Lesson 17: Divisibility Tests for 3 and 9

Student Outcomes

- Students apply divisibility rules, specifically for 3 and 9, to understand factors and multiples.

Lesson Notes

Students already have knowledge on the divisibility rules of 2, 4, 6, 8, and 10. Although those rules are not a focus for this lesson, they are revisited throughout the lesson. Also, emphasize the difference between factors and multiples throughout the lesson.

Classwork

Opening Exercise (5 minutes)

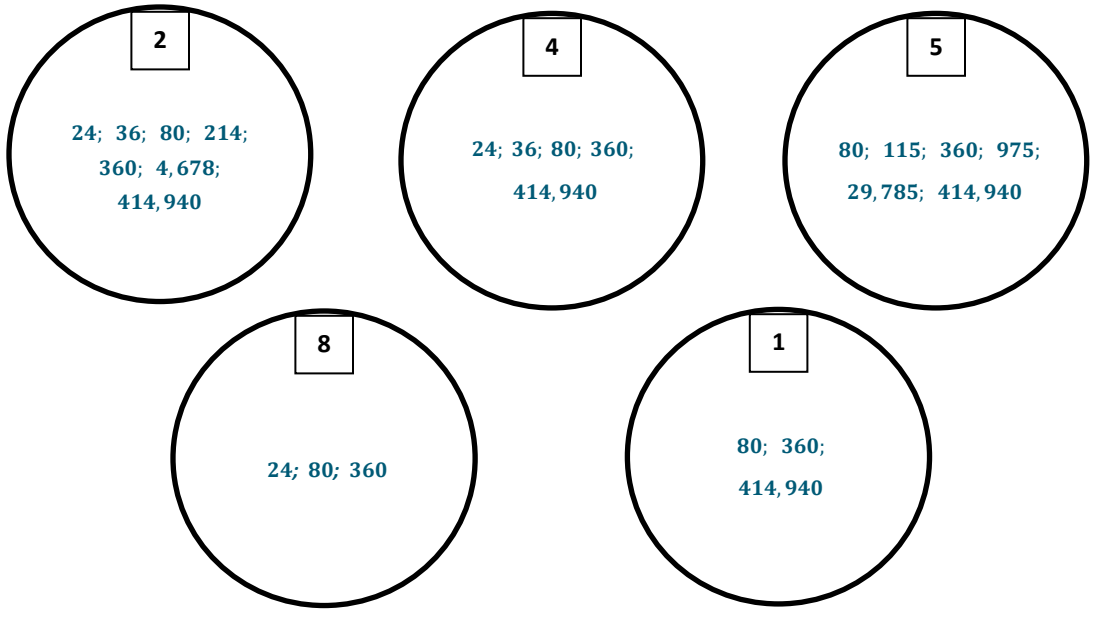
MP.1

The Opening Exercise will help students review the divisibility tests for the numbers 2, 4, 5, 8, and 10.

Opening Exercise

Below is a list of 10 numbers. Place each number in the circle(s) that is a factor of the number. You will place some numbers in more than one circle. For example, if 32 were on the list, you would place it in the circles with 2, 4, and 8 because they are all factors of 32.

$$24;\ 36;\ 80;\ 115;\ 214;\ 360;\ 975;\ 4,678;\ 29,785;\ 414,940$$



Discussion (8 minutes)

Discuss students' results from the opening exercise. Students can either share their answers, or the teacher can conduct a poll (raising hands, standing up, electronically) to determine where students placed each number.

MP.3 After sharing which numbers go in each circle, have students examine the numbers in the opening activity. Ask students to find shortcuts to determine in which group the number belongs just by looking at it.

Ask students to share their short cuts or rules and discuss the divisibility rules for each number. Have students take notes in their handbooks.

MP.8

Discussion

- **Divisibility rule for 2:** ***If and only if its unit digit is 0, 2, 4, 6, or 8.***
- **Divisibility rule for 4:** ***If and only if its last two digits are a number divisible by 4.***
- **Divisibility rule for 5:** ***If and only if its unit digit is 0 or 5.***
- **Divisibility rule for 8:** ***If and only if its last three digits are a number divisible by 8.***
- **Divisibility rule for 10:** ***If and only if its unit digit is 0.***
- **Decimal numbers with fraction parts do not follow the divisibility tests.**

Explain that students will learn two new divisibility rules today. The rules will be used to determine if numbers are divisible by 3 or 9. Start with a number students already know have factors of 3 and 9, so they can see that the rule works.

