



## Lesson 6: The Order of Operations

### Student Outcomes

- Students evaluate numerical expressions. They recognize that in the absence of parentheses, exponents are evaluated first.

### Classwork

#### Opening (5 minutes)

Take a few minutes to review the Problem Set from the previous lesson. Clarify any misconceptions about the use and evaluation of exponents.

#### Opening Exercise (5 minutes)

Post the following expression on the board, and ask students to evaluate it.

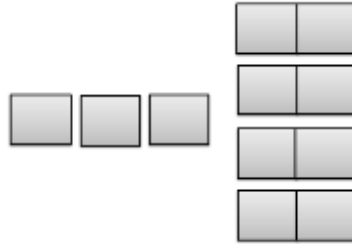
$$3 + 4 \times 2$$

Ask students to record their answer and report it using personal white boards, cards, electronic vote devices, etc. Students who arrive at an answer other than 11 or 14 should recheck their work.

#### Discussion (5 minutes)

- How did you evaluate the expression  $3 + 4 \times 2$ ?
  - I added  $3 + 4$  first for a sum of 7; then, I multiplied  $7 \times 2$  for a product of 14.*
  - I multiplied  $4 \times 2$  first for a product of 8; then, I added  $8 + 3$  for a sum of 11.*
- Only one of these answers can be correct. When we evaluate expressions, we must agree to use one set of rules so that everyone arrives at the same correct answer.
- During the last lesson, we said that addition was a shortcut to “counting on.” How could you think about subtraction?
  - Subtraction is a shortcut to “counting back.”*
- These were the first operations that you learned because they are the least complicated. Next, you learned about multiplication and division.
- Multiplication can be thought of as repeated addition. Thinking back on Lesson 4, how could you think about division?
  - Division is repeated subtraction.*
- Multiplication and division are more powerful than addition and subtraction, which led mathematicians to develop the order of operations in this way. When we evaluate expressions that have any of these four operations, we always calculate multiplication and division before doing any addition or subtraction. Since multiplication and division are equally powerful, we simply evaluate these two operations as they are written in the expression, from left to right.

- Addition and subtraction are at the same level in the order of operations and are evaluated from left to right in an expression. Now that these rules of order of operations are clear, can you go back and evaluate the expression  $3 + 4 \times 2$  as 11?



- The diagram correctly models the expression  $3 + 4 \times 2$ .
- With addition, we are finding the sum of two addends. In this example, the first addend is the number 3. The second addend happens to be the number that is the value of the expression  $4 \times 2$ ; so, before we can add, we must determine the value of the second addend.

### Example 1 (5 minutes): Expressions with Only Addition, Subtraction, Multiplication, and Division

Example 1: Expressions with Only Addition, Subtraction, Multiplication, and Division

What operations are evaluated first?

*Multiplication and division are evaluated first, from left to right.*

What operations are always evaluated last?

*Addition and subtraction are always evaluated last, from left to right.*

Ask students to evaluate the expressions.

#### Exercises 1–3

1.  $4 + 2 \times 7$

$4 + 14$

$18$

2.  $36 \div 3 \times 4$

$12 \times 4$

$48$

3.  $20 - 5 \times 2$

$20 - 10$

$10$

- In the last lesson, you learned about exponents, which are a way of writing repeated multiplication. So, exponents are more powerful than multiplication or division. If exponents are present in an expression, they are evaluated before any multiplication or division.
- We now know that when we evaluate expressions, we must agree to use one set of rules so that everyone arrives at the same correct answer. These rules are based on doing the most powerful operations first (exponents), then the less powerful ones (multiplication and division, going from left to right), and finally the least powerful ones last (addition and subtraction, going from left to right).
- Evaluate the expression  $4 + 6 \times 6 \div 8$ .
  - $4 + (6 \times 6) \div 8$
  - $4 + (36 \div 8)$
  - $4 + 4.5$
  - 8.5
- Now, evaluate the expression  $4 + 6^2 \div 8$ .
  - $4 + (6^2) \div 8$
  - $4 + (36 \div 8)$
  - $4 + 4.5$
  - 8.5
- Why was your first step to find the value of  $6^2$ ?
  - *Because exponents are evaluated first.*

### Example 2 (5 minutes): Expressions with Four Operations and Exponents

Display the following expression.

**Example 2: Expressions with Four Operations and Exponents**

$$4 + 9^2 \div 3 \times 2 - 2$$

What operation is evaluated first?

