Lesson 8: Replacing Numbers with Letters

Student Outcomes

- Students understand that a letter in an expression or an equation can represent a number. When that number is replaced with a letter, an expression or an equation is stated.
- Students discover the commutative properties of addition and multiplication, the additive identity property of zero, and the multiplicative identity property of one. They determine that $g \div 1 = g$, $g \div g = 1$, and $1 \div g = \frac{1}{g}$.

Classwork

Fluency Exercise (10 minutes): Division of Fractions II

Sprint: Refer to the Sprints and the Sprint Delivery Script sections in the Module Overview for directions on how to administer a Sprint.

Opening Exercise (5 minutes)

Write this series of equations on the board:

Opening Exercise

$$4 + 0 = 4$$
$$4 \times 1 = 4$$
$$4 \div 1 = 4$$
$$4 \times 0 = 0$$
$$1 \div 4 = \frac{1}{4}$$

Discussion (5 minutes)

MP.3

How many of these statements are true?

All of them

How many of those statements would be true if the number 4 was replaced with the number 7 in each of the number sentences?

All of them

Would the number sentences be true if we were to replace the number 4 with any other number?

- Let students make conjectures about substitutions.

> What if we replaced the number 4 with the number 0? Would each of the number sentences be true?
>
> *No. The first four are true, but the last one, dividing by zero, is not true.*

- Division by zero is *undefined*. You cannot make zero groups of objects, and group size cannot be zero.
- It appears that we can replace the number 4 with any non-zero number and each of the number sentences will be true.
- A letter in an expression can represent a number. When that number is replaced with a letter, an expression is stated.

> What if we replace the number 4 with a letter g? Please write all 4 expressions in your student materials, replacing each 4 with a g.
>
> $$g + 0 = g$$
> $$g \times 1 = g$$
> $$g \div 1 = g$$
> $$g \times 0 = 0$$
> $$1 \div g = \frac{1}{g}$$
>
> Are these all true (except for $g = 0$) when dividing?
>
> *Yes*

- Let's look at each of these a little closer and see if we can make some generalizations.

Example 1 (5 minutes): Additive Identity Property of Zero

> Example 1: Additive Identity Property of Zero
>
> $$g + 0 = g$$
>
> Remember a letter in a mathematical expression represents a number. Can we replace g with any number?
>
> *Yes*
>
> Choose a value for g, and replace g with that number in the equation. What do you observe?
>
> *The value of g does not change when 0 is added to g.*
>
> Repeat this process several times, each time choosing a different number for g.

Allow students to experiment for about a minute. Most will quickly realize the additive identity property of zero: Any number added to zero equals itself. The number's identity does not change.

> **Will all values of g result in a true number sentence?**
>
> *Yes*
>
>
> **Write the mathematical language for this property below:**
>
> $g + 0 = g$, *additive identity property of zero. Any number added to zero equals itself.*

Example 2 (5 minutes): Multiplicative Identity Property of One

> **Example 2: Multiplicative Identity Property of One**
>
> $$g \times 1 = g$$
>
> **Remember a letter in a mathematical expression represents a number. Can we replace g with any number?**
>
> *Yes*
>
>
> **Choose a value for g, and replace g with that number in the equation. What do you observe?**
>
> *The value of g does not change when g is multiplied by 1.*

Allow students to experiment for about a minute with the next question. Most will quickly realize the multiplicative identity property of one: Any number multiplied by 1 equals itself. The number's identity does not change.

> **Will all values of g result in a true number sentence? Experiment with different values before making your claim.**
>
> *Yes*
>
>
> **Write the mathematical language for this property below:**
>
> $g \times 1 = g$, *multiplicative identity property of one. Any number multiplied by one equals itself.*

Example 3 (6 minutes): Commutative Property of Addition and Multiplication

> **Example 3: Commutative Property of Addition and Multiplication**
>
> $$3 + 4 = 4 + 3$$
> $$3 \times 4 = 4 \times 3$$
>
> **Replace the 3's in these number sentences with the letter a.**
>
> $$a + 4 = 4 + a$$
> $$a \times 4 = 4 \times a$$

> **Choose a value for a, and replace a with that number in each of the equations. What do you observe?**
>
> *The result is a true number sentence.*

Allow students to experiment for about a minute with the next question. Most will quickly realize that the equations are examples of the commutative property of addition and commutative property of multiplication. These are sometimes called the "any-order properties."

