Lesson 12: Estimating Digits in a Quotient

Student Outcomes

- Students connect estimation with place value in order to determine the standard algorithm for division.

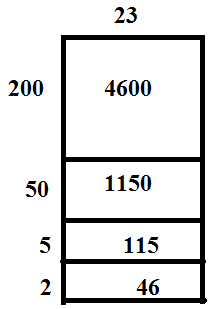
Classwork

Opening Exercise (5 minutes)

Opening Exercise

Show an example of how you would solve $5,911 \div 23$. You can use any method or model to show your work. Just be sure that you can explain how you arrived at your solution.

There are many possible models. Here is one possible solution:



$5,911 \div 23 = 257$

We may want to check our work to see if our answer is reasonable. One way to do this is to estimate our answer.

- Estimate the quotient of $5{,}911$ and 23.
  - *Answers will vary. Sample solution: First I would round the numbers to $6{,}000$ and 20.*
  - *Answer will vary. Sample solution: $6{,}000 \div 20 = 300$.*
- Using your estimation, would you say that the answer you came up with in your model is reasonable?
  - *Answers will vary. Sample solution: Yes, 257 is close to 300. This shows that my answer is reasonable.*
- Would you expect your estimated answer to be greater than or less than the actual answer? Why?
  - *Answers will vary. Sample solutions: Since I chose to round 23 to 20, I expect that the estimated answer will be greater than the actual quotient. If I had rounded up to 25, the estimated answer would be smaller than the actual quotient. (Students could also respond that $5{,}911$ was rounded to $6{,}000$. Because the number was rounded up, we know that the estimate will be larger than the actual answer.)*

- What other ways can we check our solution?
  - *I can use the inverse of division (multiplication) to check my work.*

Example 1 (10 minutes)

Example 1

We can also use estimates before we divide to help us solve division problems. In this lesson, we will be using estimation to help us divide two numbers using the division algorithm.

Estimate the quotient of $8,085 \div 33$. Then, divide.

$8,100 \div 30 = 270$.

- How could I round these numbers to get an estimate?
  - *There are many possible solutions. For example,* $8{,}000 \div 30$*;* $8{,}000 \div 35$*;* $8{,}100 \div 30$*;* $8{,}100 \div 35$.
- Why is $8{,}100$ and 30 the best option?
  - 3 *is not a factor of* 8*, but it is a factor of* 81.
- How can we use this to help us divide $8{,}085$ by 33?
  - *When I begin to divide, I use* 270 *to help me choose what numbers to divide by. My actual answer should be near* 270. *The first number I used in my area model will be* 200. *Then, I will see that the remainder is* $1{,}485$. *I know that* $30 \times 50 = 1{,}500$*, which is too big. So, I will choose one less ten and try* 40.

MP.8

Create a model to show the division of $8,085$ by 33.

	33
200	6600
40	1320
5	165

- We can keep track of the areas in the model and what we have left by making a list and subtracting. We will create a list to keep track of the amounts the same way we created the diagram.

8085		
- 6600	33 x 200	
1485		
- 1320	33 x 40	33 x 245
165		
- 165	33 x 5	
0		

- Now, we can relate this model to the standard division algorithm.

$$\begin{array}{r} 245 \\ 33\overline{)\,8085} \\ -\,6600 \\ \hline 1485 \\ -\,1320 \\ \hline 165 \\ -\,165 \\ \hline 0 \end{array}$$

At this point, you are just showing how this work is the same as the work that is shown in the model.

- What does the 2 represent on top of the division bar?
  - *We divided* 80 *hundreds by* 33*, so the* 2 *represents* 2 *hundreds.*
- How did we use the 200 in the previous model?
  - $33 \times 200 = 6{,}600$
- What does the 4 represent on top of the division bar?
  - *We divided* 148 *tens by* 33*, so the* 4 *represents* 4 *tens.*
- How was the 40 used in the previous model?
  - $33 \times 40 = 1{,}320.$

MP.8

- Now let's check our division. How can we use the quotient to check our work?
  - $245 \times 33 = 8{,}085.$

Example 2 (10 minutes)

Example 2

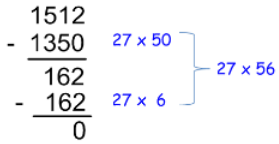
Use estimation and the standard algorithm to divide: $1,512 \div 27$.

Students will estimate the quotient first and use the estimate to help them divide using the algorithm.

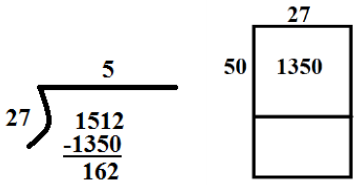
- Share an estimate that can be used to help us divide.
  - *Answers may vary.* $1{,}500 \div 30 = 50.$
- In the algorithm, we can show that there are fifty 27s by placing a 5 over the tens place. We know that 5 tens is 50. This is really showing that 151 tens $\div$ 27 = 5 tens.

$$\begin{array}{r} 5 \\ 27\overline{)\,1512} \end{array}$$

- What would typically be the next step if you were creating a model? (Students can create the model while solving to further solidify the connection.)
  - *I would multiply* 50×27*, which is* $1{,}350$*. Then, I would subtract* $1{,}350$ *from* $1{,}512$*.*

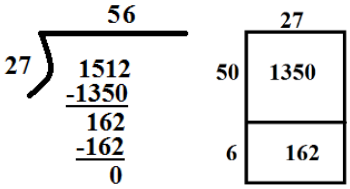


We will show these same steps in the algorithm.



