Lesson 13: From Ratio Tables to Equations Using the Value of a Ratio

Student Outcomes

- Students restate a ratio in terms of its value; e.g., if the ratio of length A to length B is $3:5$ (in the same units), students state that "length A is $\frac{3}{5}$ of length B", "length B is $\frac{5}{3}$ of length A", " length A is $\frac{3}{8}$ of the total length", and "length B is $\frac{5}{8}$ of the total length".
- Students use the value of the ratio to problem-solve by writing and solving equations.

Classwork

Exercise 1–3 (35 minutes)

Exercise 1

Each student is given a pre-made Unifix cube model consisting of one red cube and three yellow cubes to be used as a model for the scenario below.

Exercise 1

Jorge is mixing a special shade of orange paint. He mixed 1 gallon of red paint with 3 gallons of yellow paint.

Based on this ratio, which of the following statements are true?

- **$\frac{3}{4}$ of a 4-gallon mix would be yellow paint.**

  True

- **Every 1 gallon of yellow paint requires $\frac{1}{3}$ gallon of red paint.**

  True

- **Every 1 gallon of red paint requires 3 gallons of yellow paint.**

  True

- **There is 1 gallon of red paint in a 4-gallon mix of orange paint.**

  True

- **There are 2 gallons of yellow paint in an 8-gallon mix of orange paint.**

  False

Use the space below to determine if each statement is true or false.

Scaffolding:

Unifix cubes should be available for each learner. They give a color-coded manipulative model that makes the abstract story problem tangible. The cubes can be combined to give a concrete model of the chart.

Allow students to discuss each question with a partner or group. When the class comes back together as a whole group, each group is responsible for explaining to the class *one* of the statements and whether the group feels the statement is true or false and why. (The first four statements are true, while the fifth statement is false. To be made true, the fifth statement should read "There are 6 gallons of yellow paint in an 8 gallon mix of orange paint.")

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Exercise 2

Exercise 2

Based on the information on red and yellow paint given in Exercise 1, complete the table below.

Red Paint (R)	Yellow Paint (Y)
1	3
2	6
3	9
4	12
5	15

MP.7 Students should be encouraged to combine their Unifix cubes with those of a partner to model the ratio given in the second row of the table. Students should find a third partner to model the ratio given in the third row, etc.

Facilitate and lead the discussion (if necessary) to point out that we can extend the table to show total gallons.

Red Paint (R)	Yellow Paint (Y)	Relationship
1	3	$3 = 1 \times 3$
2	6	$6 = 2 \times 3$
3	9	$9 = 3 \times 3$
4	12	$12 = 4 \times 3$
5	15	$15 = 5 \times 3$

Use the table to identify the relationship between two quantities as an intermediate step in creating an equation that models that relationship.

Here is a possible conversation that could be used to help the students see the relationships:

- What information is given in the table?
  - *The table gives the number of gallons of red paint and the number of gallons of yellow paint.*
- In what context would someone use this information?
  - *This information would be useful to anyone who had a need to paint a surface and also had to mix his own paint, such as a painting contractor who prefers to mix custom colors for high-end clients.*
- We need to interpret what this table means. If I use 5 gallons of red paint, how many gallons of yellow paint would I need?
  - *I would need* 15 *gallons of yellow paint.*
- How is the amount of yellow paint related to the amount of red paint?
  - *Yes, the amount of yellow paint is always* 3 *times as much as the amount of red paint.*
- Is that true for all of the entries?
  - *Yes.*
- Now imagine that we want to make orange paint to cover an entire wing of our school, and we had 100 gallons of red paint. How could we figure out how many gallons of yellow paint to use?
  - *We could multiply* 100 *by* 3*.*

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- Now we want to write this as an equation. You have told me that I can take all the values in the first column and multiply by three to get the values in the second column. When we were given 4 gallons of red paint, we knew we would need $3 \cdot 4$ gallons of yellow paint. What if we were given R gallons of red paint, how many gallons of yellow paint would we need? So, Y, the number of gallons of yellow paint, would equal…?
  - *3 times R*
- How would we write this equation?

To get to these steps, students might need a little guidance. Help by pointing out the variables given in the table and ask them to write what R must be multiplied by to get Y.

