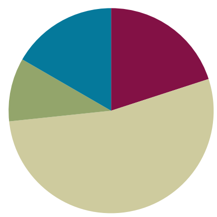
Lesson 9

Objective: Use the area model and division to show the equivalence of two fractions.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(6 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Add and Subtract **4.NBT.4** (4 minutes)
- Find Equivalent Fractions **4.NF.1** (4 minutes)
- Draw Equivalent Fractions **4.NF.1** (4 minutes)

Add and Subtract (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews the year-long Grade 4 fluency standard for adding and subtracting using the standard algorithm.

T: (Write 532 thousands 367 ones.) On your personal white board, write this number in standard form.
S: (Write 532,367.)
T: (Write 423 thousands 142 ones.) Add this number to 532,367 using the standard algorithm.
S: (Write 532,367 + 423,142 = 955,509 using the standard algorithm.)

Continue the process for 671,526 + 264,756.

T: (Write 916 thousands 450 ones.) On your boards, write this number in standard form.
S: (Write 916,450.)
T: (Write 615 thousands 137 ones.) Subtract this number from 916,450 using the standard algorithm.
S: (Write 916,450 – 615,137 = 301,313 using the standard algorithm.)

Continue with the following possible sequence: 762,162 – 335,616 and 500,000 – 358,219.

Find Equivalent Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 7.

T: (Write $\frac{1}{2} = \frac{\ \times\ }{\ \times\ } = \frac{2}{\ }$. Point to $\frac{1}{2}$.) Say the unit fraction.

S: 1 half.

T: On your personal white boards, complete the number sentence to make an equivalent fraction.

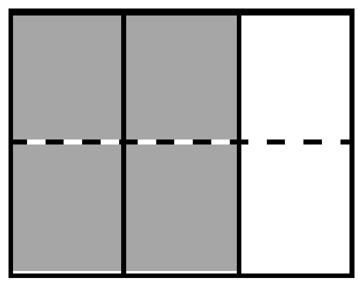
S: (Write $\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$.)

Continue with the following possible suggestions: $\frac{1}{2} = \frac{4}{8}$, $\frac{1}{3} = \frac{2}{6}$, $\frac{1}{3} = \frac{3}{9}$, $\frac{1}{4} = \frac{4}{16}$, and $\frac{1}{5} = \frac{3}{15}$.

Draw Equivalent Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 8.



T: (Write $\frac{2}{3}$.) Say the fraction.

S: 2 thirds.

T: On your personal white boards, draw a model to show $\frac{2}{3}$.

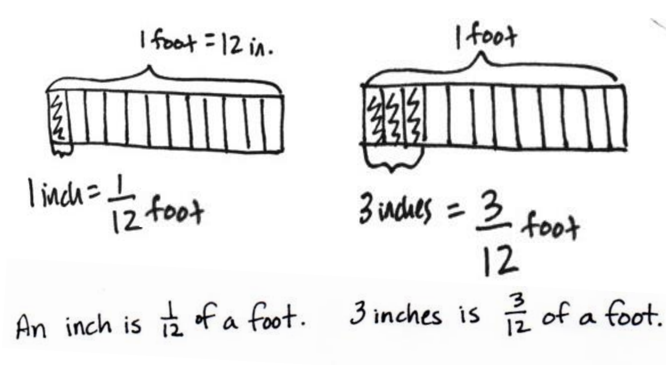
T: (Write $\frac{2}{3} = \frac{\ \times\ }{\ \times\ } = \frac{\ }{6}$.) Draw a dotted horizontal line to find the equivalent fraction. Then, complete the number sentence.

S: (Draw a dotted horizontal line, breaking 3 units into 6 smaller units. Write $\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$.)

Continue with the following possible sequence: $\frac{2}{3} = \frac{\ }{9}$, $\frac{3}{4} = \frac{\ }{12}$, $\frac{3}{5} = \frac{\ }{10}$, and $\frac{4}{5} = \frac{\ }{15}$.

