3} \times \frac{2}{5}$		24.	$5\frac{2}{5} \times 2\frac{1}{8}$	
10.	$\frac{3}{7} \times \frac{5}{8}$		25.	$4\frac{6}{7} \times 1\frac{1}{4}$	
11.	$\frac{2}{3} \times \frac{9}{10}$		26.	$2\frac{2}{3} \times 4\frac{2}{5}$	
12.	$\frac{3}{5} \times \frac{1}{6}$		27.	$6\frac{9}{10} \times 7\frac{1}{3}$	
13.	$\frac{2}{7} \times \frac{3}{4}$		28.	$1\frac{3}{8} \times 4\frac{2}{5}$	
14.	$\frac{5}{8} \times \frac{3}{10}$		29.	$3\frac{5}{6} \times 2\frac{4}{15}$	
15.	$\frac{4}{5} \times \frac{7}{8}$		30.	$4\frac{1}{3} \times 5$	

Multiplication of Fractions—Round 1 [KEY]

Directions: Determine the product of the fractions.

1.	$\frac{1}{2}\times\frac{3}{4}$	$\frac{3}{8}$	16.	$\frac{8}{9}\times\frac{3}{4}$	$\frac{24}{36}=\frac{2}{3}$
2.	$\frac{5}{6}\times\frac{5}{7}$	$\frac{25}{42}$	17.	$\frac{3}{4}\times\frac{4}{7}$	$\frac{12}{28}=\frac{3}{7}$
3.	$\frac{3}{4}\times\frac{7}{8}$	$\frac{21}{32}$	18.	$\frac{1}{4}\times\frac{8}{9}$	$\frac{8}{36}=\frac{2}{9}$
4.	$\frac{4}{5}\times\frac{8}{9}$	$\frac{32}{45}$	19.	$\frac{3}{5}\times\frac{10}{11}$	$\frac{30}{55}=\frac{6}{11}$
5.	$\frac{1}{4}\times\frac{3}{7}$	$\frac{3}{28}$	20.	$\frac{8}{13}\times\frac{7}{24}$	$\frac{56}{312}=\frac{7}{39}$
6.	$\frac{5}{7}\times\frac{4}{9}$	$\frac{20}{63}$	21.	$2\frac{1}{2}\times 3\frac{3}{4}$	$\frac{75}{8}=9\frac{3}{8}$
7.	$\frac{3}{5}\times\frac{1}{8}$	$\frac{3}{40}$	22.	$1\frac{4}{5}\times 6\frac{1}{3}$	$\frac{171}{15}=11\frac{2}{5}$
8.	$\frac{2}{9}\times\frac{7}{9}$	$\frac{14}{81}$	23.	$8\frac{2}{7}\times 4\frac{5}{6}$	$\frac{1682}{42}=40\frac{1}{21}$
9.	$\frac{1}{3}\times\frac{2}{5}$	$\frac{2}{15}$	24.	$5\frac{2}{5}\times 2\frac{1}{8}$	$\frac{459}{40}=11\frac{19}{40}$
10.	$\frac{3}{7}\times\frac{5}{8}$	$\frac{15}{56}$	25.	$4\frac{6}{7}\times 1\frac{1}{4}$	$\frac{170}{28}=6\frac{1}{14}$
11.	$\frac{2}{3}\times\frac{9}{10}$	$\frac{18}{30}=\frac{3}{5}$	26.	$2\frac{2}{3}\times 4\frac{2}{5}$	$\frac{176}{15}=11\frac{11}{15}$
12.	$\frac{3}{5}\times\frac{1}{6}$	$\frac{3}{30}=\frac{1}{10}$	27.	$6\frac{9}{10}\times 7\frac{1}{3}$	$\frac{1518}{30}=50\frac{3}{5}$
13.	$\frac{2}{7}\times\frac{3}{4}$	$\frac{6}{28}=\frac{3}{14}$	28.	$1\frac{3}{8}\times 4\frac{2}{5}$	$\frac{242}{40}=6\frac{1}{20}$
14.	$\frac{5}{8}\times\frac{3}{10}$	$\frac{15}{80}=\frac{3}{16}$	29.	$3\frac{5}{6}\times 2\frac{4}{15}$	$\frac{782}{90}=8\frac{31}{45}$
15.	$\frac{4}{5}\times\frac{7}{8}$	$\frac{28}{40}=\frac{7}{10}$	30.	$4\frac{1}{3}\times 5$	$\frac{65}{3}=21\frac{2}{3}$

Number Correct: _______

Improvement: _______

Multiplication of Fractions—Round 2

Directions: Determine the product of the fractions.