*Exponents (  $9^2 = 9 \times 9 = 81$  )*

What operations are evaluated next?

*Multiplication and division, from left to right (  $81 \div 3 = 27$ ;  $27 \times 2 = 54$  )*

What operations are always evaluated last?

*Addition and subtraction, from left to right (  $4 + 54 = 58$ ;  $58 - 2 = 56$  )*

What is the final answer?

56

*Scaffolding:*

Some students may benefit from rewriting the expression on successive lines, evaluating only one or two operations on each line.

- Evaluate the next two exercises.

While the answers are provided, it is extremely important to circulate to ensure that students are using the correct order of operations to achieve the answer. For example, in Exercise 5, they should show  $4^3$  first, followed by  $2 \times 8$ .

## Exercises 4–5

4.  $90 - 5^2 \times 3$

$$90 - 25 \times 3$$

$$90 - 75$$

$$15$$

5.  $4^3 + 2 \times 8$

$$64 + 2 \times 8$$

$$64 + 16$$

$$80$$

## Example 3 (5 minutes): Expressions with Parentheses

- The last important rule in the order of operations involves grouping symbols, usually parentheses. These tell us that in certain circumstances or scenarios, we need to do things out of the usual order. Operations inside grouping symbols are always evaluated first, before exponents and any operations.

## Example 3: Expressions with Parentheses

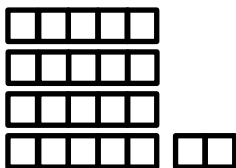
Consider a family of 4 that goes to a soccer game. Tickets are \$5.00 each. The mom also buys a soft drink for \$2.00. How would you write this expression?

$$4 \times 5 + 2$$

How much will this outing cost?

$$\$22$$

- Here is a model of the scenario:



Consider a different scenario: The same family goes to the game as before, but each of the family members wants a drink. How would you write this expression?

$$4 \times (5 + 2)$$

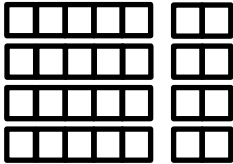
Why would you add the 5 and 2 first?

*We need to determine how much each person spends. Each person spends \$7; then, we multiply by 4 people to figure out the total cost.*

How much will this outing cost?

\$28

- Here is a model of the second scenario:



How many groups are there?

4

What is each group comprised of?

\$5 + \$2, or \$7

- The last complication that can arise is if two or more sets of parentheses are ever needed; evaluate the innermost parentheses first, and then work outward.
- Try Exercises 6 and 7.

#### Exercises 6–7

6.  $2 + (9^2 - 4)$

$2 + (81 - 4)$

$2 + 77$

79

7.  $2 \cdot (13 + 5 - 14 \div (3 + 4))$

$2 \cdot (13 + 5 - 14 \div 7)$

$2 \cdot (13 + 5 - 2)$

$2 \cdot 16$

32

If students are confused trying to divide 14 by 3, reiterate the rule about nested parentheses.

**Example 4 (5 minutes): Expressions with Parentheses and Exponents**

- Let's take a look at how parentheses and exponents work together. Sometimes a problem will have parentheses, and the values inside the parentheses have an exponent. Let's evaluate the following expression.

Place the expression on the board.

- We will evaluate the parentheses first.

**Example 4: Expressions with Parentheses and Exponents**

$$2 \times (3 + 4^2)$$

Which value will we evaluate first within the parentheses? Evaluate.

*First, evaluate  $4^2$ , which is 16; then, add 3. The value of the parentheses is 19.*

$$2 \times (3 + 4^2)$$

$$2 \times (3 + 16)$$

$$2 \times 19$$

Evaluate the rest of the expression.

$$2 \times 19 = 38$$

Place the expression on the board:

What do you think will happen when the exponent in this expression is outside of the parentheses?

$$2 \times (3 + 4)^2$$

Will the answer be the same?

*Answers will vary.*

Which should we evaluate first? Evaluate.

*Parentheses*

$$2 \times (3 + 4)^2$$

$$2 \times (7)^2$$

What happened differently here than in our last example?

*The 4 was not raised to the second power because it did not have an exponent. We simply added the values inside the parentheses.*

What should our next step be?

*We need to evaluate the exponent next.*

$$7^2 = 7 \times 7 = 49$$

Evaluate to find the final answer.

$$2 \times 49$$

$$98$$

What do you notice about the two answers?

*The final answers were not the same.*

What was different between the two expressions?