Application Problem (6 minutes)

What fraction of a foot is 1 inch? What fraction of a foot is 3 inches? (Hint: 12 inches = 1 foot.) Draw a tape diagram to model your work.



Note: Students are asked to think about fractions within a context, such as measurement, that will be useful in upcoming word problems. This measurement work will be developed more in Module 7.

Concept Development (32 minutes)

Materials: (S) Personal white board

Problem 1: Simplify $\frac{6}{12}$ by composing larger fractional units using division.

T: (Project area model showing $\frac{6}{12}$.) What fraction does the area model represent?

S: $\frac{6}{12}$.

T: Discuss with a partner. Do you see any fractions equivalent to $\frac{6}{12}$?

S: Half of the area model is shaded. The model shows $\frac{1}{2}$.

T: Which is the larger unit? Twelfths or halves?

S: Halves!

T: Circle the smaller units to make the larger units. Say the equivalent fractions.

S: $\frac{6}{12} = \frac{1}{2}$.

T: (Write $\frac{6 \div 6}{12 \div 6} = \frac{}{}$, and point to the denominator.) Twelve units were in the whole, and we made groups of 6 units. Say a division sentence to record that.

S: $12 \div 6 = 2$.

T: (Record the 2 in the denominator, and point to the numerator.) Six units were selected, and we made a group of 6 units. Say a division sentence to record that.

S: $6 \div 6 = 1$.

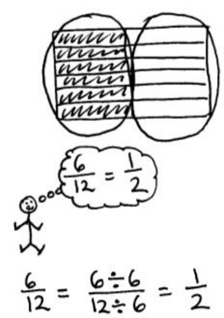
T: (Record the 1 in the numerator.) We write $\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$, dividing both the numerator and denominator by 6 to find an equivalent fraction.

T: What happened to the size of the units and the total number of units?

S: The size of the units got larger. There are fewer units in the whole. → The units are 6 times as large, but the number of units is 6 times less. → The units got larger. The number of units got smaller.

NOTES ON MULTIPLE MEANS OF EXPRESSION:

As the conceptual foundation for simplification is being set, the word *simplify* is initially avoided with students as they compose higher value units. The process is rather referred to as *composition,* the opposite of decomposition, which relates directly to their drawing, work throughout the last two lessons, and work with whole numbers. When working numerically, the process is referred to at times as *renaming,* again in an effort to relate to whole number work.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

English language learners may confuse the terms *decompose* and *compose.*

- Demonstrate that the prefix *de-* can be placed before some words to add an opposite meaning.
- Use gestures to clarify the meanings: *Decompose* is to take apart, and *compose* is to put together.
- Refresh students' memory of decomposition and composition in the context of the operations with whole numbers.

Problem 2: Simplify both $\frac{2}{8}$ and $\frac{3}{12}$ as $\frac{1}{4}$ by composing larger fractional units.

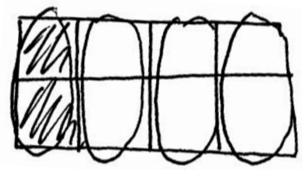
T: Draw an area model to represent $\frac{2}{8}$. Group two units to make larger units.

T: Write $\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{}{}$. How many groups of 2 are shaded?

S: 1!

T: How many groups of 2 are in the whole?

S: 4.



$$\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$$

T: (Write $\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$.) Talk to your partner about how we showed that 2 eighths is the same as 1 fourth. Discuss both the model and our use of division. (Allow students time to discuss.)

T: Draw an area model to represent $\frac{3}{12}$. Compose an equivalent fraction.

S: We can make groups of 2. → No, that won't work. Some of the groups could have shaded and unshaded units. → Groups of 3 will work. That's how many shaded units there are.

T: How many groups of 3 are shaded?

S: 1.

T: How many groups of 3 in all?

S: 4.

T: The new fraction is…?

