ENHANCE INSTRUCTION

Middle School Math Fluency Guide

Understanding the role of procedural proficiency in a standards-aligned curriculum is an important step in understanding the design and facilitation of instruction that support students' deep, authentic command of mathematics. In this UnboundEd Fluency Guide for Middle School Mathematics, we will address key aspects of procedural fluency as a critical component of mathematical achievement.[1]

Part 1: What is Procedural Fluency?

Procedural fluency is more than memorizing facts or the ability to use one specific procedure well in a given context. Procedural fluency is the skill of carrying out procedures flexibly, accurately, efficiently, and appropriately (NRC, 2001).

- Flexibility in carrying out procedures means that students know more than one approach, choose appropriate strategy, and can use one method to solve and another method to double-check
- Accuracy in carrying out procedures means that students can reliably produce the correct answer
- Efficiency in carrying out procedures means that students can carry out problem solutions easily, keep track of sub-problems, and make use of intermediate results to solve the problem
- Appropriateness in carrying out procedures means that students know when to apply a particular procedure NCTM (2014).

This does not mean that students do not have to memorize basic facts. Rather, it means that students do so as an integral part of understanding mathematical concepts and have enough practice with the procedures to be able to recall mathematical facts from memory.

How do I organize instruction to support procedural fluency?

Before we focus on building speed and accuracy, it is important to provide students with intentional practice. Developing fluency requires activities that provide more than extensive practice on a singular way of solving problems. Practice should be distributed over time and anchored on multiple representations that require students to make sense of procedures that exemplify the concepts being learned.

In order to develop students' procedural fluency, it is essential that students understand the mathematical concepts and use this understanding to determine why procedures being used to solve a mathematical task works. As students learn procedures with conceptual understanding, they build a deeper and more authentic command of mathematics. Learning with understanding is more powerful than simply memorizing because conceptual knowledge improves retention, promotes fluency, and facilitates learning related material (National Research Council, 2001, p.118). In other words, to build procedural fluency teachers should focus on developing students' conceptual understanding prior to engaging in activities that build fluency (NRC, 2001, 2005, 2012; NCTM, 2014). Therefore, designing rigorous and coherent

[1] National Council of Teachers of Mathematics (n.d.).

mathematics instruction depends on the teacher's ability to propose, sequence, and facilitate learning experiences that strike the right balance between conceptual understanding, procedural fluency, and application.

Rote memorization devoid of conceptual understanding decreases student motivation (Hiebert, 1999).

For an example of the role conceptual understanding plays in explaining why computational procedures with fraction operations work, consider this Doing What Works video[2]. For more details on developing conceptual understanding, consult one of our grade-level content guides.

How do I Build Procedural Fluency from Conceptual Understanding?

Building procedural fluency from conceptual understanding requires sequencing instruction in a manner that allows students to connect the procedures, strategies, and approaches used in problem solving to the concept being learned. By constantly examining procedures, students should be able to explain why a standard algorithm works and realize that algorithms represent efficient procedures for solving a problem. The National Council of Teachers of Mathematics[3] outlines a series of actions teacher and students should be able to engage in to build student procedural fluency. Figure 1, below, summarizes these actions.

Teacher Actions	Student Actions
• Providing students with opportunities to use their own reasoning strategies and methods for solving problems • Asking students to discuss and explain why the procedures that they are using work to solve particular problems • Connecting student-generated strategies and methods to more efficient procedures as appropriate • Using visual models to support students' understanding of general methods • Providing students with opportunities for distributed practice of procedures	• Making sure that they understand and can explain the mathematical basis for the procedures that they are using • Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems • Determining whether specific approaches generalize to a broad class of problems • Striving to use procedures appropriately and efficiently

Figure 1: Teacher and Student Actions to Build Fluency from Conceptual Understanding

Although conceptual knowledge is an essential foundation, procedural knowledge is important in its own right. All students need to have a deep and flexible knowledge of a variety of procedures, along with the ability to make critical judgments about which procedures or strategies are appropriate for use in particular situations (NRC, 2001, 2005, 2012; Star, 2005). "The truth is that in mathematics, skills and understanding are completely intertwined. In most cases, the precision and fluency in the execution of the skills are the requisite vehicles to convey the conceptual understanding" (Wu, 1999, p.1).

