



Lesson 2: The Relationship of Multiplication and Division

Student Outcomes

- Students build and clarify the relationship of multiplication and division by evaluating identities such as $a \div b \cdot b = a$ and $a \cdot b \div b = a$.

Lesson Notes

Students will use the squares that were used in Lesson 1; however, each pair of students should receive 20 squares for this lesson. Also, students will need large paper to complete the Exploratory Challenge.

Classwork

Fluency Exercise (5 minutes): Division of Fractions I

Sprint: Refer to the Sprints and Sprint Delivery Script sections in the Module Overview for directions to administer a Sprint.

Opening (2 minutes)

MP.2

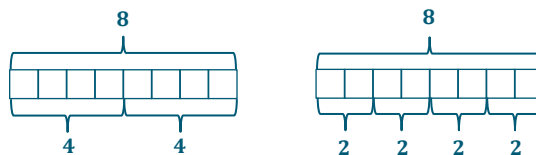
Remind students of the identities they learned the previous day. Discuss the relationship between addition and subtraction. Inform students that the relationship between multiplication and division will be discussed today. Have students make predictions about this relationship using their knowledge gained in the previous lesson.

Opening Exercise (5 minutes)

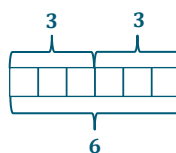
Opening Exercise

Draw a pictorial representation of the division and multiplication problems using a tape diagram.

a. $8 \div 2$


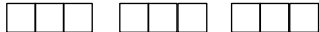



b. 3×2



Discussion (optional—see scaffolding notes)

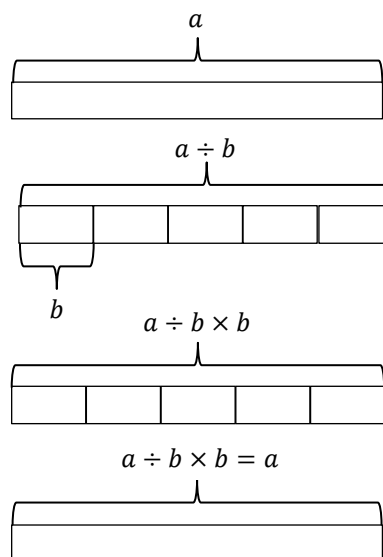
Provide each pair of students with a collection of 20 squares, which they will use to create tape diagrams throughout the lesson.

- Build a tape diagram to represent 9 units.
 - 
- Divide the 9 units into three equal groups.
 - 
- Write an expression to represent the process you modeled with the tape diagram.
 - $9 \div 3$
- Evaluate the expression.
 - 3
- Use your squares to demonstrate what it would look like to multiply 3 by 3.
 - 
- Alter our original expression, $9 \div 3$, to create an expression that represents what we did with the tape diagram.
 - $9 \div 3 \times 3$
- Evaluate the expression.
 - 9
- What do you notice about the expression of the tape diagram?
 - *Possible answer: When we divide by one number then multiply by the same number, we end up with our original number.*
- Write an equation, using variables, to represent the identities we demonstrated with tape diagrams. Draw a series of tape diagrams to demonstrate this equation.
 - *Provide students time to work in pairs.*
 - *Possible answer: $a \div b \times b = a$. Emphasize that both a 's represent the same number, and the same rule applies to the b 's.*

Scaffolding:

The discussion is provided if students struggled during Lesson 1. If the discussion is included in the lesson, the Exploratory Challenge will be shortened because students will only develop one number sentence.

MP.7



Exploratory Challenge (23 minutes)

Students will work in pairs or small groups to determine equations to show the relationship between multiplication and division. They will use tape diagrams to provide support for their findings.

Scaffolding:

If students struggle with getting started, show them the identity equations for addition and subtraction learned in Lesson 1.

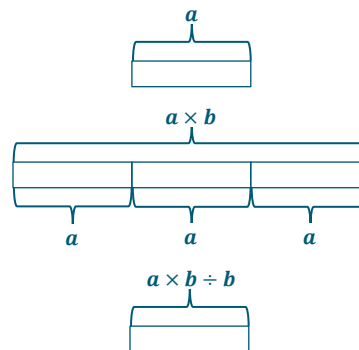
Exploratory Challenge

Work in pairs or small groups to determine equations to show the relationship between multiplication and division. Use tape diagrams to provide support for your findings.

1. Create two equations to show the relationship between multiplication and division. These equations should be identities and include variables. Use the squares to develop these equations.
2. Write your equations on large paper. Show a series of tape diagrams to defend each of your equations.

Only one number sentence is shown there; the second number sentence and series of tape diagrams are included in the optional discussion.

