Lesson 28: Two-Step Problems—All Operations

Student Outcomes

- Students calculate the solution of two-step equations by using their knowledge of order of operations and the properties of equality for addition, subtraction, multiplication, and division. Students employ tape diagrams to determine their answer.
- Students check to determine if their solution makes the equation true.

Classwork

Fluency Exercise (5 minutes): Addition of Decimals

Sprint: Refer to the Sprints and Sprint Delivery Script sections in the Module Overview for directions on how to administer a Sprint.

Mathematical Modeling Exercise (6 minutes)

Model the problems while students follow along.

Mathematical Modeling Exercise

Juan has gained $20\ \text{lb}$. since last year. He now weighs $120\ \text{lb}$. Rashod is $15\ \text{lb}$. heavier than Diego. If Rashod and Juan weighed the same amount last year, how much does Diego weigh? Let j represent Juan's weight last year in pounds, and let d represent Diego's weight in pounds.
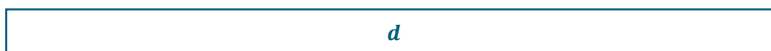
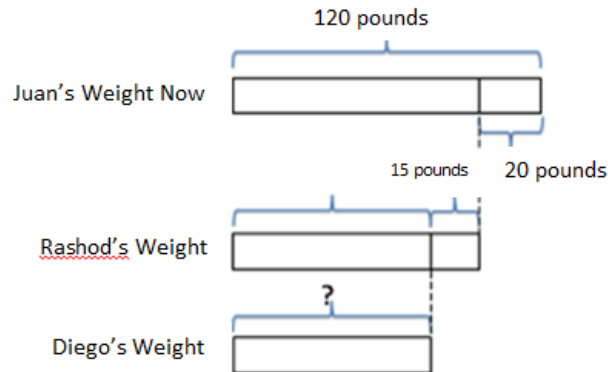Draw a tape diagram to represent Juan's weight.
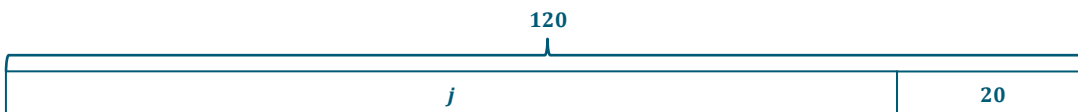
Draw a tape diagram to represent Rashod's weight.

Draw a tape diagram to represent Diego's weight.

What would combining all three tape diagrams look like?

120 pounds

Juan's Weight Now

15 pounds   20 pounds

Rashod's Weight

?

Diego's Weight

Write an equation to represent Juan's tape diagram.

$$j + 20 = 120$$

Write an equation to represent Rashod's tape diagram.

$$d + 15 + 20 = 120$$

How can we use the final tape diagram or the equations above to answer the question presented?

By combining 15 and 20 from Rashod's equation, we can use our knowledge of addition identities to determine Diego's weight.

The final tape diagram can be used to write a third equation $d + 35 = 120$. We can use our knowledge of addition identities to determine Diego's weight.

Calculate Diego's weight.

$d + 35 - 35 = 120 - 35$

$d = 85$

We can use identities to defend our thought that $d + 35 - 35 = d$.

Does your answer make sense?

Yes, if Diego weighs 85 lb., and Rashod weighs 15 lb. more than Deigo, then Rashod weighs 100 lb., which is what Juan weighed before he gained 20 lb.

Example 1 (5 minutes)

Assist students in solving the problem by providing step-by-step guidance.