> **Will all values of a result in a true number sentence? Experiment with different values before making your claim.**
>
> *Yes, any number, even zero, can be used in place of the variable a.*
>
> **Now write the equations again, this time replacing the number 4 with a variable, b.**
> $$a + b = b + a$$
> $$a \times b = b \times a$$
>
> **Will all values of a and b result in true number sentences for the first two equations? Experiment with different values before making your claim.**
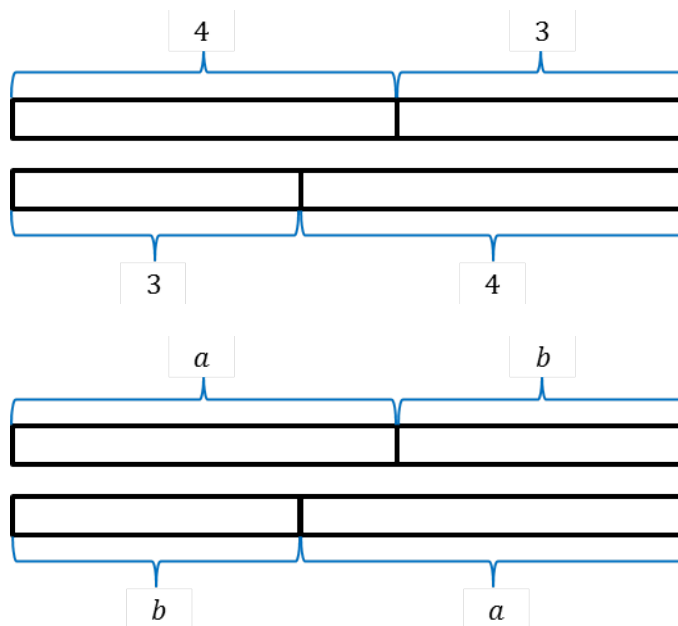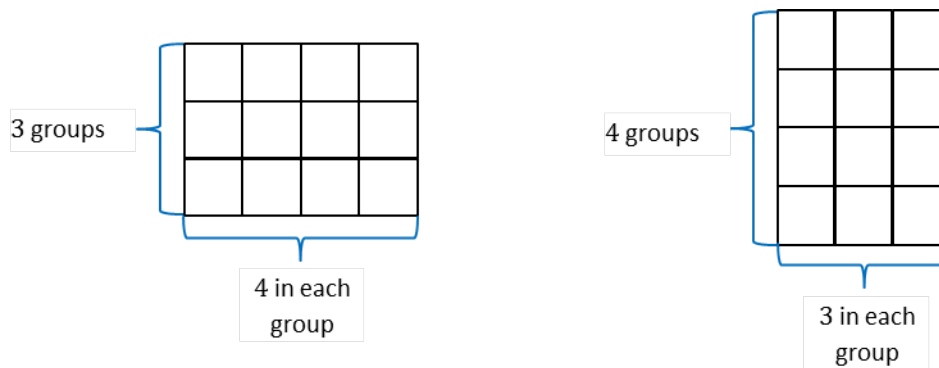>
> *Yes*
>
> **Write the mathematical language for this property below:**
>
> *$a + b = b + a$, commutative property of addition. Order does not matter when adding.*
>
> *$a \times b = b \times a$, commutative property of multiplication. Order does not matter when multiplying.*

- Models are useful for making abstract ideas more concrete.

3 groups

4 in each group

4 groups

3 in each group

Example 4 (4 minutes)

Display and discuss the models above as they relate to the commutative property of addition and the commutative property of multiplication. When finished, pose a new question:

- Will all values of a and b result in a true number sentence for the equation $a + a + a + a = b \times a$? Experiment with different values before making your claim.

Allow students to experiment for about a minute. They should discover that any value can be substituted for the variable a, but only 4 can be used for b, since there are exactly 4 copies of a in the equation.

