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Lesson 29: Multi-Step Problems—All Operations

Student Outcomes

* Students use their knowledge of simplifying expressions, order of operations, and properties of equality to calculate the solution of multi-step equations. Students use tables to determine their answer.
* Students check to determine if their solution makes the equation true.

Classwork

**Example 1 (20 minutes)**

Students participate in the discussion by answering the teacher’s questions and following along in their student materials.

Example 1

The school librarian, Mr. Marker, knows the library has books but wants to reorganize how the books are displayed on the shelves. Mr. Marker needs to know how many fiction, nonfiction, and resource books are in the library. He knows that the library has four times as many fiction books as resource books and half as many nonfiction books as fiction books. If these are the only types of books in the library, how many of each type of book are in the library?

Give students time to work individually or with a partner in order to attempt to make sense of the problem. Students may attempt to solve the problem on their own prior to the following discussion.

MP.1

Draw a tape diagram to represent the total number of books in the library.

Draw two more tape diagrams, one to represent the number of fiction books in the library and one to represent the number of resource books in the library.

* Resource Books:
* Fiction Books:

What variable should we use throughout the problem?

We should use to represent the number of resource books in the library because it represents the fewest amount of books. Choosing the variable to represent a different type of book would create fractions throughout the problem.

Write the relationship between resource books and fiction books algebraically.

If we let represent the number of resource books, then represents the number of fiction books.

Draw a tape diagram to represent the number of nonfiction books.

Nonfiction Books:

How did you decide how many sections this tape diagram would have?

There are half as many nonfiction books as fiction books. Since the fiction book tape diagram has four sections, the nonfiction book tape diagram should have two sections.

Represent the number of nonfiction books in the library algebraically.

because that is half as many as fiction books ().

Use the tape diagrams we drew to solve the problem.

We know that combining the tape diagrams for each type of book will leave us with total books.

Write an equation that represents the tape diagram.

Determine the value of .

We can gather like terms, and then solve the equation.

* What does this mean?
  + *There are resource books in the library because represented the number of resource books.*

How many fiction books are in the library?

There are fiction books in the library because .

How many nonfiction books are in the library?

There are nonfiction books in the library because .

* We can use a different math tool to solve the problem as well. If we were to make a table, how many columns would we need?
* Why do we need four columns?
  + *We need to keep track of the number of fiction, nonfiction, and resource books that are in the library, but we also need to keep track of the total number of books.*

Set up a table with four columns and label each column.

|  |  |  |  |
| --- | --- | --- | --- |
| Fiction | Nonfiction | Resource | Total |

* Highlight the important information from the word problem that will help us fill out the second row in our table.
  + *The school librarian, Mr. Marker,* ***knows the library has books*** *but wants to reorganize how the books are displayed on the shelves. Mr. Marker needs to know how many fiction, nonfiction, and resource books are in the library. He knows that the library has* ***four times as many fiction books as resource books and half as many nonfiction books as fiction books****. If these are the only types of books in the library, how many of each type of book are in the library?*
* Fill out the second row of the table using the algebraic representations.

|  |  |  |  |
| --- | --- | --- | --- |
| Fiction | Nonfiction | Resource | Total |
|  |  |  |  |

* If , how many of each type of book would be in the library?

|  |  |  |  |
| --- | --- | --- | --- |
| Fiction | Nonfiction | Resource | Total |
|  |  |  |  |
|  |  |  |  |

* How can we fill out another row of the table?
  + *Substitute different values in for .*
* Substitute in for. How many of each type of book would be in the library then?

|  |  |  |  |
| --- | --- | --- | --- |
| Fiction | Nonfiction | Resource | Total |
|  |  |  |  |
|  |  |  |  |

* Does the library have four times as many fiction books as resource books?
  + *Yes, because .*
* Does the library have half as many nonfiction books as fiction books?
  + *Yes, because half of is .*
* How do we determine how many of each type of book is in the library when there are books in the library?
  + *Continue to multiply the rows by the same value, until the total column has books.*

At this point, allow the students to work individually to determine how many fiction, nonfiction, and resource books are in the library if there are total books. Each table may look different because students may choose different values to multiply by. A sample answer is shown below.

|  |  |  |  |
| --- | --- | --- | --- |
| Fiction | Nonfiction | Resource | Total |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

How many fiction books are in the library?