2 WestEd (1995-2016).

3 National Council of Teachers of Mathematics (2014).

Part 2: Leveraging the Coherence Shift to Build Procedural Fluency

The standards[4] uses specific language to identify concepts in which students must become fluent. Whereas in elementary school students are expected to build fluency around math facts; in middle school the fluency expectations expand to include multi-step procedures and algorithms. From kindergarten to grade 6, the authors of the standards systematically used the terms "fluent" and "fluently" to indicate specific concepts and skills in which students need to become fluent. In the middle school curriculum, the standards calling for computational fluency are limited to grades 6 and 7. In grade 6, the words fluent and fluently identify multi-digit division and operations with fractions and decimals. For example, standard 6.NS.B.3, calls for students to "(f)luently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation". For an example of how this fluency has been operationalized, consider this task[5].

In grade 7, consider standard 7.EE.B.4.A, where students are called to "[s]olve equations ... fluently" – which involves using properties of equality to reason through the writing of equivalent forms of the given equation until the student can determine a value of the variable (if it exists). As students go through this process efficiently, they are engaged in more than recall.

Other standards that call for students to develop procedural fluency in middle school mathematics exist in the curriculum but, unlike 7.EE.B.4.A, they are no longer stated in terms of fluency. In grade 8, standard 8.8.EE.C.7 calls for fluency with solving linear equations in one variable, which is the bedrock of the work with algebra. We rely on the coherence and focus shifts, as well as on the recommendations regarding fluency and culminating standards from the Partnership for Assessment of Readiness for College and Careers Model Content Framework[6], to carefully analyze the progression of the standards and identify standards requiring procedural fluency development. Another key consideration was the impact that the lack of procedural fluency development would have on students' ability to successfully proceed in their learning trajectory through Middle School and beyond. Table I summarizes the concepts and standards in which students should develop procedural fluency in grades 6 through 8.

Grade	Concepts	Standards
6	Multi-digit Division Multi-digit Decimal Operation Divide fractions	6.NS.B.2 – Fluently divide multi-digit numbers using the standard algorithm. 6.NS.B.3 – Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. 6.NS.A.1 – Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

[4] CCSSO (2010)
[5] Illustrative Mathematics (2015)
[6] PARCC (2014)

7	Add, subtract, multiply, and divide rational numbers in any form Solve equations of the form *px + q = r* <u>or</u> *p(x + q) = r*	7.NS.A.1 – Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. 7.NS.A.2 – Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. 7.EE.A.1 – Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. 7.EE.B.3 – Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. 7.EE.B.4 – Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
8	Solve general one-variable linear equations, including rational coefficients and constants Solve problems involving volume of cones, cylinders, and spheres	8.EE.C.7 – Solve linear equations in one variable. 8.G.C.9 – Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Table I – Middle School Mathematics Fluency Expectations (PARCC, 2014)

Part 3: What Strategies May I Use to Build Procedural Fluency?

The most important step in implementing procedural-fluency building routines is to identify and map out the concepts and skills students will develop over a period of time. It is also critical that students do not engage in fluency building activities on concepts for which they have not yet developed sufficient understanding. Only after students have developed understanding of a concept through multiple opportunities to explore different representations and to explain why solution strategies work; should they engage in activities aimed exclusively at building procedural fluency. In order to support your efforts in this area, below we discuss strategies that support students' procedural fluency development.

Rapid White Board Exercises (RWBE)

Implementing RWBE requires material resources: namely a whiteboard and dry erase marker for each student. RWBE supports the development of procedural fluency by providing students a chance to receive immediate feedback on their work and by providing teachers multiple opportunities to assess the whole class' progress toward concept development and procedural fluency.

The implementation of RWBE consists of a series of 10 to 20 tasks on a concept, with the tasks sequenced in increasing order of difficulty. For each task, the teacher facilitates the engagement with each task using the same general steps outlined below:

- The teacher displays and reads one task at a time while students follow by reading and listening. Students only start their work after the teacher says "Go";
- As students complete their work, or upon the teacher's signal, students hold up their work toward the teacher, who proceeds to provide immediate feedback to each student on the accuracy of the answer;
- While response to correct answers need not be more elaborate than "*Yes!*", or "*Correct!*", specific feedback to incorrect answers is critical to support student conceptual development. Specific feedback should be offered to help students identify the error and regroup for the next task in the sequence. such as "*Check your multiplication facts.*" or "*Is this the correct least common denominator?*" should help students identify the error and regroup in time for the next task.

