Lesson 23: True and False Number Sentences

Student Outcomes

- Students explain what the equality and inequality symbols including $=$, $<$, $>$, $\leq$, and $\geq$ represent. They determine if a number sentence is true or false based on the given symbol.

Lesson Notes

For students to be prepared to solve equations later in this module, it is important that they understand truth values in number sentences and in equations. In the next three lessons, students will learn to differentiate between number sentences and generic equations. Later, in Lesson 26, students will learn why number sentences play a fundamental role in understanding both equations with unknown numbers and solution sets of equations that contain variables. Number sentences are special among types of equations because they have truth values. A number sentence is the most concrete version of an equation. It has the very important property that it is always true or always false, and it is this property that distinguishes it from a generic equation. Examples include $2 + 3 = 5$ (true) and $2 + 2 = 5$ (false). The property guarantees the ability to check whether or not a number is a solution to an equation with a variable. Just substitute the number into the variable; then, check to see if the resulting *number sentence* is either true or false. If the number sentence is true, the number is a solution. For that reason, number sentences are the first and most important type of equation that students need to understand. We begin Lesson 23 by first determining what is true and false and then moving to true and false number sentences using equality and inequality symbols.

Classwork

Opening (4 minutes)

Discuss the meaning of *true* and *false* by calling on students to tell if the following statements are true or false. Conclude with a discussion of what makes a number sentence true or false.

- Earth orbits the Sun.
  - *True.*
- George Washington was the first President of the United States.
  - *True.*
- There are 25 hours in a day.
  - *False.*
- $3 + 3 = 6$.
  - *True.*
- $2 + 2 = 5$.
  - *False.*
- Why is $2 + 2 = 5$ a false number sentence?
  - *Answers will vary but should include the idea that the expressions on both sides of the equal sign should evaluate to the same number; so, for this number sentence to be true, either the first or second addend should be three, resulting in the sum of five.*

Opening Exercise/Discussion (8 minutes)

Discuss what each symbol below stands for, and provide students with an example. Students can complete the table in their student materials.

Have one student come up to the front of the room and stand against the wall or board. Mark the student's height with a piece of masking tape. (It is important to pick a student who is about average height.) Measure the height of the tape mark using a tape measure, and record it as a mixed number in feet, along with the student's name, on the piece of masking tape. Next, start the following table on the board/screen:

Opening Exercise

Determine what each symbol stands for and provide an example.

Symbol	What the Symbol Stands For	Example
$=$	*is equal to*	$4\frac{7}{8} = 4.875$

- What is another example of a number sentence that includes an equal symbol?
  - *Answers will vary. You may ask more than one student.*

The student's height is the height marked by the tape on the wall. Have students stand next to the marked height. Discuss how their heights compare to the height of the tape. Are there other students in the room who have the same height?

$>$	*is greater than*	$5\frac{1}{4} > 4\frac{7}{8}$

Use the student's height measurement in the example (our example uses a student $4\frac{7}{8}$ ft. in height).

- What is another example of a number sentence that includes a greater than symbol?
  - *Answers will vary. You may ask more than one student.*

Have students taller than the tape on the wall stand near the tape. Discuss how more than one student has a height that is greater than the tape so there could be more than one number we could insert into the inequality: $> 4\frac{7}{8}$.

$<$	*is less than*	$4\frac{1}{2} < 4\frac{7}{8}$

- What is another example of a number sentence that includes a less than symbol?
  - *Answers will vary. You may ask more than one student.*

Have students shorter than the tape on the wall stand near the tape. Discuss how more than one student has a height that is less than the tape so there could be more than one number we could insert into the inequality: $< 4\frac{7}{8}$.

$\leq$	*is less than or equal to*	$4\frac{7}{8} \leq 4\frac{7}{8}$

- What is another example of a number sentence that includes a less than or equal to symbol?
  - *Answers will vary. You may ask more than one student.*

Ask students who are the exact height as the tape and students who are shorter than the tape to stand near the tape. Discuss how this symbol is different than the previous symbol.

$\geq$	*is greater than or equal to*	$5\frac{1}{4} \geq 4\frac{7}{8}$

- What is another example of a number sentence that includes a greater than or equal to symbol?
  - *Answers will vary. You may ask more than one student.*
- Which students would stand near the tape to demonstrate this symbol?
  - *Students who are the same height as or taller than the tape.*

Example 1 (5 minutes)

Display each of the equations and inequalities one by one for the students to review and determine whether they result in a true or a false number sentence.

> **Example 1**
>
> **For each equation or inequality your teacher displays, write the equation or inequality, and then substitute 3 for every x. Determine if the equation or inequality results in a true number sentence or a false number sentence.**

Display $5 + x = 8$.

- Substitute 3 for x and evaluate. Does this result in a true number sentence or a false number sentence?
  - *True.*
- Why is the number sentence a true number sentence?
  - *Each expression on either side of the equal sign evaluates to* 8. $8 = 8$.

Display $5x = 8$.