How many nonfiction books are in the library?

How many resource books are in the library?

* Let us check and make sure that our answers fit the relationship described in the word problem.

Does the library have four times as many fiction books as resource books?

Yes, because .

Does the library have half as many nonfiction books as fiction books?

Yes, because half of is .

Does the library have books?

Yes, because .

Exercises 1–4 (15 minutes)

Students work in small groups to answer the following problems using tables and algebraic methods.

Exercises 1–4

Solve each problem below using tables and algebraic methods. Then, check your answer with the word problem.

1. Indiana Ridge Middle School wanted to add a new school sport, so they surveyed the students to determine which sport is most popular. Students were able to choose among soccer, football, lacrosse, or swimming. The same number of students chose lacrosse and swimming. The number of students who chose soccer was double the number of students who chose lacrosse. The number of students who chose football was triple the number of students who chose swimming. If students completed the survey, how many students chose each sport?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Soccer | Football | Lacrosse | Swimming | Total |
|  |  |  |  |  |

The rest of the table will vary.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Soccer | Football | Lacrosse | Swimming | Total |
|  |  |  |  |  |
|  |  |  |  |  |

students chose soccer, students chose football, students chose lacrosse, and students chose swimming.

MP.1

We can confirm that these numbers satisfy the conditions of the word problem because lacrosse and swimming were chosen by the same number of students. is double , so soccer was chosen by double the number of students as lacrosse, and is triple , so football was chosen by times as many students as swimming. Also,   
.

Algebraically: Let represent the number of students who chose swimming. Then, is the number of students who chose soccer, is the number of students who chose football, and is the number of students who chose lacrosse.

Therefore, students chose swimming and students chose lacrosse. students chose soccer because , and students chose football because .

1. At Prairie Elementary School, students are asked to pick their lunch ahead of time so the kitchen staff will know what to prepare. On Monday, times as many students chose hamburgers as chose salads. The number of students who chose lasagna was one third the number of students who chose hamburgers. If students ordered lunch, how many students chose each option if hamburger, salad, and lasagna were the only three options?

|  |  |  |  |
| --- | --- | --- | --- |
| Hamburger | Salad | Lasagna | Total |
|  |  |  |  |

The rest of the table will vary.

|  |  |  |  |
| --- | --- | --- | --- |
| Hamburger | Salad | Lasagna | Total |
|  |  |  |  |
|  |  |  |  |

students chose a hamburger for lunch, students chose a salad, and students chose lasagna.

MP.1

We can confirm that these numbers satisfy the conditions of the word problem because , so hamburgers were chosen by times more students than salads. Also, , which means lasagna was chosen by one third of the number of students who chose hamburgers. Finally, , which means students completed the survey.

Algebraically: Let represent the number of students who chose a salad. Then represents the number of students who chose hamburgers, and represents the number of students who chose lasagna.

This means that students chose salad, students chose hamburgers because , and students chose lasagna because .

1. The art teacher, Mr. Gonzalez, is preparing for a project. In order for students to have the correct supplies, Mr. Gonzalez needs times more markers than pieces of construction paper. He needs the same number of bottles of glue as pieces of construction paper. The number of scissors required for the project is half the number of pieces of construction paper. If Mr. Gonzalez collected items for the project, how many of each supply did he collect?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Markers | Construction Paper | Glue Bottles | Scissors | Total |
|  |  |  |  |  |

The rest of the table will vary.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Markers | Construction Paper | Glue Bottles | Scissors | Total |
|  |  |  |  |  |
|  |  |  |  |  |

Mr. Gonzalez collected markers, pieces of construction paper, glue bottles, and scissors for the project.

We can confirm that these numbers satisfy the conditions of the word problem because Mr. Gonzalez collected the same number of pieces of construction paper and glue bottles. Also, , so Mr. Gonzalez collected times more markers than pieces of construction paper and glue bottles. Mr. Gonzalez only collected pairs of scissors, which is half of the number of pieces of construction paper. The supplies collected add up to supplies, which is the number of supplies indicated in the word problem.

