Lesson 23: Problem Solving Using Rates, Unit Rates, and Conversions

Student Outcomes

- Students solve constant rate work problems by calculating and comparing unit rates.

Materials

- Calculators

Classwork

- **If work is being done at a constant rate by one person, and at a different constant rate by another person, both rates can be converted to their unit rates and then compared directly.**
- **"Work" can include jobs done in a certain time period, rates of running or swimming, etc.**

Example 1: Fresh-Cut Grass (10 minutes)

- In the last lesson, we learned about constant speed problems. Today we will be learning about constant rate work problems. Think for a moment about what a "constant rate work" problem might be.

Allow time for speculation and sharing of possible interpretations of what the lesson title might mean. Student responses should be summarized by:

   - *Constant rate work problems let us compare two unit rates to see which situation is faster or slower*.
- In Lesson 18 we found a rate by dividing two quantities. Recall how to do this.
   - *To find a unit rate, divide the numerator by the denominator.*
- Did it matter which quantity was in the numerator and which quantity was in the denominator?
   - *Yes. To find the unit rate, it is important to have specific quantities in the numerator and denominator based on the rate unit.*
- Did the two quantities have to be two different units?
   - *Yes.*
- Suppose that on a Saturday morning you can cut 3 lawns in 5 hours, and your friend can cut 5 lawns in 8 hours. Your friend claims he is working faster than you. Who is cutting lawns at a faster rate? How do you find out?
   - *Divide the numerator by the denominator to find the unit rate.*

- Again, does it matter which quantity is represented in the numerator and which quantity is represented in the denominator?
  - *Yes. To find the amount of lawns per hour, or the rate unit of $\frac{\text{lawns}}{\text{hour}}$, the amount of lawns cut must be represented in the numerator and the amount of time in hours must be represented in the denominator.*
- What is 3 divided by 5?
  - 0.6
- How should you label the problem?
  - *The same way it is presented. Here "lawns" remains in the numerator, and "hours" remains in the denominator.*
- How should the unit rate and rate unit look when it is written completely?
  - $\frac{3}{5}\frac{\text{lawns}}{\text{hours}} = \frac{0.6}{1}\frac{\text{lawns}}{\text{hour}}$
- How should it be read?
  - *If I can cut 3 lawns in 5 hours, that equals $\frac{3}{5}$ lawns in one hour. If a calculator is used, that will be a unit rate of six-tenths. The rate unit is lawn per hour.*
- What is the unit rate of your friend's lawn cutting?
  - *My friend is cutting $\frac{5}{8}$ lawns in an hour.*

    $\frac{5}{8}\frac{\text{lawns}}{\text{hours}} = \frac{0.625}{1}\frac{\text{lawns}}{\text{hour}}$
- How is this interpreted?
  - *If my friend cuts 5 lawns in 8 hours, the unit rate is 0.625.*
- Compare the two unit rates $\frac{3}{5}$ and $\frac{5}{8}$.
  - $\frac{24}{40} < \frac{25}{40}$ *My friend is a little faster, but only $\frac{1}{40}$ of a lawn per hour, so it is very close. The unit rates have corresponding decimals 0.6 and 0.625.*

Example 1: Fresh-Cut Grass

Suppose that on a Saturday morning you can cut 3 lawns in 5 hours, and your friend can cut 5 lawns in 8 hours. Who is cutting lawns at a faster rate?

$$\frac{3}{5}\frac{\text{lawns}}{\text{hours}} = \frac{\_\_\_}{1}\frac{\text{lawns}}{\text{hour}} \qquad \frac{5}{8}\frac{\text{lawns}}{\text{hours}} = \frac{\_\_\_}{1}\frac{\text{lawns}}{\text{hour}}$$

$\frac{24}{40} < \frac{25}{40}$ My friend is a little faster, but only $\frac{1}{40}$ of a lawn per hour, so it is very close. The unit rates have corresponding decimals 0.6 and 0.625.

Example 2: Restaurant Advertising (9 minutes)

- Next, suppose you own a restaurant. You want to do some advertising, so you hire 2 students to deliver take-out menus around town. One student, Darla, delivers 350 menus in 2 hours, and another student, Drew, delivers 510 menus in 3 hours. You promise a \$10 bonus to the fastest worker since time is money in the restaurant business. Who gets the bonus?

