Lesson 16: Even and Odd Numbers

Student Outcomes

- Students generalize rules for adding and multiplying even and odd numbers.

Lesson Notes

Students will need poster paper and markers to complete the exercises.

Classwork

Opening Exercise (15 minutes)

Present each question and then allow students to share their thinking. Also, have students record notes in their student materials.

Opening Exercise

What is an even number?

Possible student responses:

- *An integer that can be evenly divided by* 2
- *A number whose unit digit is* 0*,* 2*,* 4*,* 6*, or* 8
- *All the multiples of* 2

List some examples of even numbers.

Answers will vary.

What is an odd number?

Possible student responses:

- *An integer that CANNOT be evenly divided by* 2
- *A number whose unit digit is* 1*,* 3*,* 5*,* 7*, or* 9
- *All the numbers that are NOT multiples of* 2

List some examples of odd numbers.

Answers will vary.

Present each question and then discuss the answer using models.

What happens when we add two even numbers? Will we always get an even number?

Before holding a discussion about the process to answer the following questions, have students write or share their predictions.

Exercises 1–3

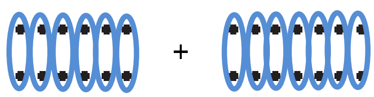
Exercises 1–3

1. **Why is the sum of two even numbers even?**

   a. **Think of the problem $12 + 14$. Draw dots to represent each number.**

   • • • • • •　　　• • • • • • •
   　　　　　　+
   • • • • • •　　　• • • • • • •

   b. **Circle pairs of dots to determine if any of the dots are left over.**

   There are no dots leftover; the answer will be even.

   c. **Will this be true every time two even numbers are added together? Why or why not?**

   Since 12 is represented by 6 sets of two dots, and 14 is represented by 7 sets of two dots, the sum will be 13 sets of two dots. This will be true every time two even numbers are added together because even numbers will never have dots left over when we are circling pairs. Therefore, the answer will always be even.

MP.3 & MP.8

Before holding a discussion about the process to answer the following questions, have students write or share their predictions.

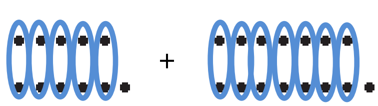
- Now, decide what happens when we add two odd numbers.

2. **Why is the sum of two odd numbers even?**

   a. **Think of the problem $11 + 15$. Draw dots to represent each number.**

   • • • • •　　　• • • • • • •
   　　　　　+
   • • • • • •　　• • • • • • • •

   b. **Circle pairs of dots to determine if any of the dots are left over.**

   When we circle groups of two dots, there is one dot remaining in each representation because each addend is an odd number. When we look at the sum, however, the two remaining dots can form a pair, leaving us with a sum that is represented by groups of two dots. The sum is, therefore, even. Since each addend is odd, there is one dot for each addend that does not have a pair. However, these two dots can be paired together, which means there are no dots without a pair, making the sum an even number.

c. **Will this be true every time two odd numbers are added together? Why or why not?**

This will be true every time two odd numbers are added together because every odd number will have one dot remaining when we circle pairs of dots. Since each number will have one dot remaining, these dots can be combined to make another pair. Therefore, no dots will be remaining, resulting in an even sum.

- Use the same method we used in the two prior examples to show that the sum of an odd number and an even number is odd. Use the problem $14 + 11$.

MP.3 & MP.8

3. **Why is the sum of an even number and an odd number odd?**
   a. **Think of the problem $14 + 11$. Draw dots to represent each number.**
   b. **Circle pairs of dots to determine if any of the dots are left over.**

Students draw dots to represent each number. After circling pairs of dots, there will be one dot left for the number 11, and the number 14 will have no dots remaining. Since there is one dot left over, the sum will be odd because every dot does not have a pair.

c. **Will this be true every time an even number and an odd number are added together? Why or why not?**

This will always be true when an even number and an odd number are added together because only the odd number will have a dot remaining after we circle pairs of dots. Since this dot does not have a pair, the sum will be odd.

d. **What if the first addend was odd and the second was even? Would the sum still be odd? Why or why not? For example, if we had $11 + 14$, would the sum be odd?**

The sum will still be odd for two reasons. First, the commutative property states that changing the order of an addition problem does not change the answer. Because an even number plus an odd number is odd, then an odd number plus an even number is also odd. Second, it does not matter which addend is odd; there will still be one dot remaining, making the sum odd.

If students are struggling, encourage them to draw dots to prove their answer. It may also be helpful to remind students of the commutative property to help them prove their answer.

Sum up the discussion by having students record notes in their handbooks.

Let's sum it up:

Even + even = even

Odd + odd = even

Odd + even = odd

Scaffolding:

- The teacher could also ask students if the same rules apply to subtraction. Using the same method for addition, have students determine if the rules apply to subtraction.
- If students are struggling with the proofs, the teacher can present each proof as students take notes in their handbooks. Or, allow students time to explore, and have a few groups who did not struggle present at the end.
- Ask early finishers if the same rule applies to division.

Exploratory Challenge/Exercises 4–6 (20 minutes–12 minutes for group work; 8 minutes for gallery walk and discussion)

MP.7

Divide students into small groups. On poster paper, each group will be asked to determine whether one of the following products is odd or even: the product of two even numbers, the product of two odd numbers, or the product of an even number and an odd number. Encourage students to use previous knowledge about even and odd numbers, the connection between addition and multiplication, and visual methods (e.g., dots) in their proofs.

Exploratory Challenge/Exercises 4–6

4. **The product of two even numbers is even.**

Answers will vary, but one example answer is provided.

