# Lesson 7: Replacing Letters with Numbers

## Student Outcomes

- Students understand that a letter represents one number in an expression. When that number replaces the letter, the expression can be evaluated to one number.

## Lesson Notes

Before this lesson, make it clear to students that just like $3 \times 3$ is $3^2$ or three squared, $\text{units} \times \text{units}$ is $\text{units}^2$ or units squared (also called square units).

It may be helpful to cut and paste some of the figures from this lesson onto either paper or an interactive whiteboard application. Each of the basic figures is depicted two ways: One has side lengths that can be counted, and the other is a similar figure without grid lines. Also, ahead of time, draw a $23 \text{ cm}$ square on a chalkboard, whiteboard, or interactive board.

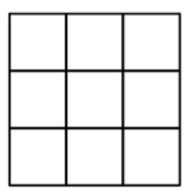
There is a square in the student materials that is approximately $23 \text{ mm}$ square, or $529 \text{ mm}^2$.

## Classwork

### Example 1 (10 minutes)

Draw or project the square shown.

**Example 1**



**What is the length of one side of this square?**

$3$ units

**What is the formula for the area of a square?**

$A = s^2$

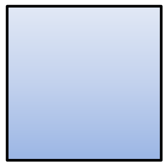
**What is the square's area as a multiplication expression?**

$3 \text{ units} \times 3 \text{ units}$

**What is the square's area?**

9 square units

**We can count the units. However, look at this other square. Its side length is $23$ cm. That is just too many tiny units to draw. What expression can we build to find this square's area?**



23 cm

$23 \text{ cm} \times 23 \text{ cm}$

**What is the area of the square? Use a calculator if you need to.**

$529 \text{ cm}^2$

- A letter represents <u>one</u> number in an expression. That number was 3 in our first square and 23 in our second square. When that number replaces the letter, the expression can be evaluated to <u>one</u> number. In our first example, the expression was evaluated to be 9, and in the second example, the expression was evaluated to be 529.

Make sure students understand that 9 is one number, but 529 is also one number. (It happens to have 3 digits, but it is still one number.)

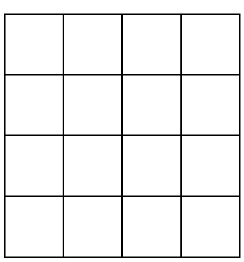
### Exercise 1 (5 minutes)

Ask students to work both problems from Exercise 1 in their student materials. Make clear to the students that these drawings are not to scale.

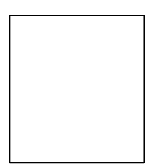
**Exercise 1**

**Complete the table below for both squares. Note: These drawings are not to scale.**

$s = 4$



$s = 25$ in.

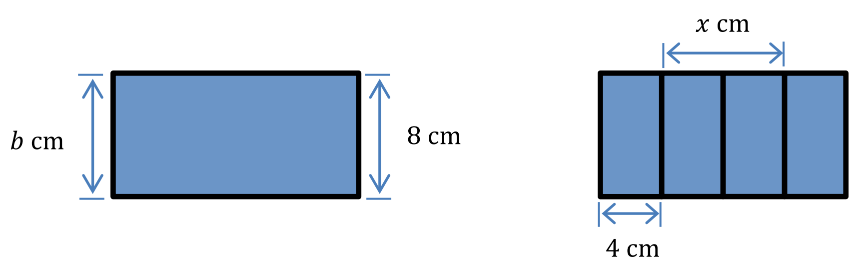


| Length of One Side of the Square | Square's Area Written as an Expression | Square's Area Written as a Number |
|---|---|---|
| $4$ units | $4$ units $\times$ $4$ units | $16$ square units |
| $25$ in. | $25$ in. $\times$ $25$ in. | $625 \text{ in}^2$ |

Make sure students have the units correctly recorded in each of the cells of the table. When units are not specified, keep the label "unit" or "square unit."

## Example 2 (10 minutes)

**Example 2**




- The formula $A = l \times w$ is an efficient way to find the area of a rectangle without being required to count the area units in a rectangle.

**What does the letter $b$ represent in this blue rectangle?**

*$b = 8$*

Give students a short time for discussion of the next question among partners, and then ask for an answer and an explanation.