Besides special attention to the quality of feedback offered to incorrect answers, teachers should also refrain from the urge to reteach. Instead, use this opportunity to determine if students would benefit from additional work to strengthen their conceptual development. Resources to implement the RWBE strategy (as well as Sprints) can be found at the Engage NY site. Alternatives to commercially available white boards can be created using transparent sheet protectors.

Number Talks

As a fluency building strategy, Number Talks[7] are powerful because they rely on students' use of number relationships and the structure of numbers to perform mental computations in a way that makes sense to them. In this sense, Number Talks also offer opportunities for students to develop skill with the Practice Standards. The low hanging fruit in the set of Standards for Mathematical Practice (SMP) is students' ability to construct viable arguments and critique the reasoning of others (SMP 3). However, other Number Talks also afford students opportunities to develop skills in other Practices. For instance, as students attend to the meaning of quantities, they develop their ability to reason abstractly and quantitatively (SMP 2) while becoming proficient in not just how to compute but also in knowing and flexibly using different properties of operations. Similarly, students attend to precision (SMP 6) as they communicate their solutions precisely to others and strive to calculate accurately and efficiently. In executing these calculations using different properties to regroup numbers, students look for and make use of structure (SMP 7)[8].

It is an instructional strategy that requires no more than 10 to 15 minutes 3 to 4 times a week to produce results over time. There are many ways to implement a Number Talks routine. The key element in the effective implementation of Number Talks revolve around engaging students in reasoning with numbers. This is a significant role reversal in the math classroom that asks students to

- *figure something out rather than be told the steps to follow*

---

[7] Humphreys, C. & Parker R. (2015).
[8] CCSSO (2010)

- *explain what they think rather than wait for the teacher to explain*
- *explain why rather than just know how*
- *test new ideas and embrace mistakes as part of the process*
- *see wrong answers and opportunities to learn something new.*

Even though there are many different ways of implementing a Number Talks routine, they generally proceed through four general steps: Think, Share, Explain, and Question. Figure I offers greater details on how to implement these steps.

Think

Students gather around the common area.

No paper & pencil needed.

Teacher writes a problem on the board – horizontally.

Share

Students show thumb up once they have an answer – additional fingers if there is more than one solution path.

Teacher calls on 4-5 students to share answers only; writes all answers on the board.

Teacher offers no indication of whether the answer shared is correct or incorrect.

Explain

Teacher selects a student to explain his/her solution and charts student thinking on the board exactly, even if there are errors. Don't try to save them.

Teacher takes time to name the strategy used.

Students use hand signals to indicate if the use of same (hang-ten) or different strategies (thumbs up).

Question

Students question each other about their thinking or the strategy used (i.e.: How did you know...?; Why did you...?; Can you explain why...; What if...?).

Teacher prompts students to compare strategies.

Teacher repeats the process with another answer or another problem.

Figure I: Routine to Implement Number Talks

Notice that in the Share step, it is important for teachers to give students enough time to think in order to dispel the prevalent notion that being good at math means being fast. Teachers should use the speed with which thumbs come up as an indication of a problem's difficulty and may encourage students who have already figured out the solution to the proposed problem to solve it using a second or third way. As students indicate the number of different solutions they have been able to come up using hand signals, they do not interfere with the thinking of others.

When facilitating the Explain step of the Number Talks, teachers select students to share and explain solution strategies and explain their reasoning while keeping an accurate record for the class. Without hinting at the accuracy of any of the proposed solutions, teachers may ask questions that invite students to share their thinking, such as:

- *Who has a strategy she or he is willing to share?*
- *Is anyone willing to convince us that your answer makes by telling us what you did?*

As teachers record students' thinking, it is important for them to remain neutral to avoid suggesting that there is "one right way" of solving the problem.

When facilitating the Question step, the goal is the help students communicate more clearly and/or to emphasize key elements of a student's strategy. Questions such as:

- *How did you know … ?*
- *Why did you … ?* or *Can you explain why … ?*
- *What if … ?*
- *Does anyone have a question for ___________ ?*
- *Can someone explain ___________'s strategy in your own words?*
- *Can you say more about … ?*
- *What connections do you notice among the strategies we have discussed?*

These questions may help make student reasoning visible not only to the student who is sharing but also to all others participating in the Number Talk.