Example 1

Marissa has twice as much money as Frank. Christina has $20 more than Marissa. If Christina has $100, how much money does Frank have? Let f represent the amount of money Frank has in dollars and m represent the amount of money Marissa has in dollars.
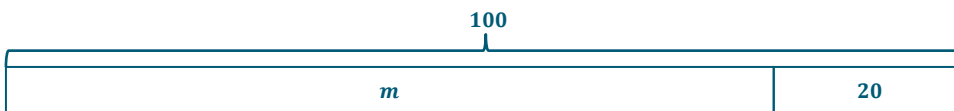
Draw a tape diagram to represent the amount of money Frank has.

f

Draw a tape diagram to represent the amount of money Marissa has.

f	f

Draw a tape diagram to represent the amount of money Christina has.

100

m	20

Which tape diagram provides enough information to determine the value of the variable m?

The tape diagram that represents the amount of money Christina has.

Write and solve the equation.

$$m + 20 = 100$$
$$m + 20 - 20 = 100 - 20$$
$$m = 80$$

The identities we have discussed throughout the module solidify that $m + 20 - 20 = m$.

What does the 80 represent?

80 *is the amount of money, in dollars, that Marissa has.*

Now that we know Marissa has $80, how can we use this information to find out how much money Frank has?

We can write an equation to represent Marissa's tape diagram since we now know the length is 80.

Write an equation.

$2f = 80$

Solve the equation.

$$2f \div 2 = 80 \div 2$$
$$f = 40$$

Once again, the identities we have used throughout the module can solidify that $2f \div 2 = f$.

What does the 40 represent?

The 40 represents the amount of money Frank has in dollars.

Does 40 make sense in the problem?

Yes, because if Frank has 40*, then Marissa has twice this, which is* 80*. Then, Christina has* 100 *because she has* 20 *more than Marissa, which is what the problem stated.*

Exercises (20 minutes; 5 minutes per station)

Students work in small groups to complete the following stations.

MP.1

Station One: **Use tape diagrams to solve the problem.**

Raeana is twice as old as Madeline, and Laura is 10 years older than Raeana. If Laura is 50 years old, how old is Madeline? Let m represent Madeline's age in years, and let r represent Raeana's age in years.
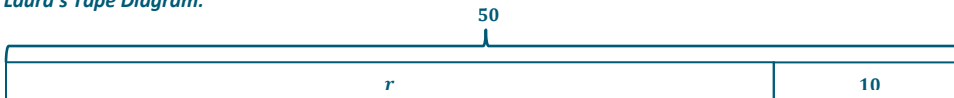
Raeana's Tape Diagram:

m	m

Madeline's Tape Diagram:

m

Laura's Tape Diagram:

50

r	10

Equation for Laura's Tape Diagram:

$$r + 10 = 50$$
$$r + 10 - 10 = 50 - 10$$
$$r = 40$$

We now know that Raeana is 40 years old, and we can use this and Raeana's tape diagram to determine the age of Madeline.

$$2m = 40$$
$$2m \div 2 = 40 \div 2$$
$$m = 20$$

Therefore, Madeline is 20 years old.

Station Two: Use tape diagrams to solve the problem.

Carli has 90 apps on her phone. Braylen has half the amount of apps as Theiss. If Carli has three times the amount of apps as Theiss, how many apps does Braylen have? Let b represent the number of Braylen's apps and t represent the number of Theiss's apps.
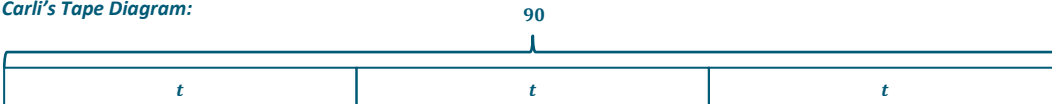
Theiss's Tape Diagram:

t

Braylen's Tape Diagram:

b	b

Carli's Tape Diagram:

90

t	t	t

Equation for Carli's Tape Diagram:

$$3t = 90$$
$$3t \div 3 = 90 \div 3$$
$$t = 30$$

We now know that Theiss has 30 apps on his phone. We can use this information to write an equation for Braylen's tape diagram and determine how many apps are on Braylen's phone.

$$2b = 30$$
$$2b \div 2 = 30 \div 2$$
$$b = 15$$

Therefore, Braylen has 15 apps on his phone.

