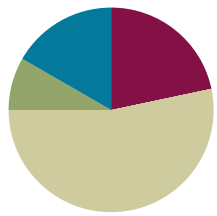
Lesson 7

Objective: Demonstrate the commutativity of multiplication, and practice related facts by skip-counting objects in array models.

Suggested Lesson Structure

Fluency Practice	(13 minutes)
Application Problem	(5 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (13 minutes)

- Group Counting **3.OA.1** (3 minutes)
- Divide Equal Groups **3.OA.2** (5 minutes)
- Multiply with Twos **3.OA.7** (5 minutes)

Group Counting (3 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by twos and threes in this activity anticipates work with those factors in Topic C.

T: Let's count by twos. (Direct students to count forward and backward to 20, emphasizing the 8 to 10, 10 to 12, and 18 to 20 transitions.)

T: Let's count by threes. (Direct students to count forward and backward to 30, periodically changing directions. Emphasize the 9 to 12, 18 to 21, and 27 to 30 transitions.)

Divide Equal Groups (5 minutes)

Materials: (S) Personal white board

Note: Students directly relate repeated addition to division. They interpret the unknown in division. This activity reviews Lesson 6.

T: (Project an array with 2 groups of 4.) Say the total as a repeated addition sentence.

S: 4 + 4 = 8.

T: Write a division sentence for 8 divided into 2 equal groups.

S: (Write 8 ÷ 2 = 4.)

T: Below that division sentence write a division sentence dividing 8 into 4 equal groups.

S: (Write 8 ÷ 4 = 2.)

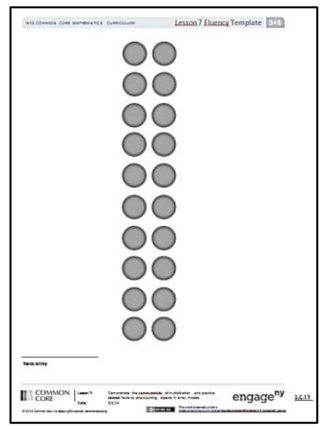
Continue with possible sequence: 5 groups of 3, 3 groups of 4, and 6 groups of 2.

Multiply with Twos (5 minutes)

Materials: (S) Personal white board, twos array (Fluency Template), blank paper

Note: Students unit count objects in an array and write multiplication sentences that match the count-by in anticipation of this lesson's objective.

Twos array fluency template



T: Slip your template into your personal white board.

T: Turn your board so that it's vertical. Use your blank paper to cover all but the first row of dots.

T: How many twos show?

S: 1 two.

T: Say the multiplication sentence to represent the array that's shown and solve.

S: 1 × 2 = 2.

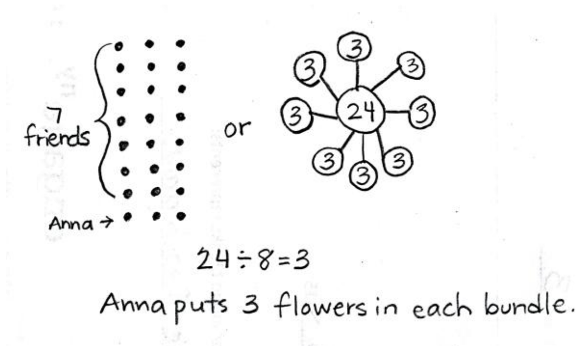
T: Uncover another row.

Continue this sequence having students uncover twos for 2 × 2, 3 × 2, 10 × 2, 4 × 2, 5 × 2, 6 × 2, 7 × 2, 9 × 2, and 8 × 2.

Application Problem (5 minutes)

Anna picks 24 flowers. She makes equal bundles of flowers and gives 1 bundle to each of her 7 friends. She keeps a bundle for herself too. How many flowers does Anna put in each bundle?

Note: This problem reviews equal groups division from Lesson 5 where the unknown represents the size of the group. The problem's complexity is in understanding that the flowers are divided equally into 8 bundles, not 7, in order to include a bundle for Anna. Students might choose to solve by drawing a division array learned in Lesson 6 or a number bond learned in Lesson 3.