Algebraically: Let represent the number of scissors needed for the project, which means represents the number of markers needed, represents the number of construction paper needed, and represents the number of glue bottles needed.

This means that pairs of scissors, markers, pieces of construction paper, and glue bottles are required for the project.

1. The math teacher, Ms. Zentz, is buying appropriate math tools to use throughout the year. She is planning on buying twice as many rulers as protractors. The number of calculators Ms. Zentz is planning on buying is one quarter of the number of protractors. If Ms. Zentz buys items, how many protractors does Ms. Zentz buy?

|  |  |  |  |
| --- | --- | --- | --- |
| Rulers | Protractors | Calculators | Total |
|  |  |  |  |

The rest of the table will vary.

|  |  |  |  |
| --- | --- | --- | --- |
| Rulers | Protractors | Calculators | Total |
|  |  |  |  |
|  |  |  |  |

Ms. Zentz will buy protractors.

MP.1

We can confirm that this number satisfies the conditions of the word problem because the number of protractors is half of the number of rulers, and the number of calculators is one fourth of the number of protractors. Also, , so the total matches the total supplies that Ms. Zentz bought.

Algebraically: Let represent the number of calculators Ms. Zentz needs for the year. Then represents the number of rulers, and represents the number of protractors Ms. Zentz will need throughout the year.

Therefore, Ms. Zentz will need calculators, rulers, and protractors throughout the year.

Allow time to answer student questions and discuss answers. In particular, encourage students to compare solution methods with one another, commenting on the accuracy and efficiency of each.

Closing (5 minutes)

* Pam says she only needed two rows in her table to solve each of the problems. How was she able to do this?
  + *Answers will vary. Pam only needed two rows on her table because she found the scale factor from the total in the first row and the total given in the problem. Once this scale factor is determined, it can be used for all the columns in the table because each table is a ratio table.*
* Is there a more efficient way to get to the answer than choosing random values by which to multiply each row?
  + *Find out how many groups of one set of materials it will take to obtain the total amount desired. Then, multiply the entire row by this number.*

Students may need to see a demonstration to fully understand the reasoning. Use the exercises to further explain.

Relate this problem-solving strategy to the ratio tables discussed throughout Module 1.

Exit Ticket (5 minutes)

Name Date

Lesson 29: Multi-Step Problems—All Operations

Exit Ticket

Solve the problem using tables and equations, and then check your answer with the word problem. Try to find the answer only using two rows of numbers on your table.

A pet store owner, Byron, needs to determine how much food he needs to feed the animals. Byron knows that he needs to order the same amount of bird food as hamster food. He needs four times as much dog food as bird food and needs half the amount of cat food as dog food. If Byron orders packages of animal food, how much dog food does he buy? Letrepresent the number of packages of bird food Byron purchased for the pet store.

Exit Ticket Sample Solutions

Solve the problem using tables and equations, and then check your answer with the word problem. Try to find the answer only using two rows of numbers on your table.

A pet store owner, Byron, needs to determine how much food he needs to feed the animals. Byron knows that he needs to order the same amount of bird food as hamster food. He needs four times as much dog food as bird food and needs half the amount of cat food as dog food. If Byron orders packages of animal food, how much dog food does he buy? Letrepresent the number of packages of bird food Byron purchased for the pet store.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Bird Food | Hamster Food | Dog Food | Cat Food | Total |
|  |  |  |  |  |

The rest of the table will vary (unless they follow suggestions from the Closing).

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Bird Food | Hamster Food | Dog Food | Cat Food | Total |
|  |  |  |  |  |
|  |  |  |  |  |

Byron would need to order packages of dog food.

The answer makes sense because Byron ordered the same amount of bird food and hamster food. The table also shows that Byron order four times as much dog food as bird food, and the amount of cat food he ordered is half the amount of dog food. The total amount of pet food Byron ordered was packages, which matches the word problem.

Algebraically: Let represent the number of packages of bird food Byron purchased for the pet store. Therefore, also represents the amount of hamster food, represents the amount of dog food, and represents the amount of cat food required by the pet store.

Therefore, Byron will order pounds of bird food, which results in pounds of dog food because .

