Lesson 5: Exponents

Student Outcomes

- Students discover that $3x = x + x + x$ is not the same thing as x^3, which is $x \cdot x \cdot x$.
- Students understand that a base number can be represented with a positive whole number, positive fraction, or positive decimal and that for any number a, we define a^m to be the product of m factors of a. The number a is the base, and m is called the exponent or power of a.

Lesson Notes

In Grade 5, students are introduced to exponents. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10 (**5.NBT.A.2**).

In this lesson, students will use new terminology (base, squared, and cubed) and practice moving between exponential notation, expanded notation, and standard notation. The following terms should be displayed, defined, and emphasized throughout Lesson 5: base, exponent, power, squared, and cubed.

Classwork

Fluency Exercise (5 minutes): Multiplication of Decimals

RWBE: Refer to the Rapid White Board Exchanges sections in the Module Overview for directions on how to administer a RWBE.

Opening Exercise (2 minutes)

Opening Exercise

As you evaluate these expressions, pay attention to how you arrive at your answers.

$\mathbf{4+4+4+4+4+4+4+4+4+4}$

$\mathbf{9+9+9+9+9}$

$\mathbf{10+10+10+10+10}$

Discussion (15 minutes)

- How many of you solved the problems by "counting on"? That is, starting with 4, you counted on 4 more each time ($5, 6, 7, \mathbf{8}, 9, 10, 11, \mathbf{12}, 13, 14, 15, \mathbf{16}\ldots$).
- If you did not find the answer that way, could you have done so?
  - *Yes, but it is time-consuming and cumbersome.*
- Addition is a faster way of "counting on."

- How else could you find the sums using addition?
  - *Count by* 4*,* 9*, or* 10*.*
- How else could you solve the problems?
  - *Multiply* 4 *times* 10*; multiply* 9 *times* 5*; or multiply* 10 *times* 5*.*
- Multiplication is a faster way to add numbers when the addends are the same.
- When we add five groups of 10, we use an abbreviation and a different notation, called multiplication.
  $10 + 10 + 10 + 10 + 10 = 5 \times 10$
- If multiplication is a more efficient way to represent addition problems involving the repeated addition of the same addend, do you think there might be a more efficient way to represent the repeated multiplication of the same factor, as in $10 \times 10 \times 10 \times 10 \times 10 = ?$

MP.2 & MP.7

Allow students to make suggestions; some will recall this from previous lessons.

$$10 \times 10 \times 10 \times 10 \times 10 = 10^5$$

- We see that when we add five groups of 10, we write 5×10, but when we multiply five copies of 10, we write 10^5. So, multiplication by 5 in the context of addition corresponds exactly to the exponent 5 in the context of multiplication.

Scaffolding:

When teaching students how to write an exponent as a *superscript*, compare and contrast the notation with how to write a *subscript*, as in the molecular formula for water, H_2O, or carbon dioxide, CO_2. Here the number is again half as tall as the capital letters, and the top of the 2 is halfway down it. The bottom of the subscript can extend a little lower than the bottom of the letter. Ignore the meaning of a chemical subscript.

Make students aware of the correspondence between addition and multiplication because what they know about *repeated addition* will help them learn exponents as *repeated multiplication* as we go forward.

- The little 5 we write is called an exponent and is written as a *superscript*. The numeral 5 is written only half as tall and half as wide as the 10, and the bottom of the 5 should be halfway up the number 10. The top of the 5 can extend a little higher than the top of the zero in 10. Why do you think we write exponents so carefully?
  - *It reduces the chance that a reader will confuse* 10^5 *with* 105*.*

Examples 1–5 (5 minutes)

Work through Examples 1–5 as a group; supplement with additional examples if needed.

Examples 1–5

Write each expression in exponential form.

1. $5 \times 5 \times 5 \times 5 \times 5 = \mathbf{5^5}$
2. $2 \times 2 \times 2 \times 2 = \mathbf{2^4}$

Write each expression in expanded form.

3. $8^3 = \mathbf{8 \times 8 \times 8}$
4. $10^6 = \mathbf{10 \times 10 \times 10 \times 10 \times 10 \times 10}$
5. $g^3 = \boldsymbol{g \times g \times g}$

- The repeated factor is called the *base,* and the exponent is also called the *power*. Say the numbers in examples 1–5 to a partner.

Check to make sure students read the examples correctly:

- *Five to the fifth power, two to the fourth power, eight to the third power, ten to the sixth power, and g to the third power.*

Go back to Examples 1–4, and use a calculator to evaluate the expressions.