Concept Development (32 minutes)

Materials: (S) Personal white board

Problem 1: Rotate arrays 90 degrees.

T: Turn your personal white board so it's horizontal. Draw a line down the middle to make two sides. On the left, skip-count by twos 4 times and write each number.

S: (Write 2, 4, 6, 8.)

T: On the right side, skip-count by fours 2 times and write each number.

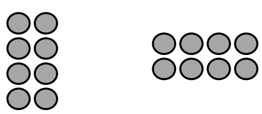
S: (Write 4, 8.)

T: How are the count-bys related?

S: The first one is 4 twos and the second one is 2 fours.

T: Under each count-by, draw an array to match it.

S: (Draw arrays shown below.)



T: What do you notice about the arrays? Do they both have 4 groups of 2?

S: (Discuss.)

T: Do they both have 2 groups of 4?

S: Yes. The one on the right has 2 rows of 4. If you turn it sideways, then the one on the left does too. Or, you can just see that it has 2 vertical rows of 4.

T: It's the same array turned different ways. We have a special name for rows when they are vertical. We call them **columns**.

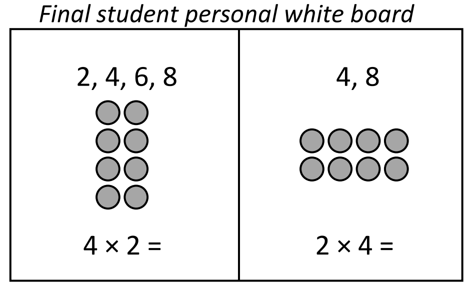
Prompt students to write and solve equations to show the total objects in each array. Continue with the following possible examples:

- 2×5 array
- 7×2 array

As you circulate, guide students to notice that factors switch places, and help them to relate the change to the rotated array. For example, write $4 \times 2 = 2 \times 4$ and ask students to discuss how they know it is true.

T: Depending on how we look at an array, columns *or* rows can be the number of groups. Discuss with your partner how you know that's true.

Final student personal white board



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

If necessary, students may keep the twos array template from the fluency activity Multiply with Two alongside their workspace to help them draw arrays from skip-counting.

NOTE ON VOCABULARY:

The word *column* was originally introduced in Grade 2, Module 6 but is treated as new vocabulary in this lesson.

S: (Discuss.)

Problem 2: Interpreting rows and columns in rotated arrays.

MP.7

T: Turn your board so it's vertical. Draw an array that shows 8 equal groups of 2. How many rows of 2 did you draw?

S: 8 rows.

T: How many columns of 8 did you draw?

S: 2 columns.

T: Write an equation to match the array. Don't solve it yet.

S: (Write 8 × 2 = _____.)

T: **Rotate** your board so that it's horizontal. How many rows of 8 do you have now?

S: 2 rows.

T: How many columns of 2?

S: 8 columns.

T: Write an equation to match the array. Don't solve it yet.

S: (Write 2 × 8 = _____.)

T: Explain to your partner using the words *columns* and *rows* why your equation changed.

S: When the array turned, the columns and rows switched. → Columns became rows and rows became columns. They both represent equal groups. It depends on how you look at the array.

T: Will 8 × 2 and 2 × 8 have the same total?

S: Yes!

T: How do you know?

S: They have the same array. → Two groups of 8 and 8 groups of 2 are the same.

Prompt students to skip-count to find the totals of the array in both positions.

Work through the following examples to build vocabulary and understanding of commutativity: 6 × 2 and 2 × 9.

T: When we multiply, changing the order of the factors doesn't change the total. We say the factors are **commutative**. That means they can switch around. Tell your partner what commutative means.

S: It means numbers can switch around. → The factors change places in a multiplication equation, but the total doesn't change. → Addition works the same way.