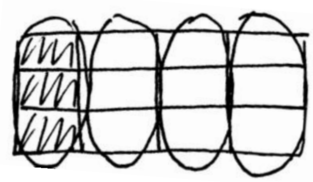
S: $\frac{1}{4}$.

T: Write the number sentence to show that you composed groups of 3.

S: (Write $\frac{3}{12} = \frac{3 \div 3}{12 \div 3} = \frac{1}{4}$.)

T: Compare the area models for $\frac{3}{12}$ and $\frac{2}{8}$.

S: They both equal $\frac{1}{4}$.



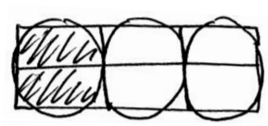
$$\frac{3}{12} = \frac{3 \div 3}{12 \div 3} = \frac{1}{4}$$

Problem 3: Simplify both $\frac{2}{6}$ and $\frac{4}{12}$ as $\frac{1}{3}$ by composing larger fractional units.

T: When we composed fractions in the last two problems, what did you notice?

S: We divided to find equivalent fractions. → We made equal groups to make large units. → We composed a unit fraction from a non-unit fraction.

T: Draw area models to show $\frac{2}{6}$ and $\frac{4}{12}$. Rename both fractions as the same unit fraction.



$$\frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$$

$$\frac{4}{12} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$$

S: I can make groups of 2 in both area models. I could make groups of 3, but I won't make equal groups of shaded and unshaded units. → Four is a factor of both 4 and 12, so I can make groups of 4. → First, I made groups of 2 when I was working with 4 twelfths, but then I noticed I could make groups of 2 again. → Hey, dividing by 2 twice is the same as dividing by 4.

T: Circle the groups, and express each composition in a number sentence using division.

S: $\frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$. $\frac{4}{12} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$.

T: How are $\frac{4}{12}$ and $\frac{2}{6}$ related?

S: When I model $\frac{4}{12}$ and $\frac{2}{6}$, I see that they both have the same area as $\frac{1}{3}$. → $\frac{1}{3} = \frac{4}{12} = \frac{2}{6}$. → The equivalent fraction for $\frac{4}{12}$ and $\frac{2}{6}$ with the largest units is $\frac{1}{3}$. → We composed $\frac{4}{12}$ and $\frac{2}{6}$ into the same unit fraction.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Challenge students working above grade level and others to couple the expressions of fraction composition with the related multiplication expression of decomposition (e.g., $\frac{4}{12} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$ and $\frac{1}{3} = \frac{1 \times 4}{3 \times 4} = \frac{4}{12}$).

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

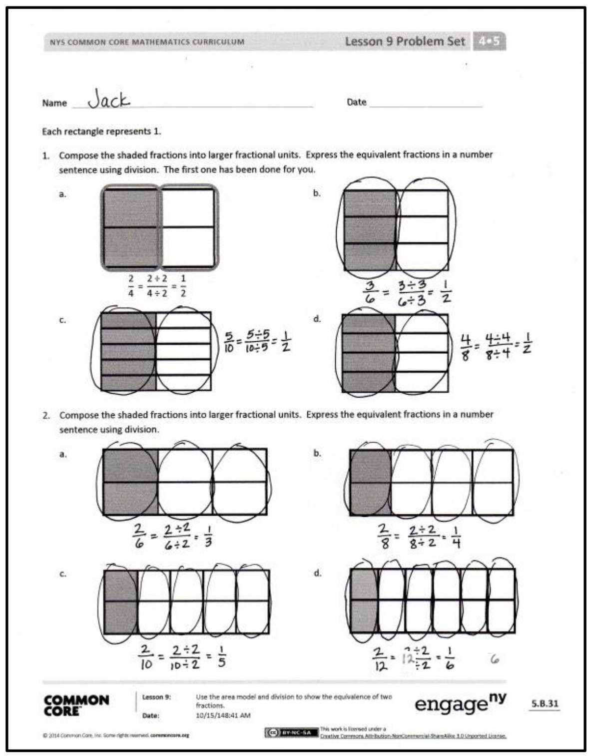
Student Debrief (10 minutes)

Lesson Objective: Use the area model and division to show the equivalence of two fractions.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 9 Problem Set 4•5

Name Jack Date

Each rectangle represents 1.