1. $5 \times 5 \times 5 \times 5 \times 5 = 5^5 = 3,125$

2. $2 \times 2 \times 2 \times 2 = 2^4 = 16$

3. $8^3 = 8 \times 8 \times 8 = 512$

4. $10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000$

What is the difference between $3g$ and g^3?

$3g = g + g + g$ or 3 times g ; $g^3 = g \times g \times g$

Take time to clarify this important distinction.

- The base number can also be written in decimal or fraction form. Try Examples 6, 7, and 8. Use a calculator to evaluate the expressions.

Example 6–8 (4 minutes)

Examples 6–8

6. Write the expression in expanded form, and then evaluate.

$(3.8)^4 = 3.8 \times 3.8 \times 3.8 \times 3.8 = 208.5136$

7. Write the expression in exponential form, and then evaluate.

$2.1 \times 2.1 = (2.1)^2 = 4.41$

8. Write the expression in exponential form, and then evaluate.

$0.75 \times 0.75 \times 0.75 = (0.75)^3 = 0.421875$

The base number can also be a fraction. Convert the decimals to fractions in Examples 7 and 8 and evaluate. Leave your answer as a fraction. Remember how to multiply fractions!

Example 7:

$$\frac{21}{10} \times \frac{21}{10} = \left(\frac{21}{10}\right)^2 = \frac{441}{100} = 4\frac{41}{100}$$

Example 8:

$$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

Note to teacher:

If students need additional help multiplying fractions, refer to the first four modules of Grade 5.

Examples 9–10 (1 minute)

Examples 9-10

9. **Write the expression in exponential form, and then evaluate.**

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

10. **Write the expression in expanded form, and then evaluate.**

$\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

- There is a special name for numbers raised to the second power. When a number is raised to the second power, it is called *squared*. Remember that in geometry, squares have the same two dimensions: length and width. For $b > 0$, b^2 is the area of a square with side length b.
- What is the value of 5 squared?
  - 25
- What is the value of 7 squared?
  - 49
- What is the value of 8 squared?
  - 64
- What is the value of 1 squared?
  - 1

A multiplication chart is included at the end of this lesson. Post or project it as needed.

- Where are square numbers found on the multiplication table?
  - *On the diagonal*
- There is also a special name for numbers raised to the third power. When a number is raised to the third power, it is called *cubed*. Remember that in geometry, cubes have the same three dimensions: length, width, and height. For $b > 0$, b^3 is the volume of a cube with edge length b.
- What is the value of 1 cubed?
  - $1 \times 1 \times 1 = 1$
- What is the value of 2 cubed?
  - $2 \times 2 \times 2 = 8$
- What is the value of 3 cubed?
  - $3 \times 3 \times 3 = 27$

MP.6

- In general, for any number x, $x^1 = x$, and for any positive integer $n > 1$, x^n is, by definition,

  $x^n = \underbrace{(x \cdot x \cdots x)}_{\text{n times}}.$
- What does the x represent in this equation?
  - *The x represents the factor that will be repeatedly multiplied by itself.*
- What does the n represent in this expression?
  - *n represents the number of times x will be multiplied.*

MP.6

- Let's look at this with some numbers. How would we represent 4^n?
  - $4^n = \underbrace{(4 \cdot 4 \cdots 4)}_{\text{n times}}$
- What does the 4 represent in this expression?
  - The 4 represents the factor that will be repeatedly multiplied by itself.
- What does the n represent in this expression?
  - *n represents the number of times 4 will be multiplied.*
- What if we were simply multiplying? How would we represent $4n$?
  - *Because multiplication is repeated addition,* $4n = \underbrace{(4 + 4 \cdots 4)}_{\text{n times}}$.
- What does the 4 represent in this expression?
  - The 4 represents the addend that will be repeatedly added to itself.
- What does the n represent in this expression?
  - *n represents the number of times 4 will be added.*

Exercises (8 minutes)

Ask students to fill in the chart, supplying the missing expressions.

Exercises

1. **Fill in the missing expressions for each row. For whole number and decimal bases, use a calculator to find the standard form of the number. For fraction bases, leave your answer as a fraction.**

Exponential Form	Expanded Form	Standard Form
3^2	3×3	9
2^6	$2 \times 2 \times 2 \times 2 \times 2 \times 2$	64
4^5	$4 \times 4 \times 4 \times 4 \times 4$	$1,024$
$\left(\frac{3}{4}\right)^2$	$\frac{3}{4} \times \frac{3}{4}$	$\frac{9}{16}$
$(1.5)^2$	1.5×1.5	2.25

2. **Write five cubed in all three forms: exponential form, expanded form, and standard form.**

   5^3; $5 \times 5 \times 5$; 125

3. **Write *fourteen and seven-tenths squared* in all three forms.**

   $(14.7)^2$; 14.7×14.7; 216.09

4. **One student thought two to the third power was equal to six. What mistake do you think he made, and how would you help him fix his mistake?**

   The student multiplied the base, 2, by the exponent, 3. This is wrong because the exponent never multiplies the base; the exponent tells how many copies of the base are to be used as factors.