Possible answer: $a \times b \div b = a$



Possible answer: $a \div b \times b = a$

MP.7

When students complete their work on the large paper, hang the papers around the room. Provide time for students to walk around and critique their peers' work. While examining the other posters, students should be comparing the equations and tape diagrams to their own.

MP.3

Use the following rubric to critique other posters.

1. Name of group you are critiquing
2. Equation you are critiquing
3. Whether or not you believe the equations are true and reasons why

Closing (5 minutes)

- What did you determine about the relationship of multiplication and division?
 - When a number is multiplied and divided by the same number, the result is the original number.
- What equations that can be used to show the relationship of multiplication and division?
 - $a \div b \cdot b = a$ and $a \cdot b \div b = a$

Exit Ticket (5 minutes)

Name _____

Date _____

Lesson 2: The Relationship of Multiplication and Division

Exit Ticket

1. Fill in the blank to make each equation true.

a. $12 \div 3 \times \underline{\hspace{1cm}} = 12$

b. $f \times h \div h = \underline{\hspace{1cm}}$

c. $45 \times \underline{\hspace{1cm}} \div 15 = 45$

d. $\underline{\hspace{1cm}} \div r \times r = p$

2. Draw a series of tape diagrams to represent the following number sentences.

a. $12 \div 3 \times 3 = 12$

b. $4 \times 5 \div 5 = 4$

Exit Ticket Sample Solutions

1. Fill in the blank to make each equation true.

a. $12 \div 3 \times \underline{\quad} = 12$

3

b. $f \times h \div h = \underline{\quad}$

f

c. $45 \times \underline{\quad} \div 15 = 45$

15

d. $\underline{\quad} \div r \times r = p$

p

2. Draw a series of tape diagrams to represent the following number sentences.

a. $12 \div 3 \times 3 = 12$



b. $4 \times 5 \div 5 = 4$



Problem Set Sample Solutions

1. Fill in each blank to make each equation true.

a. $132 \div 3 \times 3 = \underline{\quad}$

132

b. $\underline{\quad} \div 25 \times 25 = 225$

225

c. $56 \times \underline{\quad} \div 8 = 56$

8

d. $452 \times 12 \div \underline{\quad} = 452$

12

2. How is the relationship of addition and subtraction similar to the relationship of multiplication and division?

Possible answer: Both relationships create identities.

Number Correct: _____

Division of Fractions I—Round 1

Directions: Determine the quotient of the fractions.

1.	$\frac{1}{2} \div \frac{3}{5}$	
2.	$\frac{5}{6} \div \frac{1}{5}$	
3.	$\frac{3}{7} \div \frac{6}{11}$	
4.	$\frac{2}{5} \div \frac{8}{9}$	
5.	$\frac{1}{6} \div \frac{9}{10}$	
6.	$\frac{11}{12} \div \frac{8}{9}$	
7.	$\frac{5}{6} \div \frac{10}{13}$	
8.	$\frac{7}{8} \div \frac{13}{15}$	
9.	$\frac{3}{5} \div \frac{7}{9}$	
10.	$\frac{14}{17} \div \frac{13}{20}$	
11.	$3\frac{1}{2} \div 4\frac{4}{5}$	
12.	$6\frac{1}{5} \div 6\frac{3}{4}$	
13.	$2\frac{1}{4} \div 3\frac{1}{8}$	
14.	$1\frac{3}{5} \div \frac{7}{8}$	
15.	$\frac{1}{5} \div 4\frac{1}{2}$	

16.	$6\frac{7}{8} \div 11\frac{2}{3}$	
17.	$5\frac{5}{6} \div 3\frac{1}{2}$	
18.	$10\frac{5}{8} \div 12\frac{3}{7}$	
19.	$9\frac{1}{3} \div 8\frac{2}{5}$	
20.	$\frac{3}{4} \div 6\frac{7}{10}$	
21.	$2\frac{1}{3} \div 3\frac{5}{6}$	
22.	$2\frac{4}{5} \div 7\frac{9}{10}$	
23.	$5\frac{8}{9} \div 3\frac{3}{5}$	
24.	$12\frac{5}{9} \div 5$	
25.	$1\frac{5}{6} \div 2\frac{6}{7}$	
26.	$10 \div 5\frac{8}{9}$	
27.	$14\frac{3}{5} \div 10$	
28.	$7\frac{9}{11} \div 1\frac{9}{10}$	
29.	$15\frac{2}{3} \div 24$	
30.	$32 \div 12\frac{6}{7}$	

Division of Fractions I—Round 1 [KEY]

Directions: Determine the quotient of the fractions.