- Summarize your discoveries with a partner.
   - *In the equation $a + a + a + a = b \times a$, any value can be substituted for the variable a, but only 4 can be used for b, since there are exactly 4 copies of a in the equation.*

- Finally, consider the last equation, $a \div b = \frac{a}{b}$. Is this true for all values of a and b?
   - *It is true for all values of a and all values of $b \neq 0$.*

Example 4

$$3 + 3 + 3 + 3 = 4 \times 3$$

$$3 \div 4 = \frac{3}{4}$$

Replace the 3's in these number sentences with the letter a.

$$a + a + a + a = 4 \times a$$

$$a \div 4 = \frac{a}{4}$$

Choose a value for a and replace a with that number in each of the equations. What do you observe?

The result is a true number sentence.

Will all values of a result in a true number sentence? Experiment with different values before making your claim.

Yes, any number, even zero, can be used in place of the variable a.

Now write the equations again, this time replacing the number 4 with a variable, b.

$$a + a + a + a = b \times a$$

$$a \div b = \frac{a}{b}, b \neq 0$$

Will all values of a and b result in true number sentences for the equations? Experiment with different values before making your claim.

In the equation $a + a + a + a = b \times a$, any value can be substituted for the variable a, but only 4 can be used for b since there are exactly 4 copies of a in the equation.

It is true for all values of a and all values of $b \neq 0$.

Closing (2 minutes)

- Tell your partner which of these properties of numbers is the easiest for you to remember?

Allow sharing for a short time.

- Now, tell your partner which of these properties of numbers is the hardest for you to remember?

Allow sharing for a short time.

- Although these properties might seem simple, we apply them in many different ways in mathematics. If you have a good grasp on them, you will recognize them and use them in many applications.
- With a partner, create two different division problems that support the following: $g \times 1 = g$, and be ready to explain your reasoning.
  - $g \div g = 1$; $5 \div 5 = 1$; $34 \div 34 = 1$; $2\frac{7}{8} \div 2\frac{7}{8} = 1$; etc.
  - *Any nonzero number divided by itself equals* 1.
  - *If a number g is divided into g equal parts, each part will have a size equal to one.*
  - *If g items are divided into groups of size g, there will be one group.*
- What about any number divided by 1? What does this mean?
  - $g \div 1 = g$
  - *If a number g is divided into* 1 *part, then the size of that part will be g.*
  - *Or, if g items are divided into* 1 *group, there will be g items in that group.*

Exit Ticket (5 minutes)

Name _____   Date _____

Lesson 8: Replacing Numbers with Letters

Exit Ticket

1.   State the commutative property of addition, and provide an example using two different numbers.

2.   State the commutative property of multiplication, and provide an example using two different numbers.

3.   State the additive property of zero, and provide an example using any other number.

4.   State the multiplicative identity property of one, and provide an example using any other number.

Exit Ticket Sample Solutions

1. State the commutative property of addition, and provide an example using two different numbers.

   Any two different addends can be chosen, such as $5 + 6 = 6 + 5$.

2. State the commutative property of multiplication, and provide an example using two different numbers.

   Any two different factors can be chosen, such as $4 \times 9 = 9 \times 4$.

3. State the additive property of zero, and provide an example using any other number.

   Any nonzero addend can be chosen, such as $3 + 0 = 3$.

4. State the multiplicative identity property of one, and provide an example using any other number.

   Any nonzero factor can be chosen, such as $12 \times 1 = 12$.

Problem Set Sample Solutions

1. State the commutative property of addition using the variables a and b.

   $a + b = b + a$

2. State the commutative property of multiplication using the variables a and b.

   $a \times b = b \times a$

3. State the additive property of zero using the variable b.

   $b + 0 = b$

4. State the multiplicative identity property of one using the variable b.

   $b \times 1 = b$

5. Demonstrate the property listed in the first column by filling in the third column of the table.

Commutative Property of Addition	$25 + c =$	$c + 25$
Commutative Property of Multiplication	$l \times w =$	$w \times l$
Additive Property of Zero	$h + 0 =$	h
Multiplicative Identity Property of One	$v \times 1 =$	v

6. Why is there no commutative property for subtraction or division? Show examples.

   Answers will vary. Examples should show reasoning and proof that the commutative property does not work for subtraction and division. An example would be $8 \div 2$ and $2 \div 8$. $8 \div 2 = 4$, but $2 \div 8 = \frac{1}{4}$.

Division of Fractions II–Round 1

Directions: Determine the quotient of the fractions.