T: What we've explored today is called the **commutative property**.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Students need not master the words *commutative* or *commutative property* (**3.OA.5**). However, they will need to be familiar with the vocabulary moving forward in this module.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

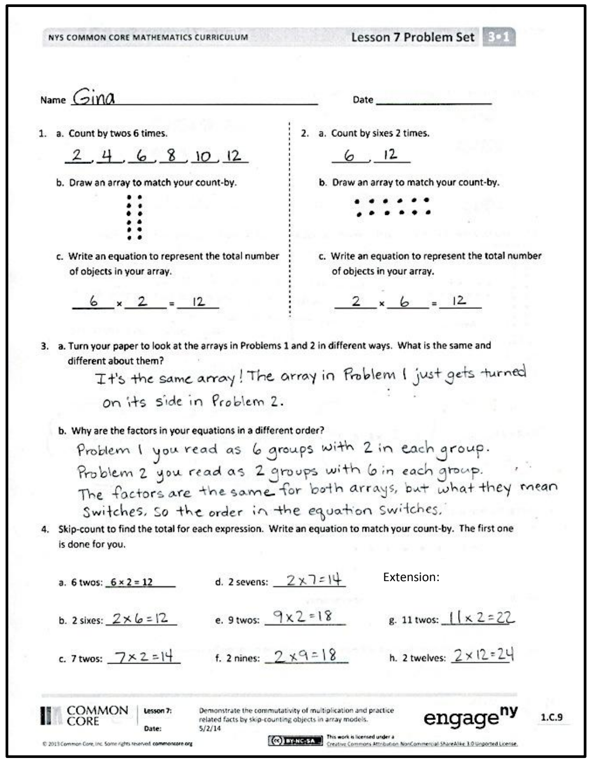
Student Debrief (10 minutes)

Lesson Objective: Demonstrate the commutativity of multiplication, and practice related facts by skip-counting objects in array models.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- Discuss the usefulness of unit counting to solve multiplication problems.
- Build fluency by having students unit count to find the answer to the following expressions without the help of an array. They can keep track of the count using fingers.
  - 3 twos, 2 threes
  - 4 twos, 2 fours
  - 2 eights, 8 twos
  - 2 tens, 10 twos
- Discuss the **commutativity** of multiplication and how it relates to equal groups, **columns**, rows, and **rotating** arrays.
- Relate the **commutative property** of multiplication



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 7 Problem Set 3•1

Name Gina Date

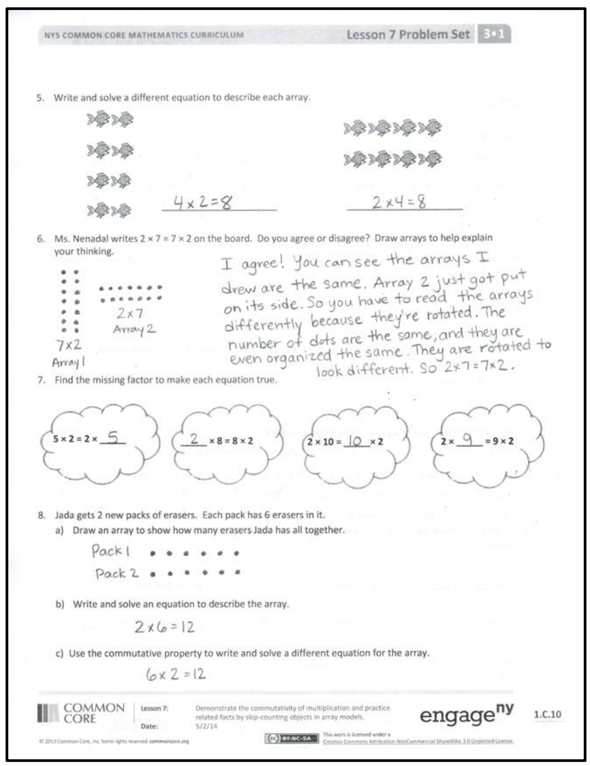
1. a. Count by twos 6 times.
2, 4, 6, 8, 10, 12
b. Draw an array to match your count-by.
c. Write an equation to represent the total number of objects in your array.
6 × 2 = 12