1.	$\frac{1}{2} \div \frac{3}{5}$	$\frac{5}{6}$
2.	$\frac{5}{6} \div \frac{1}{5}$	$\frac{25}{6} = 4\frac{1}{6}$
3.	$\frac{3}{7} \div \frac{6}{11}$	$\frac{33}{42} = \frac{11}{14}$
4.	$\frac{2}{5} \div \frac{8}{9}$	$\frac{18}{40} = \frac{9}{20}$
5.	$\frac{1}{6} \div \frac{9}{10}$	$\frac{10}{54} = \frac{5}{27}$
6.	$\frac{11}{12} \div \frac{8}{9}$	$\frac{99}{96} = 1\frac{1}{32}$
7.	$\frac{5}{6} \div \frac{10}{13}$	$\frac{65}{60} = 1\frac{1}{12}$
8.	$\frac{7}{8} \div \frac{13}{15}$	$\frac{105}{104} = 1\frac{1}{104}$
9.	$\frac{3}{5} \div \frac{7}{9}$	$\frac{27}{35}$
10.	$\frac{14}{17} \div \frac{13}{20}$	$\frac{280}{221} = 1\frac{59}{221}$
11.	$3\frac{1}{2} \div 4\frac{4}{5}$	$\frac{35}{48}$
12.	$6\frac{1}{5} \div 6\frac{3}{4}$	$\frac{124}{135}$
13.	$2\frac{1}{4} \div 3\frac{1}{8}$	$\frac{72}{100} = \frac{18}{25}$
14.	$1\frac{3}{5} \div \frac{7}{8}$	$\frac{64}{35} = 1\frac{29}{35}$
15.	$\frac{1}{5} \div 4\frac{1}{2}$	$\frac{2}{45}$
16.	$6\frac{7}{8} \div 11\frac{2}{3}$	$\frac{165}{280} = \frac{33}{56}$
17.	$5\frac{5}{6} \div 3\frac{1}{2}$	$\frac{70}{42} = 1\frac{2}{3}$
18.	$10\frac{5}{8} \div 12\frac{3}{7}$	$\frac{595}{696}$
19.	$9\frac{1}{3} \div 8\frac{2}{5}$	$\frac{140}{126} = 1\frac{1}{9}$
20.	$\frac{3}{4} \div 6\frac{7}{10}$	$\frac{30}{268} = \frac{15}{134}$
21.	$2\frac{1}{3} \div 3\frac{5}{6}$	$\frac{42}{69} = \frac{14}{23}$
22.	$2\frac{4}{5} \div 7\frac{9}{10}$	$\frac{140}{395} = \frac{28}{79}$
23.	$5\frac{8}{9} \div 3\frac{3}{5}$	$\frac{265}{162} = 1\frac{103}{162}$
24.	$12\frac{5}{9} \div 5$	$\frac{113}{45} = 2\frac{23}{45}$
25.	$1\frac{5}{6} \div 2\frac{6}{7}$	$\frac{77}{120}$
26.	$10 \div 5\frac{8}{9}$	$\frac{90}{53} = 1\frac{37}{53}$
27.	$14\frac{3}{5} \div 10$	$\frac{73}{50} = 1\frac{23}{50}$
28.	$7\frac{9}{11} \div 1\frac{9}{10}$	$\frac{860}{209} = 4\frac{24}{209}$
29.	$15\frac{2}{3} \div 24$	$\frac{47}{72}$
30.	$32 \div 12\frac{6}{7}$	$\frac{224}{90} = 2\frac{22}{45}$

Number Correct: _____

Improvement: _____

Division of Fractions I—Round 2

Directions: Determine the quotient of the fractions.

1.	$\frac{1}{2} \div \frac{3}{4}$	
2.	$\frac{3}{5} \div \frac{1}{7}$	
3.	$\frac{5}{6} \div \frac{1}{8}$	
4.	$\frac{3}{8} \div \frac{5}{9}$	
5.	$\frac{1}{4} \div \frac{2}{5}$	
6.	$\frac{7}{8} \div \frac{9}{10}$	
7.	$\frac{8}{11} \div \frac{4}{5}$	
8.	$\frac{5}{6} \div \frac{7}{12}$	
9.	$\frac{3}{4} \div \frac{8}{9}$	
10.	$\frac{7}{11} \div \frac{4}{7}$	
11.	$1\frac{3}{4} \div \frac{1}{2}$	
12.	$\frac{1}{10} \div 2\frac{3}{4}$	
13.	$5\frac{2}{3} \div \frac{7}{9}$	
14.	$\frac{5}{6} \div 6\frac{1}{2}$	
15.	$\frac{6}{7} \div 2\frac{1}{3}$	