- What do the numbers 12, 18, 30, 66, and 93 all have in common?
  - *They are divisible by* 3.
- Calculate the sum of the digits for each given number. For example, the sum of the digits in the number 12 is 3 because $1 + 2 = 3$.

Provide time for students to find the sums. Record sums on the board.

- What do all these sums have in common?
  - *They are divisible by* 3.
- When the sum of a number's digits is divisible by 3, the entire number is divisible by 3. Now let's examine a different set of numbers: 27, 36, 54, 72, and 99. What do these numbers have in common?
  - *They are divisible by* 9.
- Calculate the sum of the digits for each given number.

Provide time for students to find the sums. Record sums on the board.

- What do all the sums have in common?
  - *They are divisible by* 9.
- When the sum of the digits is divisible by 3 and 9, the entire number is divisible by 9. Let's try to use this knowledge to determine if a large number is divisible by 3, 9, or both. The number 765 is divisible by both 3 and 9. (Show students on the calculator.) Find the sum of the digits.
  - $7 + 6 + 5 = 18$.

- Are 3 and 9 both factors of 18?
  - *Yes.*
- Calculating the sum of a number's digits helps us to determine if the number is divisible by 3 or 9 or both.

Introduce the divisibility rules for 3 and 9. Have students record the rules in their handbooks.

- **Divisibility rule for 3:** ***If the sum of the digits is divisible by 3, then the number is divisible by 3.***
- **Divisibility rule for 9:** ***If the sum of the digits is divisible by 9, then the number is divisible by 9.***

Through further discussion, explain to students that if a number is divisible by 9, it is also divisible by 3, but if a number is divisible by 3, it is not necessarily divisible by 9.

- Because $9 = 3 \times 3$, any number that is divisible by 9 will also be divisible by 3.

Example 1 (5 minutes)

Example 1

This example will show you how to apply the two new divisibility rules we just discussed.

Is 378 divisible by 3 or 9? Why or why not?

a. **What are the three digits in the number 378?**

3, 7, and 8

b. **What is the sum of the three digits?**

$3 + 7 + 8 = 18$; the sum of the three digits is 18.

c. **Is 18 divisible by 9?**

Yes.

d. **Is the entire number 378 divisible by 9? Why or why not?**

The number 378 is divisible by 9 because the sum of the digits is divisible by 9.

Scaffolding:

- If needed, the teacher can also ask if 18 is divisible by 3. Students may still struggle with the connection between the multiples of 3 and 9.
- If students struggled with the opening exercise, the divisibility rules for 2, 4, 5, 8, and 10 can be reviewed in this example as well.

MP.7 This may be the place to help students recognize the difference between factors and multiples. Nine is a factor of 378 because it is the product of 9 and 42; therefore, 378 is a multiple of 9.

e. **Is the number 378 divisible by 3? Why or why not?**

Three is a factor of 378 because if 9 is a factor of 378, then 3 will also be a factor. OR

The number 378 is divisible by 3 because the sum of the digits is divisible by 3.

Example 2 (5 minutes)

The students have now seen one example of the two new divisibility rules. Allow students to work with a partner to decide whether a given number is divisible by 3 and 9. If a majority of students are still struggling, the teacher may ask the same leading questions found in Example 1.

- Is 3,822 divisible by 3 or 9? Why or why not?

Encourage students to check 9 first because if 9 is a factor, then students know that 3 is also a factor. If 3,822 is not divisible by 9, then students must check to see if 3,822 is divisible by 3.

Example 2

Is $3,822$ divisible by 3 or 9? Why or why not?

The number $3,822$ is divisible by 3, but not by 9 because the sum of the digits is $3+8+2+2=15$, and 15 is divisible by 3, but not by 9.

MP.7 This is another opportunity to emphasize the difference between factors and multiples. Three is a factor of 3,822 because the product of 3 and 1,274 is 3,822; therefore, 3,822 is a multiple of 3.

Exercises 1–5 (13 minutes)

MP.8 Students may work with partners or individually to complete the exercises. Remind students that they may circle more than one answer.

Exercises 1–5

Circle ALL the numbers that are factors of the given number. Complete any necessary work in the space provided.

1. Is $2,838$ divisible by

(3)
9
4

Explain your reasoning for your choices.

The number $2,838$ is divisible by 3 because 3 is a factor of $2,838$. I know this because the sum of the digits is 21, which is divisible by 3. The number $2,838$ is not divisible by 9 because 21 is not divisible by 9, and $2,838$ is not divisible by 4 because the last two digits (38) are not divisible by 4.

2. Is $34,515$ divisible by

(3)
(9)
(5)

Explain your reasoning for your choices.