1. Compose the shaded fractions into larger fractional units. Express the equivalent fractions in a number sentence using division. The first one has been done for you.

a. $\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$

b. $\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$

c. $\frac{5}{10} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2}$

d. $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$

2. Compose the shaded fractions into larger fractional units. Express the equivalent fractions in a number sentence using division.

a. $\frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$

b. $\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$

c. $\frac{2}{10} = \frac{2 \div 2}{10 \div 2} = \frac{1}{5}$

d. $\frac{2}{12} = \frac{2 \div 2}{12 \div 2} = \frac{1}{6}$ 6

COMMON CORE Lesson 9: Use the area model and division to show the equivalence of two fractions. Date: 10/15/148:41 AM engage ny 5.B.31

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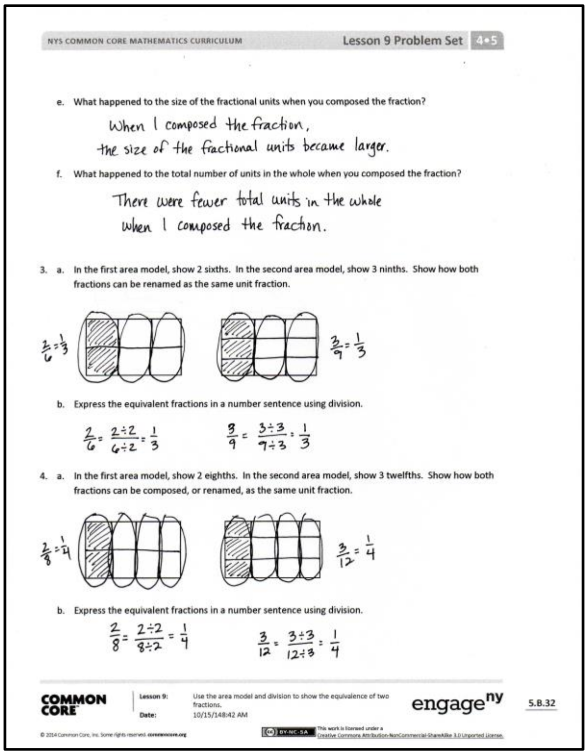
- Look at Problem 1(a–d). Write some examples of fractions where the denominator is a multiple of the numerator. (Pause.) What do we know about these fractions?
- In Problems 3 and 4, does it matter how your area models are shaded? Will it still result in a correct answer?
- Explain how two fractions can be composed into the same larger unit fraction.

- How can what you know about factors help rename a fraction in larger units?
- When we rename $\frac{3}{12}$ as $\frac{1}{4}$, why is it helpful to think about the factors of 3 and 12?
- Contrast the following: renaming fractions when you multiply versus when you divide and decomposing versus composing fractions. For each, discuss what happens to the size of the units and number of units.
- Use what you learned today to determine if $\frac{3}{8}$ can be renamed as a larger unit. Why or why not?


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 9 Problem Set 4•5

e. What happened to the size of the fractional units when you composed the fraction?

When I composed the fraction, the size of the fractional units became larger.

f. What happened to the total number of units in the whole when you composed the fraction?

There were fewer total units in the whole when I composed the fraction.

3. a. In the first area model, show 2 sixths. In the second area model, show 3 ninths. Show how both fractions can be renamed as the same unit fraction.

$\frac{2}{6}=\frac{1}{3}$ $\frac{3}{9}=\frac{1}{3}$

b. Express the equivalent fractions in a number sentence using division.