- Substitute 3 for x and evaluate. Does this result in a true number sentence or a false number sentence?
  - *False.*
- Why is the number sentence a false number sentence?
  - *Five times three equals fifteen. Fifteen does not equal eight, so the number sentence* $5(3) = 8$ *is false.*

Display $5 + x > 8$.

- Substitute 3 for x and evaluate. Does this result in a true number sentence or a false number sentence?
  - *False.*

- Why is the number sentence a false number sentence?
  - *Each expression on either side of the inequality sign evaluates to 8. However, the inequality sign states that eight is greater than eight, which is not true. We have already shown that $8 = 8$.*

Display $5x > 8$.

- Substitute 3 for x and evaluate. Does this result in a true number sentence or a false number sentence?
  - *True.*
- Why is the number sentence a true number sentence?
  - *When three is substituted, the left side of the inequality evaluates to fifteen. Fifteen is greater than eight, so the number sentence is true.*

Display $5 + x \geq 8$.

- Substitute 3 for x and evaluate. Does this result in a true number sentence or a false number sentence?
  - *True.*
- Why is the number sentence a true number sentence?
  - *Each expression on either side of the inequality sign evaluates to 8. Because the inequality sign states that the expression on the left side can be greater than or equal to the expression on the right side, the number sentence is true because we have already shown that $8 = 8$.*
- Can you find a number other than three that we can substitute for x that will result in a false number sentence?
  - *Answers will vary, but any number less than three will result in a false number sentence.*

Exercises (13 minutes)

Students work on the following exercises independently. Note that students are writing complete sentences to describe what happens when a variable is substituted with a number and whether it turns the equation into a true number sentence or a false number sentence.

MP.6

Exercises

Substitute the indicated value into the variable, and state (in a complete sentence) whether the resulting number sentence is true or false. If true, find a value that would result in a false number sentence. If false, find a value that would result in a true number sentence.

1. **$4 + x = 12$. Substitute 8 for x.**

   When 8 is substituted for x, the number sentence is true.

   Answers will vary on values to make the sentence false; any number other than 8 will make the sentence false.

2. **$3g > 15$. Substitute $4\frac{1}{2}$ for g.**

   When $4\frac{1}{2}$ is substituted for g, the number sentence is false.

   Answers will vary on values that make the sentence true; any number greater than 5 will make the sentence true.

MP.6

3. $\frac{f}{4} < 2$. Substitute 8 for f.

When 8 is substituted for f, the number sentence is false.

Answers will vary on values to make the sentence true; any number less than 8 will make the sentence true.

4. $14.2 \leq h - 10.3$. Substitute 25.8 for h.

When 25.8 is substituted for h, the number sentence is true.

Answers will vary on values to make the sentence false; any number less than 24.5 will make the sentence false.

5. $4 = \frac{8}{h}$. Substitute 6 for h.

When 6 is substituted for h, the number sentence is false.

2 is the only value that will make the sentence true.

6. $3 > k + \frac{1}{4}$. Substitute $1\frac{1}{2}$ for k.

When $1\frac{1}{2}$ is substituted for k, the number sentence is true.

Answers will vary on values to make the sentence false; the number $2\frac{3}{4}$ or any number greater than $2\frac{3}{4}$ will make the sentence false.

7. $4.5 - d > 2.5$. Substitute 2.5 for d.

When 2.5 is substituted for d, the number sentence is false.

Answers will vary on values to make the sentence true; any number less than 2 will make the number sentence true.

8. $8 \geq 32p$. Substitute $\frac{1}{2}$ for p.

When $\frac{1}{2}$ is substituted for p, the number sentence is false.

Answers will vary on values to make the sentence true; the number $\frac{1}{4}$ or any number less than $\frac{1}{4}$ will make the sentence true.

9. $\frac{w}{2} < 32$. Substitute 16 for w.

When 16 is substituted for p, the number sentence is true.

Answers will vary on values to make the sentence false; the number 64 or any other number greater than 64 will make the sentence false.

10. $18 \leq 32 - b$. Substitute 14 for b.

When 14 is substituted for b, the number sentence is true.

Answers will vary on values to make the sentence false; any number greater than 14 will make the sentence false.

Closing (10 minutes)

Take 3 minutes to discuss the answers from the exercises. From the exercises, continue the discussion from the lesson.

- Let's take a look at Exercise 1: $4 + x = 12$. We substituted 8 for x. What did we determine?
  - *When we substituted 8 for x, the number sentence was true.*
- And then we tried to find values to substitute for x to make the number sentence false. What number did you substitute for x to make this number sentence false?

Elicit responses from the class. Answers will vary, but collect all that make the number sentence false and record them on the board. Elicit responses for the next set of questions:

- Did anyone substitute with zero? A thousand? A trillion? How about a fraction? A decimal?
  - *Answers will vary.*
- If all of these responses result in a false number sentence, what can you conclude?
  - *Only one number can be substituted to make the two expressions equal, and that number is 8.*
- Look at all of the numbers that will make this number sentence false, and then look at the one number that will make this number sentence true. Why do you think the number 8 is important compared to all the others that make the number sentence false?