16.	$\frac{7}{8} \div 2\frac{1}{4}$	
17.	$\frac{3}{4} \div 2\frac{3}{5}$	
18.	$4\frac{1}{5} \div 2\frac{1}{3}$	
19.	$4\frac{3}{8} \div \frac{2}{7}$	
20.	$\frac{4}{5} \div 2\frac{1}{8}$	
21.	$1\frac{1}{2} \div 3\frac{5}{6}$	
22.	$3\frac{2}{3} \div 2\frac{1}{4}$	
23.	$4\frac{3}{5} \div 1\frac{3}{4}$	
24.	$7\frac{1}{2} \div 6\frac{1}{3}$	
25.	$3\frac{4}{5} \div 2\frac{9}{10}$	
26.	$3\frac{5}{6} \div 2\frac{1}{2}$	
27.	$3\frac{3}{4} \div 4\frac{1}{8}$	
28.	$5 \div 4\frac{5}{6}$	
29.	$3\frac{1}{4} \div 2$	
30.	$8 \div 5\frac{1}{3}$	

Division of Fractions I—Round 2 [KEY]

Directions: Determine the quotient of the fractions.

1.	$\frac{1}{2} \div \frac{3}{4}$	$\frac{4}{6} = \frac{2}{3}$
2.	$\frac{3}{5} \div \frac{1}{7}$	$\frac{21}{5} = 4\frac{1}{5}$
3.	$\frac{5}{6} \div \frac{1}{8}$	$\frac{40}{6} = 6\frac{2}{3}$
4.	$\frac{3}{8} \div \frac{5}{9}$	$\frac{27}{40}$
5.	$\frac{1}{4} \div \frac{2}{5}$	$\frac{5}{8}$
6.	$\frac{7}{8} \div \frac{9}{10}$	$\frac{70}{72} = \frac{35}{36}$
7.	$\frac{8}{11} \div \frac{4}{5}$	$\frac{40}{44} = \frac{10}{11}$
8.	$\frac{5}{6} \div \frac{7}{12}$	$\frac{60}{42} = 1\frac{3}{7}$
9.	$\frac{3}{4} \div \frac{8}{9}$	$\frac{27}{32}$
10.	$\frac{7}{11} \div \frac{4}{7}$	$\frac{49}{44} = 1\frac{5}{44}$
11.	$1\frac{3}{4} \div \frac{1}{2}$	$\frac{14}{4} = 3\frac{1}{2}$
12.	$\frac{1}{10} \div 2\frac{3}{4}$	$\frac{4}{110} = \frac{2}{55}$
13.	$5\frac{2}{3} \div \frac{7}{9}$	$\frac{153}{21} = 7\frac{2}{7}$
14.	$\frac{5}{6} \div 6\frac{1}{2}$	$\frac{10}{78} = \frac{5}{39}$
15.	$\frac{6}{7} \div 2\frac{1}{3}$	$\frac{18}{49}$

16.	$\frac{7}{8} \div 2\frac{1}{4}$	$\frac{28}{72} = \frac{7}{18}$
17.	$\frac{3}{4} \div 2\frac{3}{5}$	$\frac{15}{52}$
18.	$4\frac{1}{5} \div 2\frac{1}{3}$	$\frac{63}{35} = 1\frac{4}{5}$
19.	$4\frac{3}{8} \div \frac{2}{7}$	$\frac{245}{16} = 15\frac{5}{16}$
20.	$\frac{4}{5} \div 2\frac{1}{8}$	$\frac{32}{85}$
21.	$1\frac{1}{2} \div 3\frac{5}{6}$	$\frac{18}{46} = \frac{9}{23}$
22.	$3\frac{2}{3} \div 2\frac{1}{4}$	$\frac{44}{27} = 1\frac{17}{27}$
23.	$4\frac{3}{5} \div 1\frac{3}{4}$	$\frac{92}{35} = 2\frac{22}{35}$
24.	$7\frac{1}{2} \div 6\frac{1}{3}$	$\frac{45}{38} = 1\frac{7}{38}$
25.	$3\frac{4}{5} \div 2\frac{9}{10}$	$\frac{190}{145} = 1\frac{9}{29}$
26.	$3\frac{5}{6} \div 2\frac{1}{2}$	$\frac{46}{30} = 1\frac{8}{15}$
27.	$3\frac{3}{4} \div 4\frac{1}{8}$	$\frac{120}{132} = \frac{10}{11}$
28.	$5 \div 4\frac{5}{6}$	$\frac{30}{29} = 1\frac{1}{29}$
29.	$3\frac{1}{4} \div 2$	$\frac{13}{8} = 1\frac{5}{8}$
30.	$8 \div 5\frac{1}{3}$	$\frac{24}{16} = 1\frac{1}{2}$