1.	$\frac{4}{10} \div \frac{2}{10}$		16.	$3\frac{1}{8} \div \frac{2}{3}$	
2.	$\frac{9}{12} \div \frac{3}{12}$		17.	$1\frac{5}{6} \div \frac{1}{2}$	
3.	$\frac{6}{10} \div \frac{4}{10}$		18.	$\frac{5}{8} \div 2\frac{3}{4}$	
4.	$\frac{2}{8} \div \frac{3}{8}$		19.	$\frac{1}{3} \div 1\frac{4}{5}$	
5.	$\frac{2}{7} \div \frac{6}{7}$		20.	$\frac{3}{4} \div 2\frac{3}{10}$	
6.	$\frac{11}{9} \div \frac{8}{9}$		21.	$2\frac{1}{5} \div 1\frac{1}{6}$	
7.	$\frac{5}{13} \div \frac{10}{13}$		22.	$2\frac{4}{9} \div 1\frac{3}{5}$	
8.	$\frac{7}{8} \div \frac{13}{16}$		23.	$1\frac{2}{9} \div 3\frac{2}{5}$	
9.	$\frac{3}{5} \div \frac{7}{10}$		24.	$2\frac{2}{3} \div 3$	
10.	$\frac{9}{30} \div \frac{3}{5}$		25.	$1\frac{3}{4} \div 2\frac{2}{5}$	
11.	$\frac{1}{3} \div \frac{4}{5}$		26.	$4 \div 1\frac{2}{9}$	
12.	$\frac{2}{5} \div \frac{3}{4}$		27.	$3\frac{1}{5} \div 6$	
13.	$\frac{3}{4} \div \frac{5}{9}$		28.	$2\frac{5}{6} \div 1\frac{1}{3}$	
14.	$\frac{4}{5} \div \frac{7}{12}$		29.	$10\frac{2}{3} \div 8$	
15.	$\frac{3}{8} \div \frac{5}{2}$		30.	$15 \div 2\frac{3}{5}$	

Division of Fractions II–Round 1 [KEY]

Directions: Determine the quotient of the fractions.

1.	$\dfrac{4}{10} \div \dfrac{2}{10}$	$\dfrac{4}{2} = 2$	16.	$3\dfrac{1}{8} \div \dfrac{2}{3}$	$\dfrac{75}{16} = 4\dfrac{11}{16}$
2.	$\dfrac{9}{12} \div \dfrac{3}{12}$	$\dfrac{9}{3} = 3$	17.	$1\dfrac{5}{6} \div \dfrac{1}{2}$	$\dfrac{22}{6} = \dfrac{11}{3} = 3\dfrac{2}{3}$
3.	$\dfrac{6}{10} \div \dfrac{4}{10}$	$\dfrac{6}{4} = \dfrac{3}{2} = 1\dfrac{1}{2}$	18.	$\dfrac{5}{8} \div 2\dfrac{3}{4}$	$\dfrac{20}{88} = \dfrac{5}{22}$
4.	$\dfrac{2}{8} \div \dfrac{3}{8}$	$\dfrac{2}{3}$	19.	$\dfrac{1}{3} \div 1\dfrac{4}{5}$	$\dfrac{5}{27}$
5.	$\dfrac{2}{7} \div \dfrac{6}{7}$	$\dfrac{2}{6} = \dfrac{1}{3}$	20.	$\dfrac{3}{4} \div 2\dfrac{3}{10}$	$\dfrac{30}{92} = \dfrac{15}{46}$
6.	$\dfrac{11}{9} \div \dfrac{8}{9}$	$\dfrac{11}{8} = 1\dfrac{3}{8}$	21.	$2\dfrac{1}{5} \div 1\dfrac{1}{6}$	$\dfrac{66}{35} = 1\dfrac{31}{35}$
7.	$\dfrac{5}{13} \div \dfrac{10}{13}$	$\dfrac{5}{10} = \dfrac{1}{2}$	22.	$2\dfrac{4}{9} \div 1\dfrac{3}{5}$	$\dfrac{110}{72} = \dfrac{55}{36} = 1\dfrac{19}{36}$
8.	$\dfrac{7}{8} \div \dfrac{13}{16}$	$\dfrac{14}{13} = 1\dfrac{1}{13}$	23.	$1\dfrac{2}{9} \div 3\dfrac{2}{5}$	$\dfrac{55}{153}$
9.	$\dfrac{3}{5} \div \dfrac{7}{10}$	$\dfrac{6}{7}$	24.	$2\dfrac{2}{3} \div 3$	$\dfrac{8}{9}$
10.	$\dfrac{9}{30} \div \dfrac{3}{5}$	$\dfrac{9}{18} = \dfrac{1}{2}$	25.	$1\dfrac{3}{4} \div 2\dfrac{2}{5}$	$\dfrac{35}{48}$
11.	$\dfrac{1}{3} \div \dfrac{4}{5}$	$\dfrac{5}{12}$	26.	$4 \div 1\dfrac{2}{9}$	$\dfrac{36}{11} = 3\dfrac{3}{11}$
12.	$\dfrac{2}{5} \div \dfrac{3}{4}$	$\dfrac{8}{15}$	27.	$3\dfrac{1}{5} \div 6$	$\dfrac{16}{30} = \dfrac{8}{15}$
13.	$\dfrac{3}{4} \div \dfrac{5}{9}$	$\dfrac{27}{20} = 1\dfrac{7}{20}$	28.	$2\dfrac{5}{6} \div 1\dfrac{1}{3}$	$\dfrac{51}{24} = 2\dfrac{3}{24} = 2\dfrac{1}{8}$
14.	$\dfrac{4}{5} \div \dfrac{7}{12}$	$\dfrac{48}{35} = 1\dfrac{13}{35}$	29.	$10\dfrac{2}{3} \div 8$	$\dfrac{32}{24} = \dfrac{4}{3} = 1\dfrac{1}{3}$
15.	$\dfrac{3}{8} \div \dfrac{5}{2}$	$\dfrac{6}{40} = \dfrac{3}{20}$	30.	$15 \div 2\dfrac{3}{5}$	$\dfrac{195}{5} = 39$