It is important to realize that there is no hard and fast rule about how to conduct Number Talks. As the teacher facilitating the activity, you will have to make the call about what is best for your classroom reality. Having a plan that includes not only the rotation of problems and skills but also what to say when your Number Talk session does not naturally end after 15 minutes – and there are still students who would like to share their strategies – is part of creating a nurturing, high-functioning classroom.

Sprints

Sprints are fluency exercises that consist of two parts of related exercise sets that students complete within a minute. The goal is to motivate students to do better on the second part of the related set than he/she did in the initial one. The problems are organized in such a manner that a set of two matched tasks presents practice that progress from simple to more challenging and have the same level of difficulty between the two sets. The fast-paced structure of Sprints requires the use of problems whose solutions are suited to be accomplished within the limited space and time allotted.

An important characteristic of the implementation of Sprints is its promotion of student motivation. The goal is to have students improve from the first set to the second and not to determine who is the best in the group. Students are striving to do the best they can be compared to what they have just achieved. Therefore, it is important to acknowledge and celebrate every improvement, even if by only one item. For how to implement this strategy, we strongly encourage you to check out this video and this document with problem sets aligned with the fluency expectations for middle school mathematics.

Definition:	Facts:
Ability to carry out procedures accurately, efficiently, flexibly, and appropriately	Builds from Conceptual Understanding Connects Concept & Procedure Builds Over Time

Procedural Fluency

Examples:	Non-examples:
Explaining & Visualizing Modeling Multiple Solutions Number Talks, Sprints, Rapid White Board Exercises Sprints	Rote Memorization Dittos

Figure II: Developing Procedural Fluency in Middle School

Other considerations

The activities highlighted in this guide allow teachers to implement procedural fluency development strategies as teacher-led, whole- or small-group setting, for short periods of time (8 to 15 minutes) daily. Please notice that when sequencing fluency activities, RWBE should be implemented before Number Talks or Sprints. RWBE allow teachers to provide immediate feedback to students on an item-by-item basis, it is better suited to address performance issues; whereas Sprints are better suited to building speed and accuracy.

Technology solutions exist that could promote procedural fluency on a more individualized basis. These technology solutions, however, are better suited for a different tier of intervention. The review of these technology solutions goes beyond the scope of this guide. Nonetheless, it is important to highlight that either as a teacher-led or technological solution, the development of procedural proficiency needs to offer instruction starting from the development of conceptual understanding if they aim at procedural fluency building to foster long-term student success.

References

Hiebert, J. (1999). Relationships between research and the NCTM standards. Journal for Research in Mathematics Education, 30(1), 3–19.

Humphreys, C. & Parker R. (2015). Making Number Talks matter: Developing mathematical practices and deepening understanding, grades 4-10. Portland, ME: Stenhouse Publishers

National Council of Teachers of Mathematics (2014). Principles to Actions: Ensuring mathematical success for all. Reston, VA: Author.

National Council of Teachers of Mathematics (n.d.). Procedural Fluency in Mathematics. Available online at http://www.nctm.org/Standards-and-Positions/Position-Statements/Procedural-Fluency-in-Mathematics/

National Governors Association Center for Best Practices, Council of Chief State School Officers (CCSSO). (2010). Common Core State Standards for Mathematics. Washington, DC: Author. Available online at http://www.corestandards.org/Math/

National Research Council (2001). Adding it up: Helping children learn mathematics. Washington, DC: National Academies Press.

National Research Council (2005). How students learn: History, mathematics, and science in the classroom. Washington, DC: National Academies Press.

National Research Council (2012). Education for life and work: Developing transferable knowledge and skills for the 21st century. Washington, DC: National Academies Press.

Partnership for Assessment of Readiness for College and Careers (2014). PARCC Model Content Framework. Mathematics Grades 3-11. Version 4.0. Retrieved from http://www.parcconline.org/resources/educator-resources/model-content-frameworks.

Star, J. R. (2005). Reconceptualizing conceptual knowledge. Journal for Research in Mathematics Education, 36(5), 404–411.

WestEd (1995-2016). Making sense of computational procedures. Available online at https://dwwlibrary.wested.org/resources/1059

Wu, H. (1999). Basic skills versus conceptual understanding: A bogus dichotomy in mathematics education. American Educator, Fall 1999, pp. 1-7.