MP.8

- What would we do next?
  - 162 *ones* $\div 27$*. I know that* 30×5 *is* 150*. So, I am going to estimate that the answer is greater than* 5*. Maybe there are six* 27*s in* 162*.*
- We will show the same steps again where we check our work by multiplying and subtracting.



- Finally, we will check our work. Remind me one more time how we can check our quotient.
  - *We can multiply* 27×56 *and see if the product is* $1{,}512$*.*

Exercises 1–4 (10 minutes)

Exercises 1–4

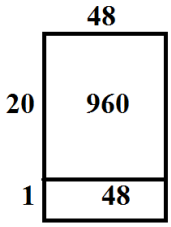
1. $1,008 \div 48$
   a. **Estimate the quotient.**

   Answers may vary.

   $1,000 \div 50 = 20$

b. Use the algorithm to divide. Draw a model to show how the steps relate to the steps used in the algorithm.

$$\begin{array}{r} 21 \\ 48\overline{)1008} \\ -960 \\ \hline 48 \\ -48 \\ \hline 0 \end{array}$$



c. Check your work.

$48 \times 21 = 1,008$

2. $2,508 \div 33$

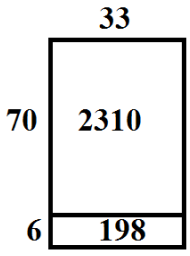
a. Estimate the quotient.

Answers may vary.

$2,400 \div 30 = 80$

b. Use the algorithm to divide. Draw a model to show how the steps relate to the steps used in the algorithm.

$$\begin{array}{r} 76 \\ 33\overline{)2508} \\ -2310 \\ \hline 198 \\ -198 \\ \hline 0 \end{array}$$



c. Check your work.

$33 \times 76 = 2,508$

3. $2,156 \div 28$

a. Estimate the quotient.

Answers may vary.

$2,100 \div 30 = 70$

b. Use the algorithm to divide.

$$\begin{array}{r} 77 \\ 28\overline{)2156} \\ -1960 \\ \hline 196 \\ -196 \\ \hline 0 \end{array}$$

c. Check your work.

$28 \times 77 = 2,156$

4. $4,732 \div 52$

a. Estimate the quotient.

Answers may vary.

$5,000 \div 50 = 100$

b. Use the algorithm to divide.

$$\begin{array}{r} 91 \\ 52\overline{)\,4732} \\ \underline{-4680} \\ 52 \\ \underline{-\,52} \\ 0 \end{array}$$

c. Check your work.

$52 \times 91 = 4,732$

Closing (5 minutes)

- How does estimation help you with the process of finding the exact quotient?
  - *My estimate gives me an idea of what number my answer should be close to. For example, if I have an estimate of* 75*, I know not to try a number in the hundreds.*
- In the previous problem, we used 100 to approximate the quotient $4,732 \div 52$. How did we know that our actual quotient would be in the 90s and not 100 as our approximation suggested? When using your estimate, how do you know if your estimate is too big?
  - *I know that* $100 \times 52 = 5,200$*, and this is greater than* $4,732$*. This tells me to start with a* 9 *in the tens place.*
- When using your estimate, how do you know if your estimate is too small?
  - *When I subtract, the difference is bigger than the divisor.*

Exit Ticket (5 minutes)

Name _____ Date_____

Lesson 12: Estimating Digits in a Quotient

Exit Ticket

1. Estimate the quotient: $1{,}908 \div 36$.

2. Use the division algorithm and your estimate to find the quotient: $1{,}908 \div 36$.

3. Use estimation to determine if $8{,}580 \div 78$ has a quotient in the 10s, 100s, or 1000s.

Exit Ticket Sample Solutions

1. Estimate the quotient: $1,908 \div 36$.

   $2,000 \div 40 = 50$

2. Use the division algorithm and your estimate to find the quotient: $1,908 \div 36$.

$$\begin{array}{r} 53 \\ 36\overline{)\,1908} \\ -1800 \\ \hline 108 \\ -\ 108 \\ \hline 0 \end{array}$$

3. Use estimation to determine if $8,580 \div 78$ has a quotient in the 10s, 100s, or 1000s.

   I would round $8,580$ to $8,800$ and 78 to 80. $8,800 \div 80 = 110$. I know that the quotient should be in the 100s.

Problem Set Sample Solutions

Complete the following steps for each problem:

a. Estimate the quotient.
b. Use the division algorithm to solve.
c. Show a model that supports your work with the division algorithm.
d. Check your work.

1. $3,312 \div 48$

   $3,500 \div 50 = 70$

$$\begin{array}{r} 69 \\ 48\overline{)\,3312} \\ -2880 \\ \hline 432 \\ -\ 432 \\ \hline 0 \end{array}$$

	48
60	2880
9	432

   $48 \times 69 = 3,312$

2. $3,125 \div 25$

$3,000 \div 30 = 100$

$$\begin{array}{r} 125 \\ 25\overline{)\,3125} \\ -2500 \\ \hline 625 \\ -500 \\ \hline 125 \\ -\,125 \\ \hline 0 \end{array}$$

	25
100	2500
20	500
5	125

$25 \times 125 = 3,125$

3. $1,344 \div 14$

$1,400 \div 14 = 100$

$$\begin{array}{r} 96 \\ 14\overline{)\,1344} \\ -1260 \\ \hline 84 \\ -84 \\ \hline 0 \end{array}$$

	14
90	1260
6	84

$14 \times 96 = 1,344$