MP.1

Station Three: Use tape diagrams to solve the problem.

Reggie ran for 180 yards during the last football game, which is 40 more yards than his previous personal best. Monte ran 50 more yards than Adrian during the same game. If Monte ran the same amount of yards Reggie ran in one game for his previous personal best, how many yards did Adrian run? Let r represent the number of yards Reggie ran during his previous personal best and a represent the number of yards Adrian ran.
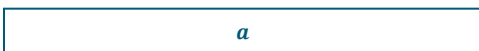
Reggie's Tape Diagram:

180

r	40

Monte's Tape Diagram:

a	50
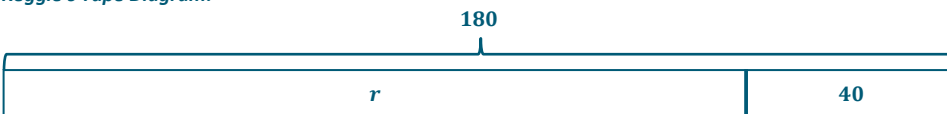
Adrian's Tape Diagram:

a

Combining all 3 tape diagrams:

180

Reggie	r	40
Monte	a	50	40
	a	90

Equation for Reggie's Tape Diagram:

$$r + 40 = 180$$

Equation for Monte's Tape Diagram:

$$a + 50 + 40 = 180$$
$$a + 90 = 180$$
$$a + 90 - 90 = 180 - 90$$
$$a = 90$$

Therefore, Adrian ran 90 yards during the football game.

Station Four: Use tape diagrams to solve the problem.

Lance rides his bike downhill at a pace of 60 miles per hour. When Lance is riding uphill, he rides 8 miles per hour slower than on flat roads. If Lance's downhill speed is 4 times faster than his flat road speed, how fast does he travel uphill? Let f represent Lance's pace on flat roads in miles per hour and u represent Lance's pace uphill in miles per hour.

MP.1

Tape Diagram for Uphill Pace:

f

u	8

Tape Diagram for Downhill:

60

f	f	f	f

Equation for Downhill Pace:

$$4f = 60$$
$$4f \div 4 = 60 \div 4$$
$$f = 15$$

Equation for Uphill Pace:

$$u + 8 = 15$$
$$u + 8 - 8 = 15 - 8$$
$$u = 7$$

Therefore, Lance travels at a pace of 7 miles per hour uphill.

Closing (4 minutes)

Use this time to go over the solutions to the stations and answer student questions.

- How did the tape diagrams help you create the expressions and equations that you used to solve the problems?
  - *Answers will vary.*

Exit Ticket (5 minutes)

Name _____   Date _____

Lesson 28: Two-Step Problems—All Operations

Exit Ticket

Use tape diagrams and equations to solve the problem with visual models and algebraic methods.

Alyssa is twice as old as Brittany, and Jazmyn is 15 years older than Alyssa. If Jazmyn is 35 years old, how old is Brittany? Let a represent Alyssa's age in years and b represent Brittany's age in years.

Exit Ticket Sample Solutions

Use tape diagrams and equations to solve the problem with visual models and algebraic methods.
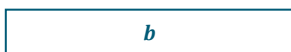
Alyssa is twice as old as Brittany, and Jazmyn is 15 years older than Alyssa. If Jazmyn is 35 years old, how old is Brittany? Let a represent Alyssa's age in years and b represent Brittany's age in years.