  - $Y = 3R$
- We were trying to find out how much yellow paint we needed given the amount of red paint. Is the formula related to the value of the ratio of the number of gallons of yellow paint to the number of gallons of red paint?
  - *The ratio of the number of gallons of yellow paint to the number of gallons of red paint is $3:1$; the value of the ratio is $\frac{3}{1}$.*
- What if we wanted an equation to tell us how much red paint to use if we are given the amount of yellow paint? How can we use the amount of yellow paint to determine the amount of red paint needed?
  - *Divide by three or multiply by $\frac{1}{3}$.*
- What is the ratio of the number of gallons of red paint to the number of gallons of yellow paint?
  - *The ratio is $1:3$ or 1 to 3, and the value of the ratio is $\frac{1}{3}$.*

MP.2

- How can I use this information to write the equation?
  - *We would take the Y-value and divide by 3; in other words, multiply by $\frac{1}{3}$. So, the equation would be $R = \frac{1}{3}Y$.*

Some suggestions for discussion questions:

- In this case the ratio of the number of gallons of red paint to the number of gallons of yellow paint is $1:3$. What if the ratio were changed to $1:4$? What would this mean in the context of our paint problem?
  - *We would use one gallon of red paint for every four gallons of yellow paint.*
- Can we still use the equation we created earlier? What would the new equation be?
  - *No. The new equation would be $Y = 4 \cdot R$*
- How can we use the ratio to write the equation?
  - *There will be 4 times as much yellow paint as there is red paint. The 4 tells us what to multiply the number of gallons of red paint by to find the number of gallons of yellow paint.*
- What if the ratio were $1:7$? What would the new equation be?
  - $Y = 7 \cdot R$

Scaffolding:

The connection to the multiplication table should be elicited: rows 1 and 3 show the relationship in this ratio. Students might also find that equivalent fractions can be seen this way.

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Exercise 3

Students can try the first question on their own, or you can discuss the question if you feel students need further instructions with the concept. Otherwise, students start the exercise on their own, in partners, or in small groups.

- Jorge now plans to mix red paint and blue paint to create purple paint. The color of purple he has decided to make combines red paint and blue paint in the ratio $4:1$. If Jorge can only purchase paint in one gallon containers, construct a ratio table for all possible combinations for red and blue paint that will give Jorge no more than 25 gallons of purple paint.
- Write an equation that will let Jorge calculate the amount of red paint he will need for any given amount of blue paint.
- Write an equation that will let Jorge calculate the amount of blue paint he will need for any given amount of red paint.
- If Jorge has 24 gallons of red paint, how much blue paint will he have to use to create the desired color of purple?
- If Jorge has 24 gallons of blue paint, how much red paint will he have to use to create the desired color of purple?

MP.5 Allow students to make a table or drawing.

- Remember that we sometimes use variables to represent numbers. Let's use B and R for the amounts of blue paint and red paint, respectively.
- No matter how much blue paint I use, I need 4 times as much red paint. So, for one gallon of blue paint, I need (1×4) 4 gallons of red paint. That is a ratio of $1:4$. The value of the ratio is $\frac{1}{4}$.
- Where do we see the ratio in the equations?
  - *We determine the amount of red paint by multiplying the unknown amount of blue paint by 4. So, for every 1 gallon of blue paint, we need 4 gallons of red paint. To determine the amount of blue paint, we need to find $\frac{1}{4}$ of the amount of red paint.*

Scaffolding:

The connection to the multiplication table should be elicited: columns 1 and 4 show the relationship in this ratio.

Exercise 3

Blue (B)	Red (R)	Relationship
1	4	$4 = 1 \times 4$
2	8	$8 = 2 \times 4$
3	12	$12 = 3 \times 4$
4	16	$16 = 4 \times 4$
5	20	$20 = 5 \times 4$

Continue to allow students time to work on the remainder of the problems. As you are working with the students, be sure to remind them of the value of the ratio and how it is used to make the equation.

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a. Using the same relationship of red to blue from above, create a table that models the relationship of the three colors blue, red, and purple (total) paint. Let B represent the number of gallons of blue paint, let R represent the number of gallons of red paint, and let T represent the total number of gallons of (purple) paint. Then write an equation that models the relationship between the blue paint and the total amount of paint and answer the questions.

Blue (B)	Red (R)	Total Paint (T)
1	4	5
2	8	10
3	12	15
4	16	20
5	20	25

Equation: $T = 5B$

Value of the ratio of total paint to blue paint: $\frac{5}{1}$.

How is the value of the ratio related to the equation?

The value of the ratio is used to determine the total paint value by multiplying it with the blue paint value.

b. During a particular U.S. Air Force training exercise, the ratio of the number of men to the number of women was $6:1$. Use the ratio table provided below to create at least two equations that model the relationship between the number of men and the number of women participating in this training exercise.