Using the problem 6×14, students know that this is equivalent to six groups of fourteen, or $14 + 14 + 14 + 14 + 14 + 14$. Students also know that the sum of two even numbers is even; therefore, when adding the addends two at a time, the sum will always be even. This means the sum of six even numbers will be even, making the product even since it will be equivalent to the sum.

Using the problem 6×14, students can use the dots from previous examples.

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•••••••   •••••••   •••••••   •••••••   •••••••   •••••••

From here, students can circle dots and see that there will be no dots remaining, so the answer must be even.

5. **The product of two odd numbers is odd.**

Answers will vary, but an example answer is provided.

Using the problem 5×15, students know that this is equivalent to five groups of fifteen, or $15 + 15 + 15 + 15 + 15$. Students also know that the sum of two odd numbers is even, and the sum of an odd and even number is odd. When adding two of the addends together at a time, the answer will rotate between even and odd. When the final two numbers are added together, one will be even and the other odd. Therefore, the sum will be odd, which makes the product odd since it will be equivalent to the sum.

Using the problem 5×15, students may also use the dot method.

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••••••••   ••••••••   ••••••••   ••••••••   ••••••••

After students circle the pairs of dots, one dot from each set of 15 will remain, for a total of 5 dots. Students can group these together and circle more pairs, as shown below.

•••••

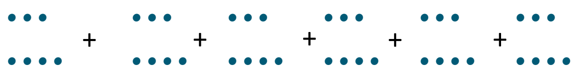
Since there is still one dot remaining, the product of two odd numbers is odd.

6. **The product of an even number and an odd number is even.**

Answers will vary, but one example is provided.

Using the problem 6×7, students know that this is equivalent to the sum of six sevens, or $7 + 7 + 7 + 7 + 7 + 7$. Students also know that the sum of two odd numbers is even, and the sum of two even numbers is even. Therefore, when adding two addends at a time, the results will be an even number. Calculating the sum of these even numbers will also be even, which means the total sum will be even. This also implies the product will be even since the sum and product are equivalent.

Using the problem 6×7, students may also use the dot method.



After students circle the pairs of dots, one dot from each set of 7 will remain, for a total of 6 dots. Students can group these together and circle more pairs, as shown below.

Since there are no dots remaining, the product of an even number and an odd number is even.

After students complete their posters, hang the posters up around the room. Conduct a gallery walk to allow groups to examine each poster and take notes in their student materials. In the end, students should have a proof for all three exercises in their student handbook.

Allow time for a discussion and an opportunity for students to ask any unanswered questions.

Closing (5 minutes)

- How does knowing whether a sum or product will be even or odd assist in division?
  - *Possible student response: When dividing, it is helpful to know whether the sum or product of two numbers is even or odd because it narrows down the possible factors. For example, if a dividend is odd, then we know the factors must also be odd because the product of two odd numbers is odd.*

Lesson Summary

Adding:

- **The sum of two even numbers is even.**
- **The sum of two odd numbers is even.**
- **The sum of an even number and an odd number is odd.**

Multiplying:

- **The product of two even numbers is even.**
- **The product of two odd numbers is odd.**
- **The product of an even number and an odd number is even.**

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 16: Even and Odd Numbers

Exit Ticket

Determine whether each sum or product will be even or odd. Explain your reasoning.

1. $56{,}426 + 17{,}895$

2. $317{,}362 \times 129{,}324$

3. $10{,}481 + 4{,}569$

4. $32{,}457 \times 12{,}781$

5. Show or explain why $12 + 13 + 14 + 15 + 16$ will result in an even sum.

Exit Ticket Sample Solutions

Determine whether each sum or product will be even or odd. Explain your reasoning.

1. $56,426 + 17,895$

   The sum is odd because the sum of an even number and an odd number is odd.

2. $317,362 \times 129,324$

   The product is even because the product of two even numbers is even.

3. $10,481 + 4,569$

   The sum is even because the sum of two odd numbers is even.

4. $32,457 \times 12,781$

   The product is odd because the product of two odd numbers is odd.

5. Show or explain why $12 + 13 + 14 + 15 + 16$ will result in an even sum.

   $12 + 13$ will be odd because even $+$ odd is odd.

   Odd number $+14$ will be odd because odd $+$ even is odd.

   Odd number $+15$ will be even because odd $+$ odd is even.

   Even number $+16$ will be even because even $+$ even is even.

   OR

   Students may group even numbers together, $12 + 14 + 16$, which would result in an even number. Then, when students combine the two odd numbers, $13 + 15$, the result would be another even number. We know that the sum of two evens would result in another even number.

Problem Set Sample Solutions

Without solving, tell whether each sum or product is even or odd. Explain your reasoning.

1. $346 + 721$

   The sum is odd because the sum of an even and an odd number is odd.

2. $4,690 \times 141$

   The product is even because the product of an even and an odd number is even.

3. $1,462,891 \times 745,629$

   The product is odd because the product of two odd numbers is odd.

4. $425,922 + 32,481,064$

The sum is even because the sum of two even numbers is even.

5. $32 + 45 + 67 + 91 + 34 + 56$

The first two addends will be odd because even and an odd is odd.

Odd number $+67$ *will be even because the sum of two odd numbers is even.*

Even number $+91$ *will be odd because the sum of an even and an odd number is odd.*

Odd number $+34$ *will be odd because the sum of an odd and an even number is odd.*

Odd number $+56$ *will be odd because the sum of an odd and an even number is odd.*

Therefore, the final sum will be odd.