



## Lesson 10: The Structure of Ratio Tables—Additive and Multiplicative

### Student Outcomes

- Students identify both the additive and multiplicative structure of a ratio table and use the structure to make additional entries in the table.
- Students use ratio tables to solve problems.

### Classwork

#### Exploratory Challenge (35 minutes)

##### Exploratory Challenge

Imagine that you are making a fruit salad. For every quart of blueberries you add, you would like to put in 3 quarts of strawberries. Create three ratio tables that show the amounts of blueberries and strawberries you would use if you needed to make fruit salad for greater numbers of people.

Table 1 should contain amounts where you have added fewer than 10 quarts of blueberries to the salad.

Table 2 should contain amounts of blueberries between 10 and 50 quarts.

Table 3 should contain amounts of blueberries greater than 100 quarts.

*Student answers may vary. Here are possible solutions:*

Table 1	
Quarts of Blueberries	Quarts of Strawberries
1	3
2	6
3	9
4	12
5	15

Table 2	
Quarts of Blueberries	Quarts of Strawberries
10	30
20	60
30	90
40	120
50	150

Table 3	
Quarts of Blueberries	Quarts of Strawberries
100	300
200	600
300	900
400	1,200
500	1,500

*The answers to the questions will depend on the variation of the table that students have created.*

- a. Describe any patterns you see in the tables. Be specific in your descriptions.

*The value in the second column is always three times as much as the corresponding value in the first column. In the first table, the entries in the first column increase by 1, and the entries in the second column increase by 3. In the second table, the entries in the first column increase by 10, and the entries in the second column increase by 30. In the third table, the entries in the first column increase by 100, and the entries in the second column increase by 300.*

- b. How are the amounts of blueberries and strawberries related to each other?

*The amount of strawberries is always three times the amount of blueberries. Students could also respond that the ratio of the number of quarts of blueberries to the number of quarts of strawberries is always equivalent to 1:3.*

- c. How are the values in the blueberries column related to each other?

*Answers will vary. However, students could use the chart paper and write on the table to see the patterns. Most tables should have addition repeating throughout.*

- d. How are the values in the strawberries column related to each other?

*Answers will vary. However, students could use the chart paper and write on the table to see the patterns. Most tables should have addition repeating throughout.*

- e. If we know we want to add 7 quarts of blueberries to the fruit salad in Table 1, how can we use the table to help us determine how many strawberries to add?

*We could extend our table until we get to 7 in the blueberry column.*

- f. If we know we used 70 quarts of blueberries to make our salad, how can we use a ratio table to find out how many quarts of strawberries were used?

*We could start with the ratio 1:3 that was given in the description and then multiply by ten to get 10 and 30. These would be the first values in my table. Then, I would count up by tens in the blueberries column and count up by 30s in the strawberries column.*

Students create the three ratio tables on the student pages. Have index cards ready that say Table 1, Table 2, and Table 3 to hand out to students so that students can place the assigned table on chart paper (15 minutes). After the charts are created, have students focus on how they created the tables and discuss the structure of the tables with a partner or small group. After the students have had a chance to work, pull the class together as a whole group for a discussion about the structure of the tables and how the tables are related. Use the questions below to guide the discussion.

- How are all three of the tables related?
  - *Each table represents the same ratio of the number of quarts of blueberries to the number of quarts of strawberries, which is 1:3.*
- What operation(s) did you use to determine the values for quarts of blueberries and quarts of strawberries?
  - *Adding or multiplying (Have students elaborate where they used each operation.)*
- How is the number of quarts of strawberries related to the number of quarts of blueberries?
  - *The number of quarts of strawberries is always three times the number of quarts of blueberries or the number of quarts of blueberries is one-third the number of quarts of strawberries. Students could also respond that the ratio of the number of quarts of blueberries to the number of quarts of strawberries is always equivalent to 1:3.*

B	S
1 x 3 =	3
2 x 3 =	6
3 x 3 =	9
4 x 3 =	12

MP.7

Students write directly on the chart paper to check and see whether this is true for every entry in each table. Extend this question to ask the students why the rows have the same ratio of the number of quarts of blueberries to the number of quarts of strawberries (or why the rows do not have the same ratio, if a mistake was made) or if the answer makes sense.