The number $34,515$ is divisible by 3 and 9 because both 3 and 9 are factors of $34,515$. I know this because the sum of the digits is 18, and 18 is divisible by both 3 and 9. The number $34,515$ is also divisible by 5 because the unit digit is a 5.

3. Is $10,534,341$ divisible by

(3)
9
2

Explain your reasoning for your choices.

The number $10,534,341$ is divisible by 3, but not 9, because 3 is a factor of $10,534,341$, but 9 is not. I know this because the sum of the digits is 21, which is divisible by 3, but not 9. The number $10,534,341$ is not divisible by 2 because it does not end with 0, 2, 4, 6, or 8.

4. Is $4,320$ divisible by

(3)
(9)
(10)

Explain your reasoning for your choices.

The number $4,320$ is divisible by 3 and 9 because 3 and 9 are factors of $,320$. I know this because the sum of the digits is 9, which is divisible by 3 and 9. The number $4,320$ is also divisible by 10 because 10 is a factor of $,320$. I know this because the unit digit is 0.

5. Is $6,240$ divisible by

(3)
9
(8)

Explain your reasoning for your choices.

The number $6,240$ is divisible by 3, but not divisible by 9, because 3 is a factor of $6,240$, but 9 is not. I know this because the sum of the digits is 12, which is divisible by 3 but not divisible by 9. The number $6,240$ is divisible by 8 because the last three digits (240) is divisible by 8.

Closing (4 minutes)

- Without completing the division, how can you determine if a number is divisible by 3?
  - *Calculate the sum of the digits; if the sum of the digits is divisible by 3, the entire number is divisible by 3.*
- If a number is divisible by 9, will it be divisible by 3? Explain your answer.
  - *If a number is divisible by 9, the sum of the digits will be divisible by 9. Any number that is divisible by 9 is also divisible by 3 since $9 = 3 \times 3$.*
- If a number is divisible by 3, will it be divisible by 9? Explain your answer.
  - *If a number is divisible by 3, it may not be divisible by 9 because 3 has more multiples than 9.*

Lesson Summary

To determine if a number is divisible by 3 or 9:

- **Calculate the sum of the digits.**
- **If the sum of the digits is divisible by 3, the entire number is divisible by 3.**
- **If the sum of the digits is divisible by 9, the entire number is divisible by 9.**

Note: If a number is divisible by 9, the number is also divisible by 3.

Exit Ticket (5 minutes)

Name _____ Date_____

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Exit Ticket

1. Is $26{,}341$ divisible by 3? If it is, write the number as the product of 3 and another factor. If not, explain.

2. Is $8{,}397$ divisible by 9? If it is, write the number as the product of 9 and another factor. If not, explain.

3. Explain why $186{,}426$ is divisible by both 3 and 9.

Exit Ticket Sample Solutions

1. Is $26,341$ divisible by 3? If it is, write the number as the product of 3 and another factor. If not, explain.

   The number $26,341$ is not divisible by 3 because the sum of the digits is 16, which is not divisible by 3.

2. Is $8,397$ divisible by 9? If it is, write the number as the product of 9 and another factor. If not, explain.

   The number $8,397$ is divisible by 9 because the sum of the digits is 27, which is divisible by 9. Nine is a factor of $8,397$ because $9 \times 933 = 8,397$.

3. Explain why $186,426$ is divisible by both 3 and 9.

   The number $186,426$ is divisible by both 3 and 9 because the sum of the digits is 27, which is divisible by both 3 and 9.

Problem Set Sample Solutions

1. Is $32,643$ divisible by both 3 and 9? Why or why not?

   The number $32,643$ is divisible by both 3 and 9 because the sum of the digits is 18, which is divisible by 3 and 9.

2. Circle all the factors of $424,380$ from the list below.

   (2) (3) (4) (5) 8 9 (10)

3. Circle all the factors of $322,875$ from the list below.

   2 (3) 4 (5) 8 (9) 10

4. Write a 3 digit number that is divisible by both 3 and 4. Explain how you know this number is divisible by 3 and 4.

   Answers will vary. Possible student response: The sum of the digits is divisible by 3, and that's how I know the number is divisible by 3. The last 2 digits are divisible by 4, so the entire number is divisible by 4.

5. Write a 4 digit number that is divisible by both 5 and 9. Explain how you know this number is divisible by 5 and 9.

   Answers will vary. Possible student response: The number ends with a 5 or 0, so the entire number is divisible by 5. The sum of the digits is divisible by 9, so the entire number is divisible by 9.