Division of Fractions II–Round 2

Directions: Determine the quotient of the fractions.

1.	$\dfrac{10}{2} \div \dfrac{5}{2}$	
2.	$\dfrac{6}{5} \div \dfrac{3}{5}$	
3.	$\dfrac{10}{7} \div \dfrac{2}{7}$	
4.	$\dfrac{3}{8} \div \dfrac{5}{8}$	
5.	$\dfrac{1}{4} \div \dfrac{3}{12}$	
6.	$\dfrac{7}{5} \div \dfrac{3}{10}$	
7.	$\dfrac{8}{15} \div \dfrac{4}{5}$	
8.	$\dfrac{5}{6} \div \dfrac{5}{12}$	
9.	$\dfrac{3}{5} \div \dfrac{7}{9}$	
10.	$\dfrac{3}{10} \div \dfrac{3}{9}$	
11.	$\dfrac{3}{4} \div \dfrac{7}{9}$	
12.	$\dfrac{7}{10} \div \dfrac{3}{8}$	
13.	$4 \div \dfrac{4}{9}$	
14.	$\dfrac{5}{8} \div 7$	
15.	$9 \div \dfrac{2}{3}$	

16.	$\dfrac{5}{8} \div 1\dfrac{3}{4}$	
17.	$\dfrac{1}{4} \div 2\dfrac{2}{5}$	
18.	$2\dfrac{3}{5} \div \dfrac{3}{8}$	
19.	$1\dfrac{3}{5} \div \dfrac{2}{9}$	
20.	$4 \div 2\dfrac{3}{8}$	
21.	$1\dfrac{1}{2} \div 5$	
22.	$3\dfrac{1}{3} \div 1\dfrac{3}{4}$	
23.	$2\dfrac{2}{5} \div 1\dfrac{1}{4}$	
24.	$3\dfrac{1}{2} \div 2\dfrac{2}{3}$	
25.	$1\dfrac{4}{5} \div 2\dfrac{3}{4}$	
26.	$3\dfrac{1}{6} \div 1\dfrac{3}{5}$	
27.	$3\dfrac{3}{5} \div 2\dfrac{1}{8}$	
28.	$5 \div 1\dfrac{1}{6}$	
29.	$3\dfrac{3}{4} \div 5\dfrac{1}{2}$	
30.	$4\dfrac{2}{3} \div 5\dfrac{1}{4}$	

Lesson 8: Replacing Numbers with Letters
Date: 11/19/14

Division of Fractions II–Round 2 [KEY]

Directions: Determine the quotient of the fractions.