- How are the amounts of blueberries related to each other in the table?
  - *Answers will vary. Students should notice that there is a pattern in the blueberries column. The paper that each group of students made could be different. So how they created the pattern will vary. However, students could use the chart paper and write on the table to see the patterns. Most tables should have addition repeating throughout.*

	<i>B</i>	<i>S</i>
+10	10	30
+10	20	60
+10	30	90
	40	120

How are the amounts of strawberries related to each other in the table?

<i>B</i>	<i>S</i>
10	30
20	60
30	90
40	120

Extend this question to further ask the students how the two patterns are related to each other.

	<i>B</i>	<i>S</i>
+10	10	30
+10	20	60
+10	30	90
+10	40	120

- The change in the amount of blueberries compared to the change in the amount of strawberries is 10 to 30 or 1 to 3, the same ratio we started with.
- If we know we want to add 7 quarts of blueberries in Table 1, how can we use the table to help us get the amount of strawberries needed?
  - *We could extend our table until we got to seven in the blueberries column.*

MP.7

- What if we were making enough fruit salad to serve a large number of people and added 70 quarts of blueberries? How could we create a table to find the value for strawberries?
  - *We could start with the ratio 1: 3 that was given in the description and then multiply by ten to get 10 and 30. These would be the first values in my table. Then, I would count up by tens in the blueberries column and count up by 30s in the strawberries column.*

### Exercise 1

Students examine tables that were made incorrectly and make comments on what was done wrong when making the tables. Students also note the ratio that was used to create the ratio table and then create a correct table.

#### Exercise 1

The following tables were made incorrectly. Find the mistakes that were made, create the correct ratio table, and state the ratio that was used to make the correct ratio table.

a.

Hours	Pay in Dollars
3	24
5	40
7	52
9	72

Hours	Pay in Dollars
3	24
5	40
7	56
9	72

Ratio 1: 8 (Solutions may vary.)

b.

Blue	Yellow
1	5
4	8
7	13
10	16

Blue	Yellow
1	5
4	20
7	35
10	50

Ratio 1: 5 (Solutions may vary.)

Hours	Pay in Dollars
3	24
5	40
7	52
9	72

Students should be able to note that the entry (7, 52) is incorrect. It should be (7, 56).

Students should also note that each entry should have a ratio of 1 to 8 showing that 1 hour pays \$8.

Blue	Yellow
1	5
4	8
7	13
10	16

Students may notice that each of the entries has a different ratio. The problem with this table is that the same amount was added repeatedly to both the Blue column and the Yellow column. Because the first ratio is 1:5, to get a ratio of 4:20, we would add three to the Blue column and fifteen to the Yellow column. Adding three to the Blue column and then adding fifteen to the Yellow column creates a ratio of 1:5, just as we started with.

### Closing (5 minutes)

Questions to Review:

- In a vertically oriented ratio table, how are the values across the rows related?
  - *The values across the rows form a ratio of  $a:b$ . So, the value of the second column will be determined by multiplying the value in the first column by  $\frac{b}{a}$ , and the value of the first column will be determined by multiplying the value in the second column by  $\frac{a}{b}$ .*
- In a vertically oriented ratio table, how are the values related as we move down a column?
  - *The values in the column depend on how the table was created, but they could be increasing by the same sum or by the same multiple. For example, the values in the first column could be increasing by 5 each time. So, the values could go from 6, 11, 16, 21, 26, etc. or the numbers could be formed by multiplying. In other words, the values could go from 6, 12, 24, 48, etc. if the values were multiplied by 2 each time.*
- Is there a way to use addition to figure out the next row in a ratio table?
  - *I can use the ratio to help me use addition to get the next row. For example, if the ratio of  $x:y$  is 2:5, I can add 2 to the value in the first column and add 5 to the value in the second column to get the next row in the table. I cannot just add the same thing to both the values in the first and second columns.*
- Is there a way to use multiplication to figure out the next row in a ratio table?
  - *If I use multiplication to get the next row in the table, I need to multiply both the values in the first column and the values in the second column by the same number. So, if the original row is (4, 5) and I want to multiply by 3 to get the next row, I would multiply  $4 \times 3$  and  $5 \times 3$  to get a row that is (12, 15). Unlike the addition method, I would do the same thing to both the values in the first column and the values in the second column.*

## Lesson Summary

Ratio tables are constructed in a special way.