Brittany's Tape Diagram:

b

Alyssa's Tape Diagram:

b	b

Jazmyn's Tape Diagram:

35

a	15
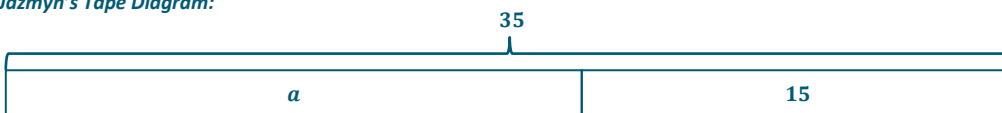
Equation for Jazmyn's Tape Diagram:

$$a + 15 = 35$$
$$a + 15 - 15 = 35 - 15$$
$$a = 20$$

Now that we know Alyssa is 20 years old, we can use this information and Alyssa's tape diagram to determine Brittany's age.

$$2b = 20$$
$$2b \div 2 = 20 \div 2$$
$$b = 10$$

Therefore, Brittany is 10 years old.

Problem Set Sample Solutions

Use tape diagrams to solve each problem.

1. Dwayne scored 55 points in the last basketball game, which is 10 points more than his previous personal best. Lebron scored 15 points more than Chris in the same game. Lebron scored the same number of points as Dwayne's previous personal best. Let d represent the number of points Dwayne scored during his previous personal best and c represent the number of Chris's points.

    a. How many points did Chris score during the game?

	55		
Dwayne	d		10
Lebron	c	15	10
	c	25	

Equation for Dwayne's Tape Diagram: $d + 10 = 55$

Equation for Lebron's Tape Diagram:

$$c + 15 + 10 = 55$$
$$c + 25 = 55$$
$$c + 25 - 25 = 55 - 25$$
$$c = 30$$

Therefore, Chris scored 30 points in the game.

    b. If these are the only three players who scored, what was the team's total number of points at the end of the game?

Dwayne scored 55 points. Chris scored 30 points. Lebron scored 45 points (answer to Dwayne's equation). Therefore, the total number of points scored is $55 + 30 + 45 = 130$.

2. The number of customers at Yummy Smoothies varies throughout the day. During the lunch rush on Saturday, there were 120 customers at Yummy Smoothies. The number of customers at Yummy Smoothies during dinner time was 10 customers fewer than the number during breakfast. The number of customers at Yummy Smoothies during lunch was 3 times more than during breakfast. How many people were at Yummy Smoothies during breakfast? How many people were at Yummy Smoothies during dinner? Let d represent the number of customers at Yummy Smoothies during dinner and b represent the number of customers at Yummy Smoothies during breakfast.

Tape Diagrams for Lunch:

120

| b | b | b |

Tape Diagram for Dinner:

b

| d | 10 |
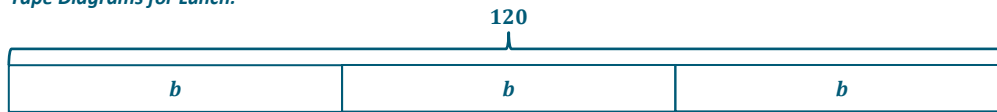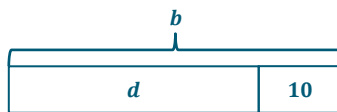
Equation for Lunch's Tape Diagram:

$$3b = 120$$
$$3b \div 3 = 120 \div 3$$
$$b = 40$$

Now that we know 40 customers were at Yummy Smoothies for breakfast, we can use this information and the tape diagram for dinner to determine how many customers were at Yummy Smoothies during dinner.

$$d + 10 = 40$$
$$d + 10 - 10 = 40 - 10$$
$$d = 30$$

Therefore, 30 customers were at Yummy Smoothies during dinner and 40 customers during breakfast.

3. Karter has 24 t-shirts. Karter has 8 fewer pairs of shoes than pairs of pants. If the number of t-shirts Karter has is double the number of pants he has, how many pairs of shoes does Karter have? Let p represent the number of pants Karter has and s represent the number of pairs of shoes he has.