$\frac{2}{6}=\frac{2\div2}{6\div2}=\frac{1}{3}$ $\frac{3}{9}=\frac{3\div3}{9\div3}=\frac{1}{3}$

4. a. In the first area model, show 2 eighths. In the second area model, show 3 twelfths. Show how both fractions can be composed, or renamed, as the same unit fraction.

$\frac{2}{8}=\frac{1}{4}$ $\frac{3}{12}=\frac{1}{4}$

b. Express the equivalent fractions in a number sentence using division.

$\frac{2}{8}=\frac{2\div2}{8\div2}=\frac{1}{4}$ $\frac{3}{12}=\frac{3\div3}{12\div3}=\frac{1}{4}$

COMMON CORE Lesson 9: Use the area model and division to show the equivalence of two fractions. Date: 10/15/148:42 AM engage ny 5.B.32

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Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name ____ Date ____________________

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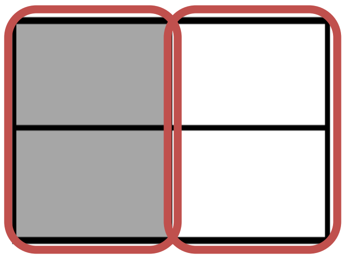
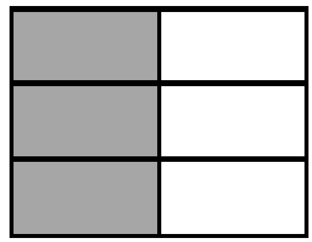
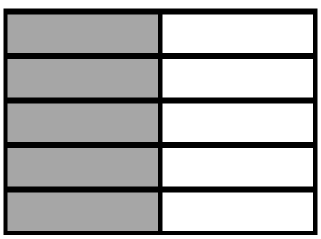
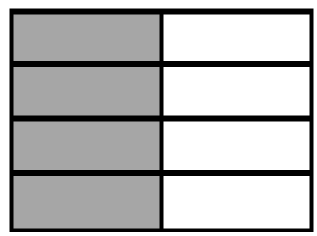
1. Compose the shaded fractions into larger fractional units. Express the equivalent fractions in a number sentence using division. The first one has been done for you.

a. 

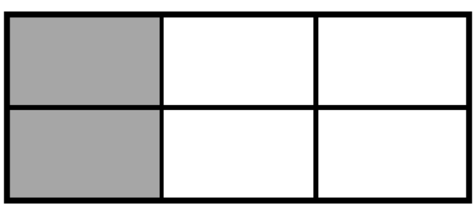
$$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

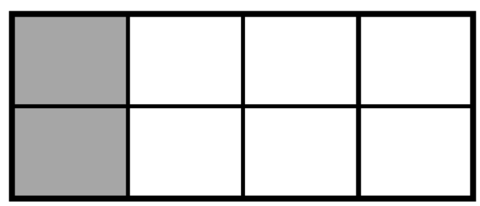
b. 

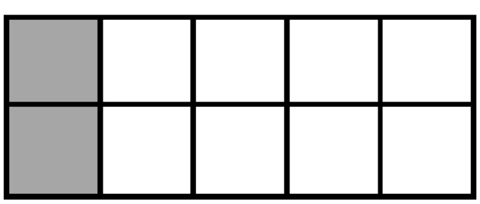
c. 

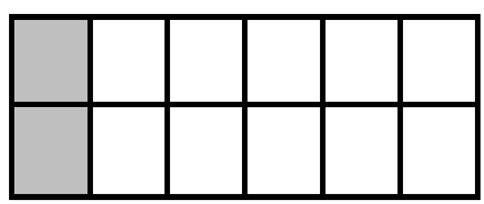
d. 

2. Compose the shaded fractions into larger fractional units. Express the equivalent fractions in a number sentence using division.

a. 

b. 

c. 

d. 

e. What happened to the size of the fractional units when you composed the fraction?

f. What happened to the total number of units in the whole when you composed the fraction?