Problem Set Sample Solutions

Solve the problems, and then check your answers with the word problem.

1. On average, a baby uses three times the number of large diapers as small diapers and double the number of medium diapers as small diapers.
   1. If the average baby uses diapers size large and small, how many of each size would be used?

|  |  |  |  |
| --- | --- | --- | --- |
| Small | Medium | Large | Total |
|  |  |  |  |
|  |  |  |  |

An average baby would use small diapers, medium diapers, and large diapers.

The answer makes sense because the number of large diapers is times more than small diapers. The number of medium diapers is double the number of small diapers, and the total number of diapers is .

* 1. Support your answer with equations.

Let represent the number of small diapers a baby needs. Therefore, represents the number of medium diapers, and represents the amount of large diapers a baby needs.

Therefore, a baby requires small diapers, medium diapers (because ), and large diapers (because ), which matches the answer in part (a).

1. Tom has three times as many pencils as pens but has a total of writing utensils.
   1. How many pencils does Tom have?

|  |  |  |
| --- | --- | --- |
| Pencils | Pens | Total |
|  |  |  |
|  |  |  |

* 1. How many more pencils than pens does Tom have?

. Tom has more pencils than pens.

1. Serena’s mom is planning her birthday party. She bought balloons, plates, and cups. Serena’s mom bought twice as many plates as cups. The number of balloons Serena’s mom bought was half the number of cups.
   1. If Serena’s mom bought items, how many of each item did she buy?

|  |  |  |  |
| --- | --- | --- | --- |
| Balloons | Plates | Cups | Total |
|  |  |  |  |
|  |  |  |  |

Serena’s mom bought balloons, plates, and cups.

* 1. Tammy brought balloons to the party. How many total balloons were at Serena’s birthday party?

. There were total balloons at the party.

* 1. If half the plates and all but four cups were used during the party, how many plates and cups were used?

. Twenty-four plates were used during the party.

. Twenty cups were used during the party.

1. Elizabeth has a lot of jewelry. She has four times as many earrings as watches but half the number of necklaces as earrings. Elizabeth has the same number of necklaces as bracelets.
   1. If Elizabeth has pieces of jewelry, how many earrings does she have?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Earrings | Watches | Necklaces | Bracelets | Total |
|  |  |  |  |  |
|  |  |  |  |  |

Elizabeth has earrings, watches, necklaces, and bracelets.

* 1. Support your answer with an equation.

Let represent the number of watches Elizabeth has. Therefore, represents the number of earrings Elizabeth has, and represents both the number of necklaces and bracelets she has.

Therefore, Elizabeth has watches, earrings because , and necklaces and bracelets each because .

1. Claudia was cooking breakfast for her entire family. She made double the amount of chocolate chip pancakes as she did regular pancakes. She only made half as many blueberry pancakes as she did regular pancakes. Claudia also knows her family loves sausage, so she made triple the amount of sausage as blueberry pancakes.
   1. How many of each breakfast item did Claudia make if she cooked items in total?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Chocolate Chip Pancakes | Regular Pancakes | Blueberry Pancakes | Sausage | Total |
|  |  |  |  |  |
|  |  |  |  |  |

Claudia cooked chocolate chip pancakes, regular pancakes, blueberry pancakes, and pieces of sausage.

* 1. After everyone ate breakfast, there were chocolate chip pancakes, regular pancakes, blueberry pancake, and no sausage left. How many of each item did the family eat?

The family ate chocolate chip pancakes, regular pancakes, blueberry pancakes, and pieces of sausage during breakfast.

1. During a basketball game, Jeremy scored triple the number of points as Donovan. Kolby scored double the number of points as Donovan.
   1. If the three boys scored points, how many points did each boy score?

|  |  |  |  |
| --- | --- | --- | --- |
| Jeremy | Donovan | Kolby | Total |
|  |  |  |  |
|  |  |  |  |

Jeremy scored points, Donovan scored points, and Kolby scored points.

* 1. Support your answer with an equation.

Let represent the number of points Donovan scored, which means represents the number of points Jeremy scored, and represents the number of points Kolby scored.

Therefore, Donovan scored points, Jeremy scored points because , and Kolby scored points because .