Tape Diagram for T-Shirts:

24

| p | p |

Tape Diagram for Shoes:
p

| s | 8 |
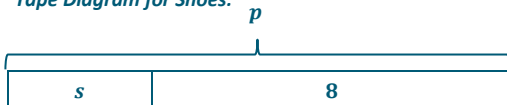
Equation for T-Shirts Tape Diagram:

$$2p = 24$$
$$2p \div 2 = 24 \div 2$$
$$p = 12$$

Equation for Shoes Tape Diagram:

$$s + 8 = 12$$
$$s + 8 - 8 = 12 - 8$$
$$s = 4$$

Karter has 4 pairs of shoes.

4. Darnell completed 35 push-ups in one minute, which is 8 more than his previous personal best. Mia completed 6 more push-ups than Katie. If Mia completed the same amount of push-ups as Darnell completed during his previous personal best, how many push-ups did Katie complete? Let d represent the number of push-ups Darnell completed during his previous personal best and k represent the number of push-ups Katie completed.

35		
d		8
k	6	8
k	14	

$d + 8 = 35$

$$k + 6 + 8 = 35$$
$$k + 14 = 35$$
$$k + 14 - 14 = 35 - 14$$
$$k = 21$$

Katie completed 21 *push-ups.*

5. Justine swims freestyle at a pace of 150 laps per hour. Justine swims breaststroke 20 laps per hour slower than she swims butterfly. If Justine's freestyle speed is three times faster than her butterfly speed, how fast does she swim breaststroke? Let b represent Justine's butterfly speed in laps per hour and r represent Justine's breaststroke speed in laps per hour.

Tape Diagram for Breaststroke:

b	
r	20

Tape Diagram for Freestyle:

150		
b	b	b

$$3b = 150$$
$$3b \div 3 = 150 \div 3$$
$$b = 50$$

Therefore, Justine swims butterfly at a pace of 50 *laps per hour.*

$$r + 20 = 50$$
$$r + 20 - 20 = 50 - 20$$
$$r = 30$$

Therefore, Justine swims breaststroke at a pace of 30 *laps per hour.*

Addition of Decimals I–Round 1

Directions: Determine the sum of the decimals.

1.	1.3 + 2.1		18.	14.08 + 34.27	
2.	3.6 + 2.2		19.	24.98 + 32.05	
3.	8.3 + 4.6		20.	76.67 + 40.33	
4.	14.3 + 12.6		21.	46.14 + 32.86	
5.	21.2 + 34.5		22.	475.34 + 125.88	
6.	14.81 + 13.05		23.	561.09 + 356.24	
7.	32.34 + 16.52		24.	872.78 + 135.86	
8.	56.56 + 12.12		25.	788.04 + 324.69	
9.	78.03 + 21.95		26.	467 + 32.78	
10.	32.14 + 45.32		27.	583.84 + 356	
11.	14.7 + 32.8		28.	549.2 + 678.09	
12.	24.5 + 42.9		29.	497.74 + 32.1	
13.	45.8 + 32.4		30.	741.9 + 826.14	
14.	71.7 + 32.6		31.	524.67 + 764	
15.	102.5 + 213.7		32.	821.3 + 106.87	
16.	365.8 + 127.4		33.	548 + 327.43	
17.	493.4 + 194.8		34.	108.97 + 268.03	

Addition of Decimals I–Round 1 [KEY]

Directions: Determine the sum of the decimals.