3. a. In the first area model, show 2 sixths. In the second area model, show 3 ninths. Show how both fractions can be renamed as the same unit fraction.

b. Express the equivalent fractions in a number sentence using division.

4. a. In the first area model, show 2 eighths. In the second area model, show 3 twelfths. Show how both fractions can be composed, or renamed, as the same unit fraction.

b. Express the equivalent fractions in a number sentence using division.

Name ____ Date ______________________

a. In the first area model, show 2 sixths. In the second area model, show 4 twelfths. Show how both fractions can be composed, or renamed, as the same unit fraction.

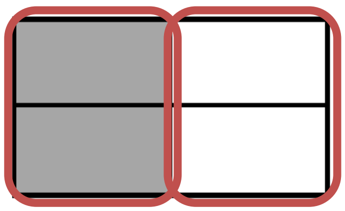
b. Express the equivalent fractions in a number sentence using division.

Name ____ Date _____________________

Each rectangle represents 1.

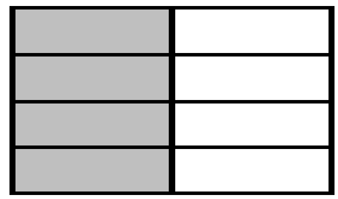
1. Compose the shaded fractions into larger fractional units. Express the equivalent fractions in a number sentence using division. The first one has been done for you.

a.



$$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

b.



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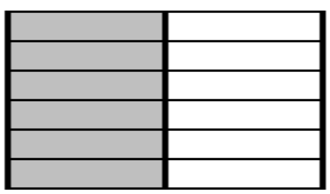

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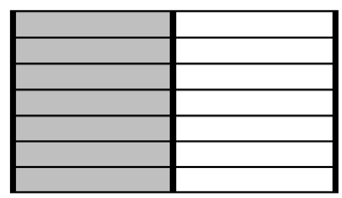
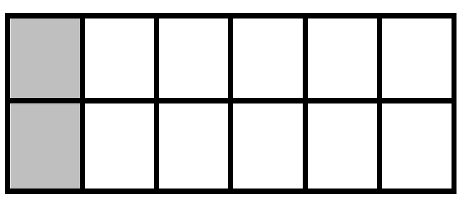


2. Compose the shaded fractions into larger fractional units. Express the equivalent fractions in a number sentence using division.

a.



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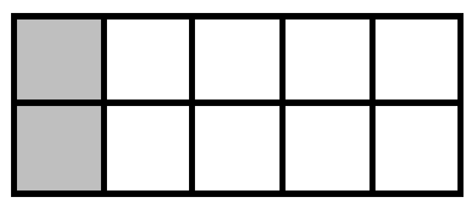

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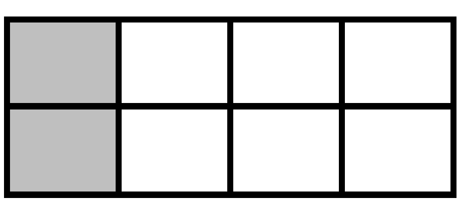

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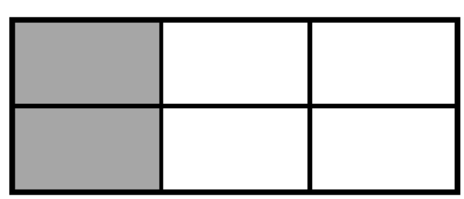

e. What happened to the size of the fractional units when you composed the fraction?

f. What happened to the total number of units in the whole when you composed the fraction?

3. a. In the first area model, show 4 eighths. In the second area model, show 6 twelfths. Show how both fractions can be composed, or renamed, as the same unit fraction.

b. Express the equivalent fractions in a number sentence using division.

4. a. In the first area model, show 4 eighths. In the second area model, show 8 sixteenths. Show how both fractions can be composed, or renamed, as the same unit fraction.

b. Express the equivalent fractions in a number sentence using division.