1.	$\dfrac{10}{2} \div \dfrac{5}{2}$	$\dfrac{10}{5} = 2$	16.	$\dfrac{5}{8} \div 1\dfrac{3}{4}$	$\dfrac{20}{56} = \dfrac{5}{14}$
2.	$\dfrac{6}{5} \div \dfrac{3}{5}$	$\dfrac{6}{3} = 2$	17.	$\dfrac{1}{4} \div 2\dfrac{2}{5}$	$\dfrac{5}{48}$
3.	$\dfrac{10}{7} \div \dfrac{2}{7}$	$\dfrac{10}{2} = 5$	18.	$2\dfrac{3}{5} \div \dfrac{3}{8}$	$\dfrac{104}{15} = 6\dfrac{14}{15}$
4.	$\dfrac{3}{8} \div \dfrac{5}{8}$	$\dfrac{3}{5}$	19.	$1\dfrac{3}{5} \div \dfrac{2}{9}$	$\dfrac{72}{10} = 7\dfrac{2}{10} = 7\dfrac{1}{5}$
5.	$\dfrac{1}{4} \div \dfrac{3}{12}$	$\dfrac{3}{3} = 1$	20.	$4 \div 2\dfrac{3}{8}$	$\dfrac{32}{19} = 1\dfrac{6}{19}$
6.	$\dfrac{7}{5} \div \dfrac{3}{10}$	$\dfrac{14}{3} = 4\dfrac{2}{3}$	21.	$1\dfrac{1}{2} \div 5$	$\dfrac{3}{10}$
7.	$\dfrac{8}{15} \div \dfrac{4}{5}$	$\dfrac{8}{12} = \dfrac{2}{3}$	22.	$3\dfrac{1}{3} \div 1\dfrac{3}{4}$	$\dfrac{40}{21} = 1\dfrac{19}{21}$
8.	$\dfrac{5}{6} \div \dfrac{5}{12}$	$\dfrac{10}{5} = 2$	23.	$2\dfrac{2}{5} \div 1\dfrac{1}{4}$	$\dfrac{48}{25} = 1\dfrac{23}{25}$
9.	$\dfrac{3}{5} \div \dfrac{7}{9}$	$\dfrac{27}{35}$	24.	$3\dfrac{1}{2} \div 2\dfrac{2}{3}$	$\dfrac{21}{16} = 1\dfrac{5}{16}$
10.	$\dfrac{3}{10} \div \dfrac{3}{9}$	$\dfrac{27}{30} = \dfrac{9}{10}$	25.	$1\dfrac{4}{5} \div 2\dfrac{3}{4}$	$\dfrac{36}{55}$
11.	$\dfrac{3}{4} \div \dfrac{7}{9}$	$\dfrac{27}{28}$	26.	$3\dfrac{1}{6} \div 1\dfrac{3}{5}$	$\dfrac{95}{48} = 1\dfrac{47}{48}$
12.	$\dfrac{7}{10} \div \dfrac{3}{8}$	$\dfrac{56}{30} = \dfrac{28}{15} = 1\dfrac{13}{15}$	27.	$3\dfrac{3}{5} \div 2\dfrac{1}{8}$	$\dfrac{144}{85} = 1\dfrac{59}{85}$
13.	$4 \div \dfrac{4}{9}$	$\dfrac{16}{9} = 1\dfrac{7}{9}$	28.	$5 \div 1\dfrac{1}{6}$	$\dfrac{30}{7} = 4\dfrac{2}{7}$
14.	$\dfrac{5}{8} \div 7$	$\dfrac{5}{56}$	29.	$3\dfrac{3}{4} \div 5\dfrac{1}{2}$	$\dfrac{30}{44} = \dfrac{15}{22}$
15.	$9 \div \dfrac{2}{3}$	$\dfrac{18}{3} = 6$	30.	$4\dfrac{2}{3} \div 5\dfrac{1}{4}$	$\dfrac{56}{63} = \dfrac{8}{9}$