Each pair of values in the table will be equivalent to the same ratio.

red	white
3	12
6	24
1	4
21	84

$6 : 24$

$1 : 4$

$21 : 84$

$1 : 4$

You can use repeated addition or multiplication to create a ratio table.

There is a constant value that we can multiply the values in the first column by to get the values in the second column.

red	white
3 $\times 4$	12
6 $\times 4$	24
12 $\times 4$	48
21 $\times 4$	84

If you add a certain number to each entry in one column, you may not be able to add that same number to the entries in the other column and keep the same ratio. Instead, the numbers you add to the entries must be related to the ratio used to make the table. However, if you multiply the entries in one column by a certain number, you can multiply the entries in the other column by the same number to create equivalent ratios.

red	white
3	12
6	24
12	48
21	84

$\times 7$   $\times 7$

## Exit Ticket (5 minutes)

Name \_\_\_\_\_

Date \_\_\_\_\_

## Lesson 10: The Structure of Ratio Tables—Additive and Multiplicative

### Exit Ticket

Show more than one way you could use the structure of the table to find the unknown value. Fill in the unknown values.

Number of Weeks	Amount of Money in Account
2	\$350
4	\$700
6	\$1,050
8	
10	

## Exit Ticket Sample Solutions

Show more than one way you could use the structure of the table to find the unknown value. Fill in the unknown values.

Number of Weeks	Amount of Money in Account
2	\$350
4	\$700
6	\$1,050
8	\$1,400
10	\$1,750

*I can add two to the weeks each time to get the next number. I can add \$350 to the money to get the next values.*

*In the rows, we have 2: 350, which is equal to 1: 175. So the money is 175 times larger than the week. I can just multiply the week by 175 to get the amount of money in the account.*

*The ratio used to create the table was 1: 175.*

## Problem Set Sample Solutions

1. a. Create a ratio table for making lemonade with a lemon juice-to-water ratio of 1: 3. Show how much lemon juice would be needed if you use 36 cups of water to make lemonade.

Lemon Juice (cups)	Water (cups)
1	3
2	6
3	9
4	12
12	36

*12 cups of lemon juice would be needed if 6 cups of water is used.*

- b. How is the value of the ratio used to create the table?

*The value of the ratio is  $\frac{1}{3}$ . If we know the amount of lemon juice, we can multiply that amount by 3 to get the amount of water. If we know the amount of water, we can multiply that amount by  $\frac{1}{3}$  (or divide by 3) to get the amount of lemon juice.*

2. Ryan made a table to show how much blue and red paint he mixed to get the shade of purple he will use to paint the room. He wants to use the table to help him make larger and smaller batches of purple paint.

Blue	Red
12	3
20	5
28	7
36	9

- a. What ratio was used to create this table? Support your answer.

*The ratio of the amount of blue paint to the amount of red paint is 4: 1. I know this because 12: 3, 20: 5, 28: 7, and 36: 9 are all equivalent to 4: 1.*



- b. How are the values in each row related to each other?

*In each row, the amount of red paint is  $\frac{1}{4}$  times the amount of blue paint, or the amount of blue paint is 4 times the amount of red paint.*

- c. How are the values in each column related to each other?

*The values in the columns are increasing using the ratio. Since the ratio of the amount of blue paint to the amount of red paint is 4:1, we have used  $4 \times 2$ :  $1 \times 2$ , or 8:2, and repeatedly added to form the table. 8 was added to the entries in the blue column, while 2 was added to the entries in the red column.*