**With a partner, answer the following question: Given that the second rectangle is divided into four <u>equal</u> parts, what number does the $x$ represent?**

*$x = 8$*

**How did you arrive at this answer?**

*We reasoned that each width of the $4$ congruent rectangles must be the same. Two $4$ cm lengths equals $8$ cm.*

**What is the total length of the second rectangle? Tell a partner how you know.**

*The length consists of $4$ segments that each has a length of $4$ cm. $4 \times 4 \text{ cm} = 16 \text{ cm}$.*

**If the two large rectangles have equal lengths and widths, find the area of each rectangle.**

$128\ \text{cm}^2$

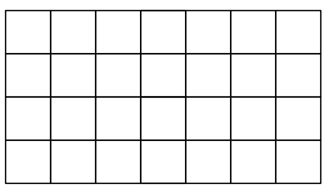
**Discuss with your partner how the formulas for the area of squares and rectangles can be used to evaluate area for a particular figure.**

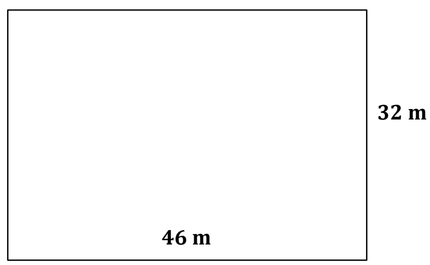
- Remember, a letter represents one number in an expression. When that number replaces the letter, the expression can be evaluated to one number.

## Exercise 2 (5 minutes)

Ask students to complete the table for both rectangles in their student materials. Using a calculator is appropriate.

**Exercise 2**



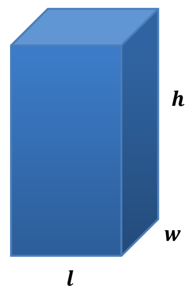


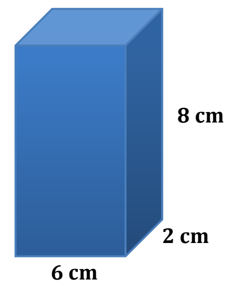

| Length of Rectangle | Width of Rectangle | Rectangle's Area Written as an Expression | Rectangle's Area Written as a Number |
|---|---|---|---|
| $7$ units | $4$ units | $7$ units $\times$ $4$ units | $28$ square units |
| $46\ \text{m}$ | $32\ \text{m}$ | $46\ \text{m} \times 32\ \text{m}$ | $1{,}472\ \text{m}^2$ |

## Example 3 (3 minutes)

- The formula $V = l \times w \times h$ is a quick way to determine the volume of right rectangular prisms.
- Take a look at the right rectangular prisms in your student materials.

**Example 3**




**What does the $l$ represent in the first diagram?**

*The length of the rectangular prism.*

**What does the $w$ represent in the first diagram?**

*The width of the rectangular prism.*

**What does the $h$ represent in the first diagram?**

*The height of the rectangular prism.*

- Notice that the right rectangular prism in the second diagram is an exact copy of the first diagram.

**Since we know the formula to find the volume is $V = l \times w \times h$, what number can we substitute for the $l$ in the formula? Why?**

*$6$, because the length of the second right rectangular prism is $6$ cm.*

**What other number can we substitute for the $l$?**

*No other number can replace the $l$. Only one number can replace one letter.*

**What number can we substitute for the $w$ in the formula? Why?**

*$2$, because the width of the second right rectangular prism is $2$ cm.*

**What number can we substitute for the $h$ in the formula?**

*$8$, because the height of the second right rectangular prism is $8$ cm.*

**Determine the volume of the second right rectangular prism by replacing the letters in the formula with their appropriate numbers.**

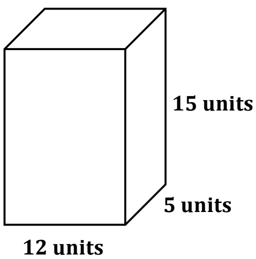
$V = l \times w \times h$; $V = 6\text{ cm} \times 2\text{ cm} \times 8\text{ cm} = 96\text{ cm}^3$

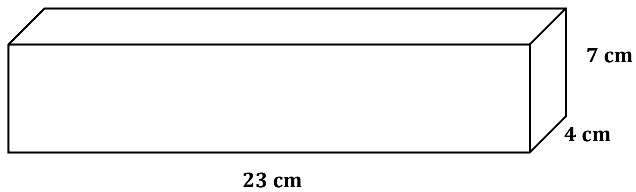
### Exercise 3 (5 minutes)

Ask students to complete the table for both figures in their student materials. Using a calculator is appropriate.

**Exercise 3**

**Complete the table for both figures. Using a calculator is appropriate.**




| Length of Rectangular Prism | Width of Rectangular Prism | Height of Rectangular Prism | Rectangular Prism's Volume Written as an Expression | Rectangular Prism's Volume Written as a Number |
|---|---|---|---|---|
| 12 units | 5 units | 15 units | 12 units $\times$ 5 units $\times$ 15 units | 900 cubic units |
| 23 cm | 4 cm | 7 cm | 23 cm $\times$ 4 cm $\times$ 7 cm | $644\text{ cm}^3$ |

### Closing (2 minutes)

- How many numbers are represented by one letter in an expression?
  - *One.*
- When that number replaces the letter, the expression can be evaluated to what?
  - *One number.*

**Lesson Summary**

**EXPRESSION: An *expression* is a numerical expression, or it is the result of replacing some (or all) of the numbers in a numerical expression with variables.**

**There are two ways to build expressions:**

1. **We can start out with a numerical expression, such as $\frac{1}{3} \cdot (2 + 4) + 7$, and replace some of the numbers with letters to get $\frac{1}{3} \cdot (x + y) + z$.**
2. **We can build such expressions from scratch, as in $x + x(y - z)$, and note that if numbers were placed in the expression for the variables $x$, $y$, and $z$, the result would be a numerical expression.**

The key is to strongly link expressions back to computations with numbers.