1.	$\frac{5}{6} \times \frac{1}{4}$		16.	$\frac{3}{7} \times \frac{2}{9}$	
2.	$\frac{2}{3} \times \frac{5}{7}$		17.	$\frac{4}{5} \times \frac{10}{13}$	
3.	$\frac{1}{3} \times \frac{2}{5}$		18.	$\frac{2}{9} \times \frac{3}{8}$	
4.	$\frac{5}{7} \times \frac{5}{8}$		19.	$\frac{1}{8} \times \frac{4}{5}$	
5.	$\frac{3}{8} \times \frac{7}{9}$		20.	$\frac{3}{7} \times \frac{2}{15}$	
6.	$\frac{3}{4} \times \frac{5}{6}$		21.	$1\frac{1}{2} \times 4\frac{3}{4}$	
7.	$\frac{2}{7} \times \frac{3}{8}$		22.	$2\frac{5}{6} \times 3\frac{3}{8}$	
8.	$\frac{1}{4} \times \frac{3}{4}$		23.	$1\frac{7}{8} \times 5\frac{1}{5}$	
9.	$\frac{5}{8} \times \frac{3}{10}$		24.	$6\frac{2}{3} \times 2\frac{3}{8}$	
10.	$\frac{6}{11} \times \frac{1}{2}$		25.	$7\frac{1}{2} \times 3\frac{6}{7}$	
11.	$\frac{6}{7} \times \frac{5}{8}$		26.	$3 \times 4\frac{1}{3}$	
12.	$\frac{1}{6} \times \frac{9}{10}$		27.	$2\frac{3}{5} \times 5\frac{1}{6}$	
13.	$\frac{3}{4} \times \frac{8}{9}$		28.	$4\frac{2}{5} \times 7$	
14.	$\frac{5}{6} \times \frac{2}{3}$		29.	$1\frac{4}{7} \times 2\frac{1}{2}$	
15.	$\frac{1}{4} \times \frac{8}{11}$		30.	$3\frac{5}{6} \times \frac{3}{10}$	

Multiplication of Fractions—Round 2 [KEY]

Directions: Determine the product of the fractions.

1.	$\frac{5}{6}\times\frac{1}{4}$	$\frac{5}{24}$	16.	$\frac{3}{7}\times\frac{2}{9}$	$\frac{6}{63}=\frac{2}{21}$
2.	$\frac{2}{3}\times\frac{5}{7}$	$\frac{10}{21}$	17.	$\frac{4}{5}\times\frac{10}{13}$	$\frac{40}{65}=\frac{8}{13}$
3.	$\frac{1}{3}\times\frac{2}{5}$	$\frac{2}{15}$	18.	$\frac{2}{9}\times\frac{3}{8}$	$\frac{6}{72}=\frac{1}{12}$
4.	$\frac{5}{7}\times\frac{5}{8}$	$\frac{25}{56}$	19.	$\frac{1}{8}\times\frac{4}{5}$	$\frac{4}{40}=\frac{1}{10}$
5.	$\frac{3}{8}\times\frac{7}{9}$	$\frac{21}{72}=\frac{7}{24}$	20.	$\frac{3}{7}\times\frac{2}{15}$	$\frac{6}{105}=\frac{2}{35}$
6.	$\frac{3}{4}\times\frac{5}{6}$	$\frac{15}{24}=\frac{5}{8}$	21.	$1\frac{1}{2}\times4\frac{3}{4}$	$\frac{57}{8}=7\frac{1}{8}$
7.	$\frac{2}{7}\times\frac{3}{8}$	$\frac{6}{56}=\frac{3}{28}$	22.	$2\frac{5}{6}\times3\frac{3}{8}$	$\frac{459}{48}=9\frac{9}{16}$
8.	$\frac{1}{4}\times\frac{3}{4}$	$\frac{3}{16}$	23.	$1\frac{7}{8}\times5\frac{1}{5}$	$\frac{390}{40}=9\frac{3}{4}$
9.	$\frac{5}{8}\times\frac{3}{10}$	$\frac{15}{80}=\frac{3}{16}$	24.	$6\frac{2}{3}\times2\frac{3}{8}$	$\frac{380}{24}=15\frac{5}{6}$
10.	$\frac{6}{11}\times\frac{1}{2}$	$\frac{6}{22}=\frac{3}{11}$	25.	$7\frac{1}{2}\times3\frac{6}{7}$	$\frac{405}{14}=28\frac{13}{14}$
11.	$\frac{6}{7}\times\frac{5}{8}$	$\frac{30}{56}=\frac{15}{28}$	26.	$3\times4\frac{1}{3}$	$\frac{39}{3}=13$
12.	$\frac{1}{6}\times\frac{9}{10}$	$\frac{9}{60}=\frac{3}{20}$	27.	$2\frac{3}{5}\times5\frac{1}{6}$	$\frac{403}{30}=13\frac{13}{30}$
13.	$\frac{3}{4}\times\frac{8}{9}$	$\frac{24}{36}=\frac{2}{3}$	28.	$4\frac{2}{5}\times7$	$\frac{154}{5}=30\frac{4}{5}$
14.	$\frac{5}{6}\times\frac{2}{3}$	$\frac{10}{18}=\frac{5}{9}$	29.	$1\frac{4}{7}\times2\frac{1}{2}$	$\frac{55}{14}=3\frac{13}{14}$
15.	$\frac{1}{4}\times\frac{8}{11}$	$\frac{8}{44}=\frac{2}{11}$	30.	$3\frac{5}{6}\times\frac{3}{10}$	$\frac{69}{60}=1\frac{3}{20}$