1.	1.3 + 2.1	**3.4**	18.	14.08 + 34.27	**48.35**
2.	3.6 + 2.2	**5.8**	19.	24.98 + 32.05	**57.03**
3.	8.3 + 4.6	**12.9**	20.	76.67 + 40.33	**117**
4.	14.3 + 12.6	**26.9**	21.	46.14 + 32.86	**79**
5.	21.2 + 34.5	**55.7**	22.	475.34 + 125.88	**601.22**
6.	14.81 + 13.05	**27.86**	23.	561.09 + 356.24	**917.33**
7.	32.34 + 16.52	**48.86**	24.	872.78 + 135.86	**1,008.64**
8.	56.56 + 12.12	**68.68**	25.	788.04 + 324.69	**1,112.73**
9.	78.03 + 21.95	**99.98**	26.	467 + 32.78	**499.78**
10.	32.14 + 45.32	**77.46**	27.	583.84 + 356	**939.84**
11.	14.7 + 32.8	**47.5**	28.	549.2 + 678.09	**1,227.29**
12.	24.5 + 42.9	**67.4**	29.	497.74 + 32.1	**529.84**
13.	45.8 + 32.4	**78.2**	30.	741.9 + 826.14	**1,568.04**
14.	71.7 + 32.6	**104.3**	31.	524.67 + 764	**1,288.67**
15.	102.5 + 213.7	**316.2**	32.	821.3 + 106.87	**928.17**
16.	365.8 + 127.4	**493.2**	33.	548 + 327.43	**875.43**
17.	493.4 + 194.8	**688.2**	34.	108.97 + 268.03	**377**

Number Correct: _____

Improvement: _____

Addition of Decimals I–Round 2

Directions: Determine the sum of the decimals.

1.	3.4 + 1.2		18.	67.82 + 37.9	
2.	5.6 + 3.1		19.	423.85 + 47.5	
3.	12.4 + 17.5		20.	148.9 + 329.18	
4.	10.6 + 11.3		21.	4 + 3.25	
5.	4.8 + 3.9		22.	103.45 + 6	
6.	4.56 + 1.23		23.	32.32 + 101.8	
7.	32.3 + 14.92		24.	62.1 + 0.89	
8.	23.87 + 16.34		25.	105 + 1.45	
9.	102.08 + 34.52		26.	235.91 + 12	
10.	35.91 + 23.8		27.	567.01 + 432.99	
11.	62.7 + 34.89		28.	101 + 52.3	
12.	14.76 + 98.1		29.	324.69 + 567.31	
13.	29.32 + 31.06		30.	245 + 0.987	
14.	103.3 + 32.67		31.	191.67 + 3.4	
15.	217.4 + 87.79		32.	347.1 + 12.89	
16.	22.02 + 45.8		33.	627 + 4.56	
17.	168.3 + 89.12		34.	0.157 + 4.56	

Addition of Decimals I–Round 2 [KEY]

Directions: Determine the sum of the decimals.

1.	3.4 + 1.2	**4.6**	18.	67.82 + 37.9	**105.72**
2.	5.6 + 3.1	**8.7**	19.	423.85 + 47.5	**471.35**
3.	12.4 + 17.5	**29.9**	20.	148.9 + 329.18	**478.08**
4.	10.6 + 11.3	**21.9**	21.	4 + 3.25	**7.25**
5.	4.8 + 3.9	**8.7**	22.	103.45 + 6	**109.45**
6.	4.56 + 1.23	**5.79**	23.	32.32 + 101.8	**134.12**
7.	32.3 + 14.92	**47.22**	24.	62.1 + 0.89	**62.99**
8.	23.87 + 16.34	**40.21**	25.	105 + 1.45	**106.45**
9.	102.08 + 34.52	**136.6**	26.	235.91 + 12	**247.91**
10.	35.91 + 23.8	**59.71**	27.	567.01 + 432.99	**1,000**
11.	62.7 + 34.89	**97.59**	28.	101 + 52.3	**153.3**
12.	14.76 + 98.1	**112.86**	29.	324.69 + 567.31	**892**
13.	29.32 + 31.06	**60.38**	30.	245 + 0.987	**245.987**
14.	103.3 + 32.67	**135.97**	31.	191.67 + 3.4	**195.07**
15.	217.4 + 87.79	**305.19**	32.	347.1 + 12.89	**359.99**
16.	22.02 + 45.8	**67.82**	33.	627 + 4.56	**631.56**
17.	168.3 + 89.12	**257.42**	34.	0.157 + 4.56	**4.717**