The description for *expression* given above is meant to work nicely with how students in Grade 6 and Grade 7 learn to manipulate expressions. In these grades, a lot of time is spent *building expressions* and *evaluating expressions*. Building and evaluating helps students see that expressions are really just a slight abstraction of arithmetic in elementary school. Building often occurs by thinking about examples of numerical expressions first, and then replacing the numbers with letters in a numerical expression. The act of evaluating for students at this stage means they replace each of the variables with specific numbers and then compute to obtain a number.

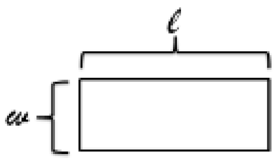
## Exit Ticket (5 minutes)

Name ______________________________ Date ________________

# Lesson 7: Replacing Letters with Numbers

## Exit Ticket

1. In the drawing below, what do the letters $l$ and $w$ represent?



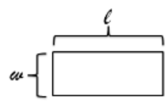

2. What does the expression $l + w + l + w$ represent?

3. What does the expression $l \cdot w$ represent?

4. The rectangle below is congruent to the rectangle shown in Problem 1. Use this information to evaluate the expressions from Problems 2 and 3.

## Exit Ticket Sample Solutions

1. In the drawing below, what do the letters $l$ and $w$ represent?



   *Length and width of the rectangle*

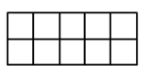
2. What does the expression $l + w + l + w$ represent?

   *Perimeter of the rectangle, or the sum of the sides of the rectangle*

3. What does the expression $l \cdot w$ represent?

   *Area of the rectangle*

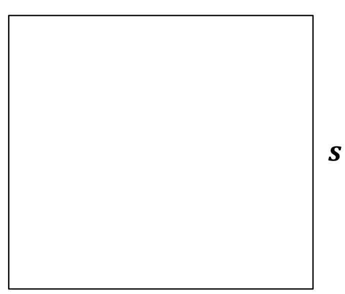
4. The rectangle below is congruent to the rectangle shown in Problem 1. Use this information to evaluate the expressions from Problems 2 and 3.



   $l = 5$ and $w = 2$ $\quad$ $P = 14$ units $\quad$ $A = 10$ units$^2$

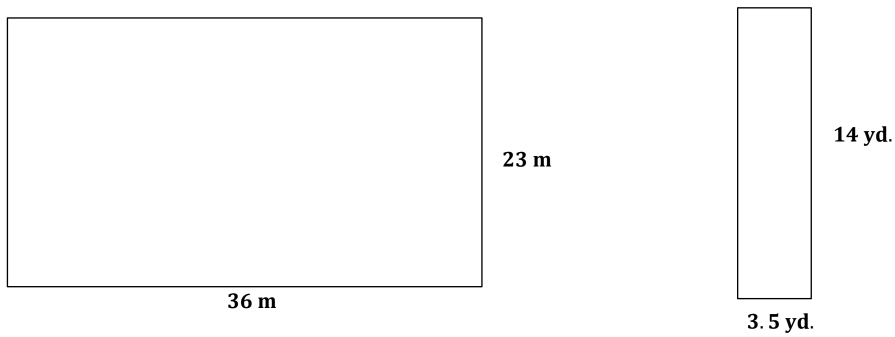
## Problem Set Sample Solutions

1. Replace the side length of this square with $4$ in., and find the area.



   *The student should draw a square, label the side $4$ in., and calculate the area to be $16$ in$^2$.*

2. Complete the table for each of the given figures.




| Length of Rectangle | Width of Rectangle | Rectangle's Area Written as an Expression | Rectangle's Area Written as a Number |
|---|---|---|---|
| $36 \text{ m}$ | $23 \text{ m}$ | $36 \text{ m} \times 23 \text{ m}$ | $828 \text{ m}^2$ |
| $14 \text{ yd.}$ | $3.5 \text{ yd.}$ | $14 \text{ yd.} \times 3.5 \text{ yd.}$ | $49 \text{ yd}^2$ |

3. Find the perimeter of each quadrilateral in Problems 1 and 2.

$P = 16 \text{ in.}$ $\quad P = 118 \text{ m}$ $\quad P = 35 \text{ yd.}$

4. Using the formula $V = l \times w \times h$, find the volume of a right rectangular prism when the length of the prism is $45 \text{ cm}$, the width is $12 \text{ cm}$, and the height is $10 \text{ cm}$.

$V = l \times w \times h$; $V = 45 \text{ cm} \times 12 \text{ cm} \times 10 \text{ cm} = 5,400 \text{ cm}^3$