Women (W)	Men (M)
1	6
2	12
3	18
4	24
5	30

Equations:

$M = 6W$

$W = \left(\frac{1}{6}\right)M$

$\frac{M}{W} = 6$

$\frac{W}{M} = \frac{1}{6}$

Scaffolding:

The connection to the multiplication table should be elicited: columns 1 and 6 show the relationship in this ratio.

If 200 women participated in the training exercise, use one of your equations to calculate the number of men who participated.

I can substitute 200 for the value of women and multiply by 6, the value of the ratio, to get the number of men. There would be $1,200$ men participating in the training exercise.

c. Malia is on a road trip. During the first five minutes of Malia's trip, she sees 18 cars and 6 trucks. Assuming this ratio of cars to trucks remains constant over the duration of the trip, complete the ratio table using this comparison. Let T represent the number of trucks she sees, and let C represent the number of cars she sees.

Trucks (T)	Cars (C)
1	3
3	9
6	18
12	36
20	60

What is the value of the ratio of the number of cars to the number of trucks?

$\frac{3}{1}$

What equation would model the relationship between cars and trucks?

$C = 3T$ *and* $T = \left(\frac{1}{3}\right)C$

At the end of the trip, Malia had counted $1,254$ trucks.
How many cars did she see?

$C = 1,254 \cdot 3$; $C = 3,762$ *cars*

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d. Kevin is training to run a half-marathon. His training program recommends that he run for 5 minutes and walk for 1 minute. Let R represent the number of minutes running, and let W represent the number of minutes walking.

Minutes Running (R)	5	10	20	40	50
Minutes Walking (W)	1	2	4	8	10

What is the value of the ratio of the number of minutes walking to the number of minutes running?

$\frac{1}{5}$

What equation could you use to calculate the minutes spent walking if you know the minutes spent running?

$W = \frac{1}{5}R$; Answers will vary.

Closing (5 minutes)

MP.5 Have students explain the relationship between the ratio and the equation. Students can include examples, tables, equations, or other representations to justify their reasoning.

Lesson Summary

The value of a ratio can be determined using a ratio table. This value can be used to write an equation that also represents the ratio.

Example:

1	4
2	8
3	12
4	16

The multiplication table can be a valuable resource to use in seeing ratios. Different rows can be used to find equivalent ratios.

Exit Ticket (5 minutes)

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Name ____ Date____________________

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Exit Ticket

A carpenter uses four nails to install each shelf. Complete the table to represent the relationship between the number of nails (N) and the number of shelves (S). Write the ratio that describes the number of nails per number of shelves. Write as many different equations as you can that describe the relationship between the two quantities.

Shelves (S)	**Nails** (N)
1	4
2	
	12
	16
5	

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Exit Ticket Sample Solutions

A carpenter uses four nails to install each shelf. Complete the table to represent the relationship between the number of nails (N) and the number of shelves (S). Write the ratio that describes the number of nails per number of shelves. Write as many different equations as you can that describe the relationship between the two quantities.

Shelves (S)	Nails (N)
1	4
2	8
3	12
4	16
5	20

$\left(\frac{N}{S}\right) = \left(\frac{4}{1}\right)$

Equations:

$N = 4S$

$S = \left(\frac{1}{4}\right)N$

Problem Set Sample Solutions

A cookie recipe calls for 1 cup of white sugar and 3 cups of brown sugar.

Make a table showing the comparison of the amount of white sugar to the amount of brown sugar.

White Sugar (W)	Brown Sugar (B)
1	3
2	6
3	9
4	12
5	15

1. **Write the value of the ratio of the amount of white sugar to the amount of brown sugar.**

   $\frac{1}{3}$

2. **Write an equation that shows the relationship of the amount of white sugar to the amount of brown sugar.**

   $B = 3W$ *or* $W = \frac{1}{3}B$

3. **Explain how the value of the ratio can be seen in the table.**

   The values in the first row show the values in the ratio. The ratio of the amount of brown sugar to the amount of white sugar is $3:1$. The value of the ratio is $\frac{3}{1}$.

4. **Explain how the value of the ratio can be seen in the equation.**

   The amount of brown sugar is represented as B in the equation. The amount of white sugar is represented as W. The value is represented because the amount of brown sugar is three times as much as the amount of white sugar, or $B = 3W$.

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Using the same recipe, compare the amount of white sugar to the amount of total sugars used in the recipe.

Make a table showing the comparison of the amount of white sugar to the amount of total sugar.

White Sugar (W)	Total Sugar (T)
1	**4**
2	**8**
3	**12**
4	**16**
5	**20**

5. **Write the value of the ratio of the amount of total sugar to the amount of white sugar.**

   $\frac{4}{1}$

6. **Write an equation that shows the relationship of total sugar to white sugar.**

   $T = 4W$

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