Elicit various responses. The goal is for students to understand that since all numbers other than 8 results in false statements, those numbers do not contribute valuable information about the equation in the same way that the number 8 does.

- What about inequalities? Let's take another look at Exercise 2: $3g > 15$. We substituted $4\frac{1}{2}$ for g and determined that after we evaluated the inequality, it created a false number sentence because $13\frac{1}{2}$ is not greater than 15. What number did you substitute for g to make this number sentence true?

Elicit responses from the class. Answers will vary, but collect all that make the number sentence true and record them on the board. Elicit responses for the next set of questions:

- What about 14, 16, 18, or 20? 100? 200? How about 5.1? 5.01? 5.001? 5.0000000000000001?
  - *Answers will vary.*
- What can you conclude about the substituted numbers that will make this number sentence true?
  - *To make this number sentence true, any number greater than five can be substituted for the variable, whether it is a whole number, fraction, or decimal.*
- Which substituted numbers made this number sentence false?
  - *Answers will vary but must be five or any number less than five.*
- Visualize a number line in your mind. If we can only substitute numbers greater than five on the number line to make this number sentence true, what would that number line look like?

Answers will vary. The goal is for students to visualize that only part of the number line works for the inequality in order to create a true number sentence, while the other part does not work and makes the number sentence false.

Lesson Summary

NUMBER SENTENCE: A *number sentence* is a statement of equality (or inequality) between two numerical expressions.

TRUTH VALUES OF A NUMBER SENTENCE: A number sentence that is an equation is said to be *true* if both numerical expressions evaluate to the same number; it is said to be *false* otherwise. True and false are called *truth values*.

Number sentences that are inequalities also have truth values. For example, $3 < 4$, $6 + 8 > 15 - 12$, and $(15 + 3)^2 < 1000 - 32$ are all true number sentences, while the sentence $9 > 3(4)$ is false.

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 23: True and False Number Sentences

Exit Ticket

Substitute the value for the variable and state in a complete sentence whether the resulting number sentence is true or false. If true, find a value that would result in a false number sentence. If false, find a value that would result in a true number sentence.

1. $15a \geq 75$. Substitute 5 for a.

2. $23 + b = 30$. Substitute 10 for b.

3. $20 > 86 - h$. Substitute 46 for h.

4. $32 \geq 8m$. Substitute 5 for m.

Exit Ticket Sample Solutions

Substitute the value for the variable and state in a complete sentence whether the resulting number sentence is true or false. If true, find a value that would result in a false number sentence. If false, find a value that would result in a true number sentence.

1. $15a \geq 75$. Substitute 5 for a.

   When 5 is substituted in for a, the number sentence is true. Answers will vary, but any value for a less than 5 will result in a false number sentence.

2. $23 + b = 30$. Substitute 10 for b.

   When 10 is substituted in for b, the number sentence is false. The only value for b that will result in a true number sentence is 7.

3. $20 > 86 - h$. Substitute 46 for h.

   When 46 is substituted in for h, the number sentence will be false. Answers will vary, but any value for h greater than 66 will result in a true number sentence.

4. $32 \geq 8m$. Substitute 5 for m.

   When 5 is substituted in for m, the number sentence is false. Answers will vary, but the value of 4 and any value less than 4 for m will result in a true number sentence.

Problem Set Sample Solutions

Substitute the value for the variable, and state (in a complete sentence) whether the resulting number sentence is true or false. If true, find a value that would result in a false number sentence. If false, find a value that would result in a true number sentence.

1. $3\frac{5}{6} = 1\frac{2}{3} + h$. Substitute $2\frac{1}{6}$ for h.

   When $2\frac{1}{6}$ is substituted in for h, the number sentence is true. Answers will vary, but any value for h other than $2\frac{1}{6}$ will result in a false number sentence.

2. $39 > 156g$. Substitute $\frac{1}{4}$ for g.

   When $\frac{1}{4}$ is substituted in for g, the number sentence is false. Answers will vary, but any value for g less than $\frac{1}{4}$ will result in a true number sentence.

3. $\frac{f}{4} \leq 3$. Substitute 12 for f.

   When 12 is substituted in for f, the number sentence is true. Answers will vary, but any value for f greater than 12 will result in a false number sentence.

4. $121 - 98 \geq r$. Substitute 23 for r.

When 23 is substituted in for r, the number sentence is true. Answers will vary, but any value for r greater than 23 will result in a false number sentence.

5. $\frac{54}{q} = 6$. Substitute 10 for q.

When 10 is substituted in for q, the number sentence is false. The number 9 is the only value for q that will result in a true number sentence.

Create a number sentence using the given variable and symbol. The number sentence you write must be true for the given value of the variable.

6. Variable: d Symbol: $\geq$ The sentence is true when 5 is substituted for d.

7. Variable: y Symbol: $\neq$ The sentence is true when 10 is substituted for y.

8. Variable: k Symbol: $<$ The sentence is true when 8 is substituted for k.

9. Variable: a Symbol: $\leq$ The sentence is true when 9 is substituted for a.

Answers will vary for Problems 6–9.