- How should the unit rates and the rate units look when they are written completely?
  - $\frac{350}{2}\frac{\text{menus}}{\text{hours}} = \frac{175}{1}\frac{\text{menus}}{\text{hour}}$; $\frac{510}{3}\frac{\text{menus}}{\text{hours}} = \frac{170}{1}\frac{\text{menus}}{\text{hour}}$
- Compare the unit rates for each student. Who works faster at the task and gets the bonus cash?
  - *Darla's unit rate is* $\frac{175}{1}\frac{\text{menus}}{\text{hour}}$ *and Drew's unit rate is* $\frac{170}{1}\frac{\text{menus}}{\text{hour}}$. *Since Darla is able to deliver* 5 *more menus an hour than Drew, she should get the bonus.*
- Will the unit labels in the numerator and denominator always match in the work rates we are comparing?
  - *Yes.*

Example 2: Restaurant Advertising

$$\frac{\_\_\_}{\_\_\_}\frac{\textbf{menus}}{\textbf{hours}} = \frac{\_\_\_}{\textbf{1}}\frac{\textbf{menus}}{\textbf{hour}} \qquad \frac{\_\_\_}{\_\_\_}\frac{\textbf{menus}}{\textbf{hours}} = \frac{\_\_\_}{\textbf{1}}\frac{\textbf{menus}}{\textbf{hour}}$$

$$\frac{350}{2}\frac{\textit{menus}}{\textit{hours}} = \frac{175}{1}\frac{\textit{menus}}{\textit{hour}} \qquad \frac{510}{3}\frac{\textit{menus}}{\textit{hours}} = \frac{170}{1}\frac{\textit{menus}}{\textit{hour}}$$

Set up a problem for the student that does not keep the units in the same arrangement:

$$\frac{350}{2}\frac{\text{menus}}{\text{hours}} = \frac{175}{1}\frac{\text{menus}}{\text{hour}} \qquad \frac{3}{510}\frac{\text{hours}}{\text{menus}} = \frac{1}{170}\frac{\text{hour}}{\text{menus}}$$

- What happens if they do not match and one is inverted?
  - *It will be difficult to compare the rates. We would have to say* 175 *menus would be delivered per hour by Darla, and it would take an hour for Drew to deliver* 170 *menus. Mixing up the units makes the explanations awkward.*
- Will time always be in the denominator?
  - *Yes.*
- Do you always divide the numerator by the denominator to find the unit rate?
  - *Yes.*

Example 3: Survival of the Fittest (9 minutes)

- Which runs faster: a cheetah that can run 60 feet in 4 seconds or gazelle that can run 100 feet in 8 seconds?

Example 3: Survival of the Fittest

$$\frac{\_\_\_}{\_\_\_}\frac{\textbf{feet}}{\textbf{seconds}} = \frac{\_\_\_}{\textbf{1}}\frac{\textbf{feet}}{\textbf{second}} \qquad \frac{\_\_\_\_\_\_\_\_\ \textbf{feet}}{\_\_\_\_\_\_\_\_\ \textbf{seconds}} = \frac{\_\_\_\_\_\_\_\_\ \textbf{feet}}{\textbf{1 second}}$$

$$\frac{60}{4}\frac{\textit{feet}}{\textit{seconds}} = \frac{15}{1}\frac{\textit{feet}}{\textit{second}} \qquad \frac{100}{8}\frac{\textit{feet}}{\textit{seconds}} = \frac{12.5}{1}\frac{\textit{feet}}{\textit{second}}$$

The cheetah runs faster.

Example 4: Flying Fingers (7 minutes)

- What if the units of time are not the same in the two rates? What will this mean for the rate units? The secretary in the main office can type 225 words in 3 minutes, while the computer teacher can type 105 words in 90 seconds. Can we still compare the unit rates? Who types at a faster rate?

Ask half of the class to solve this problem using words per minute and the other half using words per second. Ask for a volunteer from each group to display and explain their solutions.

Example 4: Flying Fingers

$$\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_ \qquad \_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_$$

$\frac{225}{3}\frac{\textit{words}}{\textit{minutes}} = \frac{75}{1}\frac{\textit{words}}{\textit{minute}}$ $\frac{105}{1.5}\frac{\textit{words}}{\textit{minutes}} = \frac{70}{1}\frac{\textit{words}}{\textit{minute}}$ *The secretary types faster.*

$\frac{225}{180}\frac{\textit{words}}{\textit{seconds}} = \frac{1.25}{1}\frac{\textit{words}}{\textit{second}}$ $\frac{105}{90}\frac{\textit{words}}{\textit{seconds}} = \frac{1.166667}{1}\frac{\textit{words}}{\textit{second}}$ *The secretary types faster.*