2. a. Count by sixes 2 times.
6, 12
b. Draw an array to match your count-by.
c. Write an equation to represent the total number of objects in your array.
2 × 6 = 12

3. a. Turn your paper to look at the arrays in Problems 1 and 2 in different ways. What is the same and different about them?
It's the same array! The array in Problem 1 just gets turned on its side in Problem 2.
b. Why are the factors in your equations in a different order?
Problem 1 you read as 6 groups with 2 in each group. Problem 2 you read as 2 groups with 6 in each group. The factors are the same for both arrays, but what they mean switches, so the order in the equation switches.

4. Skip-count to find the total for each expression. Write an equation to match your count-by. The first one is done for you.

a. 6 twos: 6 × 2 = 12
b. 2 sixes: 2 × 6 = 12
c. 7 twos: 7 × 2 = 14
d. 2 sevens: 2 × 7 = 14
e. 9 twos: 9 × 2 = 18
f. 2 nines: 2 × 9 = 18
Extension:
g. 11 twos: 11 × 2 = 22
h. 2 twelves: 2 × 12 = 24



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 7 Problem Set 3•1

5. Write and solve a different equation to describe each array.
4 × 2 = 8
2 × 4 = 8

6. Ms. Nenadal writes 2 × 7 = 7 × 2 on the board. Do you agree or disagree? Draw arrays to help explain your thinking.
7x2 Array 1
2x7 Array 2
I agree! You can see the arrays I drew are the same. Array 2 just got put on its side. So you have to read the arrays differently because they're rotated. The number of dots are the same, and they are even organized the same. They are rotated to look different. So 2×7 = 7×2.

7. Find the missing factor to make each equation true.
5 × 2 = 2 × 5
2 × 8 = 8 × 2
2 × 10 = 10 × 2
2 × 9 = 9 × 2

8. Jada gets 2 new packs of erasers. Each pack has 6 erasers in it.
a) Draw an array to show how many erasers Jada has all together.
Pack 1
Pack 2
b) Write and solve an equation to describe the array.
2 × 6 = 12
c) Use the commutative property to write and solve a different equation for the array.
6 × 2 = 12

to the commutative property of addition to help students connect it to prior learning.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

Name ______________________________ Date ________________

1. a. Count by twos 6 times.

   _____, _____, _____, _____, _____, _____

   b. Draw an array to match your count-by.

   c. Write an equation to represent the total number of objects in your array.

   _______ × _______ = _______

2. a. Count by sixes 2 times.

   _______, _______

   b. Draw an array to match your count-by.

   c. Write an equation to represent the total number of objects in your array.

   _______ × _______ = _______

3. a. Turn your paper to look at the arrays in Problems 1 and 2 in different ways. What is the same and what is different about them?

   b. Why are the factors in your equations in a different order?

4. Skip-count to find the total for each expression. Write an equation to match your count-by. The first one is done for you.

   a. 6 twos: $6 \times 2 = 12$

   b. 2 sixes: ____________

   c. 7 twos: ____________

   d. 2 sevens: ____________

   e. 9 twos: ____________

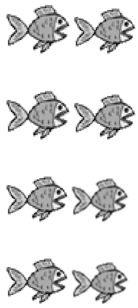
   f. 2 nines: ____________

   Extension:

   g. 11 twos: ____________

   h. 2 twelves: ____________

5. Write and solve a different equation to describe each array.



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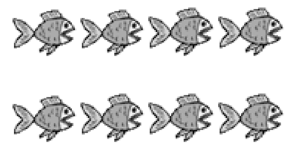

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6. Ms. Nenadal writes 2 × 7 = 7 × 2 on the board. Do you agree or disagree? Draw arrays to help explain your thinking.

7. Find the missing factor to make each equation true.

5 × 2 = 2 × ______

______ × 8 = 8 × 2

2 × 10 = ______ × 2

2 × ______ = 9 × 2

8. Jada gets 2 new packs of erasers. Each pack has 6 erasers in it.
   a. Draw an array to show how many erasers Jada has altogether.

   b. Write and solve an equation to describe the array.

   c. Use the commutative property to write and solve a different equation for the array.