Closing (2 minutes)

- We use multiplication as a quicker way to do repeated addition if the addends are the same. We use exponents as a quicker way to multiply if the factors are the same.
- Carefully write exponents as superscripts to avoid confusion.

Lesson Summary

EXPONENTIAL NOTATION FOR WHOLE NUMBER EXPONENTS: Let m be a nonzero whole number. For any number a, the expression a^m is the product of m factors of a, i.e.,

$$a^m = \underbrace{a \cdot a \cdot \cdots \cdot a}_{\text{m times}}.$$

The number a is called the *base,* and m is called the *exponent* or *power* of a.

When m is 1, "the product of one factor of a" just means a, i.e., $a^1 = a$. Raising any nonzero number a to the power of 0 is defined to be 1, i.e., $a^0 = 1$ for all $a \neq 0$.

Exit Ticket (3 minutes)

Name ____ Date____________________

Lesson 5: Exponents

Exit Ticket

1. What is the difference between $6z$ and z^6?

2. Write 10^3 as a multiplication expression having repeated factors.

3. Write $8 \times 8 \times 8 \times 8$ using exponents.

Exit Ticket Sample Solutions

1. What is the difference between $6z$ and z^6?

   $6z = z + z + z + z + z + z$ *or* 6 *times* z ; $z^6 = z \times z \times z \times z \times z \times z$

2. Write 10^3 as a series of products.

   $10 \times 10 \times 10$

3. Write $8 \times 8 \times 8 \times 8$ using an exponent.

   8^4

Problem Set Sample Solutions

1. Complete the table by filling in the blank cells. Use a calculator when needed.

Exponential Form	Expanded Form	Standard Form
3^5	$3 \times 3 \times 3 \times 3 \times 3$	243
4^3	$4 \times 4 \times 4$	64
$(1.9)^2$	1.9×1.9	3.61
$\left(\frac{1}{2}\right)^5$	$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$	$\frac{1}{32}$

2. Why do whole numbers raised to an exponent get greater, while fractions raised to an exponent get smaller?

   As whole numbers are multiplied by themselves, products are larger because there are more groups. As fractions of fractions are taken, the product is smaller. A part of a part is less than how much we started with.

3. The powers of 2 that are in the range 2 through $1,000$ are 2, 4, 8, 16, 32, 64, 128, 256, and 512. Find all the powers of 3 that are in the range 3 through $1,000$.

   $3, 9, 27, 81, 243, 729$

4. Find all the powers of 4 in the range 4 through $1,000$.

   $4, 16, 64, 256$

5. Write an equivalent expression for $n \times a$ using only addition.

   $\underbrace{(a + a + \cdots a)}_{\text{n times}}$

6. Write an equivalent expression for w^b using only multiplication.

   $w^b = \underbrace{(w \cdot w \cdots w)}_{\text{b times}}$

a. Explain what w is in this new expression.

w is the factor that will be repeatedly multiplied by itself.

b. Explain what b is in this new expression.

b is the number of times w will be multiplied.

7. What is the advantage of using exponential notation?

It is a shorthand way of writing a multiplication expression if the factors are all the same.

8. What is the difference between $4x$ and x^4? Evaluate both of these expressions when $x = 2$.

$4x$ means four times x, this is the same as $x + x + x + x$. On the other hand, x^4 means x to the fourth power, or $x \times x \times x \times x$.

When $x = 2$, $4x = 4 \times 2 = 8$.

When $x = 2$, $x^4 = 2 \times 2 \times 2 \times 2 = 16$.

Multiplication of Decimals

Progression of Exercises

1. $0.5 \times 0.5 =$

   $\mathbf{0.25}$

2. $0.6 \times 0.6 =$

   $\mathbf{0.36}$

3. $0.7 \times 0.7 =$

   $\mathbf{0.49}$

4. $0.5 \times 0.6 =$

   $\mathbf{0.3}$

5. $1.5 \times 1.5 =$

   $\mathbf{2.25}$

6. $2.5 \times 2.5 =$

   $\mathbf{6.25}$

7. $0.25 \times 0.25 =$

   $\mathbf{0.0625}$

8. $0.1 \times 0.1 =$

   $\mathbf{0.01}$

9. $0.1 \times 123.4 =$

   $\mathbf{12.34}$

10. $0.01 \times 123.4 =$

   $\mathbf{1.234}$