- Do we have to convert one time unit?
  - *Yes.*
- What will happen if we do not convert one time unit so that they match?
  - *We cannot compare the rates. It is not easy to tell which is faster: 70 words per minute or 1.25 words per second.*
- Does it matter which one you change?
  - *No. Either change 90 seconds to 1.5 minutes or change 3 minutes to 180 seconds, as long as the rate units are the same when you are finished.*
- Can you choose the one that makes the problem easier for you?
  - *Yes.*
- Is there an advantage in choosing one method over the other?
  - *Changing seconds to minutes avoids repeating decimals.*
- Looking back on our work so far what is puzzling you? What questions do you have?
- Describe how this type of problem is similar to unit pricing problems.
  - *Unit pricing problems use division, and so do work rate problems.*
- Describe how work problems are different than unit price problems.
  - *Unit price problems always have cost in the numerator; work rate problems always have time in the denominator.*

Closing (5 minutes)

- Rate problems, including constant rate problems, always count or measure something happening per unit of time. The time is always in the denominator.
- Sometimes the units of time in the denominators of the rates being compared are not the same. One must be converted to the other before calculating the unit rate of each.

Lesson Summary

- Rate problems, including constant rate problems, always count or measure something happening per unit of time. The time is always in the denominator.
- Sometimes the units of time in the denominators of the rates being compared are not the same. One must be converted to the other before calculating the unit rate of each.

Exit Ticket (5 minutes)

Name ____ Date____________________

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Exit Ticket

A 6th grade math teacher can grade 25 homework assignments in 20 minutes.

Is he working at a faster rate or slower rate than grading 36 homework assignments in 30 minutes?

Exit Ticket Sample Solutions

A 6th grade math teacher can grade 25 homework assignments in 20 minutes.

Is he working at a faster rate or slower rate than grading 36 homework assignments in 30 minutes?

$\frac{25 \text{ assignments}}{20 \text{ minutes}} = \frac{1.25 \text{ assignments}}{1 \text{ minute}}$ $\frac{36 \text{ assignments}}{30 \text{ minutes}} = \frac{1.2 \text{ assignments}}{1 \text{ minute}}$

It is faster to grade 25 assignments in 20 minutes.

Problem Set Sample Solutions

1. Who walks at a faster rate: someone who walks 60 feet in 10 seconds or someone who walks 42 feet in 6 seconds?

   $\frac{60 \text{ feet}}{10 \text{ seconds}} = 6 \frac{\text{feet}}{\text{second}}$

   $\frac{42 \text{ feet}}{6 \text{ seconds}} = 7 \frac{\text{feet}}{\text{second}}$ → *Faster*

2. Who walks at a faster rate: someone who walks 60 feet in 10 seconds or someone who takes 5 seconds to walk 25 feet? Review the lesson summary before answering!

   $\frac{60 \text{ feet}}{10 \text{ seconds}} = 6 \frac{\text{feet}}{\text{second}}$ → *Faster*

   $\frac{25 \text{ feet}}{5 \text{ seconds}} = 5 \frac{\text{feet}}{\text{second}}$

3. Which parachute has a slower decent: a red parachute that falls 10 feet in 4 seconds or a blue parachute that falls 12 feet in 6 seconds?

   Red: $\frac{10 \text{ feet}}{4 \text{ seconds}} = 2.5 \frac{\text{feet}}{\text{second}}$

   Blue: $\frac{12 \text{ feet}}{6 \text{ seconds}} = 2 \frac{\text{feet}}{\text{second}}$ → *Slower*

4. During the winter of 2012–2013, Buffalo, New York received 22 inches of snow in 12 hours. Oswego, New York received 31 inches of snow over a 15-hour period. Which city had a heavier snowfall rate? Round your answers to the nearest hundredth.

   $\frac{22 \text{ inches}}{12 \text{ hours}} = 1.83 \frac{\text{inches}}{\text{hour}}$

   $\frac{31 \text{ inches}}{15 \text{ hours}} = 2.07 \frac{\text{inches}}{\text{hour}}$ → *Heavier*

5. A striped marlin can swim at a rate of 70 miles per hour. Is this a faster or slower rate than a sailfish, which takes 30 minutes to swim 40 miles?

Marlin: 70 mph → slower

Sailfish:

$$\frac{40}{30}\frac{\text{miles}}{\cancel{\text{minutes}}} \times \frac{60}{1}\frac{\cancel{\text{minutes}}}{\text{hour}} = \frac{2,400}{30}\frac{\text{miles}}{\text{hour}} = 80 \text{ mph}$$

6. One math student, John, can solve 6 math problems in 20 minutes while another student, Juaquine, can solve the same 6 math problems at a rate of 1 problem per 4 minutes. Who works faster?

$$\frac{6}{20}\frac{\text{problems}}{\text{minutes}} = 0.3\,\frac{\text{problems}}{\text{minute}}$$ → *Faster*

$$\frac{1}{4}\frac{\text{problem}}{\text{minutes}} = 0.25\,\frac{\text{problems}}{\text{minute}}$$