Name ____ Date ____________________

$$\boxed{2 \times 5 = 5 \times 2}$$

Do you agree or disagree with the statement in the box? Draw arrays and use skip-counting to explain your thinking.

Name ____ Date ___________________

1. a. Count by twos 7 times.

   _____, _____, _____, _____, _____ ,_____, _____

   b. Draw an array to match your count-by.

   c. Write an equation to represent the total number of objects in your array.

   _______ × _______ = _______

2. a. Count by sevens 2 times.

   ______, ______

   b. Draw an array to match your count-by.

   c. Write an equation to represent the total number of objects in your array.

   _______ × _______ = _______

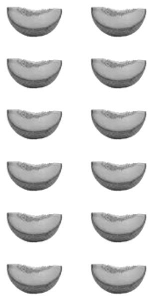
3. a. Turn your paper to look at the arrays in Problems 1 and 2 in different ways. What is the same and what is different about them?

   b. Why are the factors in your equations in a different order?

4. Skip-count to find the total for each expression. Write an equation to match your count-by. The first one is done for you.

   a. 2 twos: $2 \times 2 = 4$
   b. 3 twos: ____________
   c. 2 threes: ____________
   d. 2 fours: ______________
   e. 4 twos: ______________
   f. 5 twos: ______________
   g. 2 fives: ______________
   h. 6 twos: ______________
   i. 2 sixes: ______________

5. Write and solve a different equation to describe each array.



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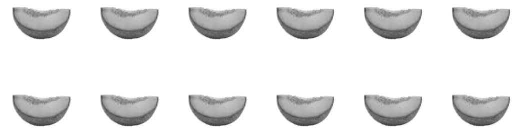

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6. Angel writes 2 × 8 = 8 × 2 in his notebook. Do you agree or disagree? Draw arrays to help explain your thinking.

7. Find the missing factor to make each equation true.

2 × 6 = 6 × ______

______ × 2 = 2 × 7

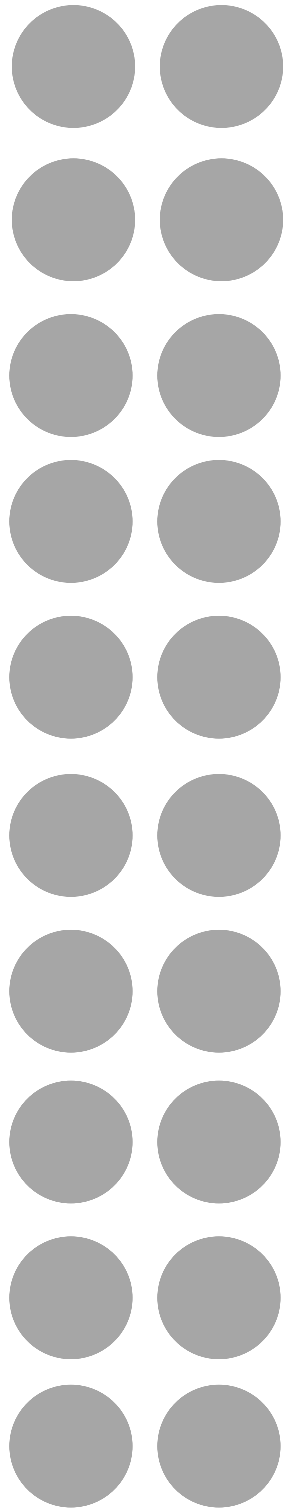
9 × 2 = ______ × 9

2 × ______ = 10 × 2

8. Tamia buys 2 bags of candy. Each bag has 7 pieces of candy in it.
   a. Draw an array to show how many pieces of candy Tamia has altogether.

   b. Write and solve an equation to describe the array.

   c. Use the commutative property to write and solve a different equation for the array.



twos array