*Answers may vary. In the first problem, a value inside the parentheses had an exponent, and that value was evaluated first because it was inside of the parentheses. In the second problem, the exponent was outside of the parentheses, which made us evaluate what was in the parentheses first; then, we raised that value to the power of the exponent.*

What conclusions can you draw about evaluating expressions with parentheses and exponents?

*Answers may vary. Regardless of the location of the exponent in the expression, evaluate the parentheses first. Sometimes there will be values with exponents inside the parentheses. If the exponent is outside the parentheses, evaluate the parentheses first, and then evaluate to the power of the exponent.*

- Try Exercises 8 and 9.

#### Exercises 8–9

8.  $7 + (12 - 3^2)$

$$7 + (12 - 9)$$

$$7 + 3$$

$$10$$

9.  $7 + (12 - 3)^2$

$$7 + 9^2$$

$$7 + 81$$

$$88$$

#### Closing (5 minutes)

- When we evaluate expressions, we use one set of rules so that everyone arrives at the same correct answer. Grouping symbols, like parentheses, tell us to evaluate whatever is inside them before moving on. These rules are based on doing the most powerful operations first (exponents), then the less powerful ones (multiplication and division, going from left to right), and finally the least powerful ones last (addition and subtraction, going from left to right).

## Lesson Summary

**NUMERICAL EXPRESSION:** A *numerical expression* is a number, or it is any combination of sums, differences, products, or divisions of numbers that evaluates to a number.

Statements like “ $3 +$ ” or “ $3 \div 0$ ” are not numerical expressions because neither represents a point on the number line. Note: Raising numbers to whole number powers are considered numerical expressions as well, since the operation is just an abbreviated form of multiplication, e.g.,  $2^3 = 2 \cdot 2 \cdot 2$ .

**VALUE OF A NUMERICAL EXPRESSION:** The *value of a numerical expression* is the number found by evaluating the expression.

For example:  $\frac{1}{3} \cdot (2 + 4) + 7$  is a numerical expression, and its value is 9.

Note: Please do not stress words over meaning here. It is okay to talk about the *number computed*, *computation*, *calculation*, etc., to refer to the value as well.

## Exit Ticket (5 minutes)



Name \_\_\_\_\_

Date \_\_\_\_\_

## Lesson 6: The Order of Operations

### Exit Ticket

1. Evaluate this expression:  $39 \div (2 + 1) - 2 \times (4 + 1)$ .

2. Evaluate this expression:  $12 \times (3 + 2^2) \div 2 - 10$ .

3. Evaluate this expression:  $12 \times (3 + 2)^2 \div 2 - 10$ .

## Exit Ticket Sample Solutions

1. Evaluate this expression:  $39 \div (2 + 1) - 2 \times (4 + 1)$ .

$$39 \div 3 - 2 \times 5$$

$$13 - 10$$

$$3$$

2. Evaluate this expression:  $12 \times (3 + 2^2) \div 2 - 10$ .

$$12 \times (3 + 4) \div 2 - 10$$

$$12 \times 7 \div 2 - 10$$

$$84 \div 2 - 10$$

$$42 - 10$$

$$32$$

3. Evaluate this expression:  $12 \times (3 + 2)^2 \div 2 - 10$ .

$$12 \times 5^2 \div 2 - 10$$

$$12 \times 25 \div 2 - 10$$

$$300 \div 2 - 10$$

$$150 - 10$$

$$140$$

## Problem Set Sample Solutions

Evaluate each expression.

1.  $3 \times 5 + 2 \times 8 + 2$

$$15 + 16 + 2$$

$$33$$

2.  $(\$1.75 + 2 \times \$0.25 + 5 \times \$0.05) \times 24$

$$(\$1.75 + \$0.50 + \$0.25) \times 24$$

$$\$2.50 \times 24$$

$$\$60.00$$

3.  $(2 \times 6) + (8 \times 4) + 1$

$$12 + 32 + 1$$

$$45$$

4.  $((8 \times 1.95) + (3 \times 2.95) + 10.95) \times 1.06$

$$(15.6 + 8.85 + 10.95) \times 1.06$$

$$35.4 \times 1.06$$

$$37.524$$

5.  $((12 \div 3)^2 - (18 \div 3^2)) \times (4 \div 2)$

$$(4^2 - (18 \div 9)) \times (4 \div 2)$$

$$(16 - 2) \times 2$$

$$14 \times 2$$

$$28$$