

Name _____

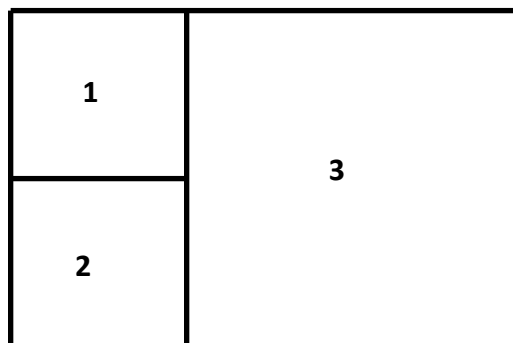
Date _____

1. A rectangle with positive area has length represented by the expression $3x^2 + 5x - 8$ and width by $2x^2 + 6x$. Write expressions in terms of x for the perimeter and area of the rectangle. Give your answers in standard polynomial form and show your work.
 - a. Perimeter:
 - b. Area:
 - c. Are both your answers polynomials? Explain why or why not for each.
 - d. Is it possible for the perimeter of the rectangle to be 16 units? If so, what value(s) of x will work? Use mathematical reasoning to explain how you know you are correct.

- e. For what value(s) of the domain will the area equal zero?
- f. The problem states that the area of the rectangle is positive. Find and check two positive domain values that will produce a positive area.
- g. Is it possible that negative domain values could produce a positive function value (area)? Explain why or why not in the context of the problem.

2. A father divided his land so that he could give each of his two sons a plot of his own and keep a larger plot for himself. The sons' plots are represented by squares 1 and 2 in the figure below. All three shapes are squares. The area of square 1 equals that of square 2, and each can be represented by the expression $4x^2 - 8x + 4$.

- a. Find the side length of the father's plot, which is square 3, and show or explain how you found it.



- b. Find the area of the father's plot, and show or explain how you found it.

- c. Find the total area of all three plots by adding the three areas, and verify your answer by multiplying the outside dimensions. Show your work.

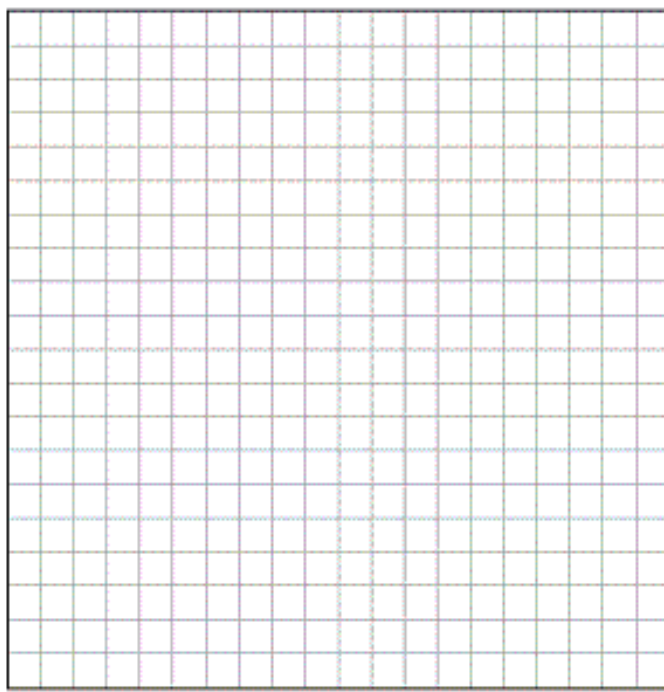
3. The baseball team pitcher was asked to participate in a demonstration for his math class. He took a baseball to the edge of the roof of the school building and threw it up into the air at a slight angle so that the ball eventually fell all the way to the ground. The class determined that the motion of the ball from the time it was thrown could be modeled closely by the function

$$h(t) = -16t^2 + 64t + 80,$$

where $h(t)$ represents the height of the ball in feet after t seconds.

- a. Determine whether the function has a maximum value or a minimum value. Explain your answer mathematically.
- b. Find the maximum or minimum value of the function. After how many seconds did the ball reach this value? Show how you found your answers.
- c. For what interval of the domain is the function increasing (i.e., ball going up)? For what interval of the domain is the function decreasing (i.e., ball going down)? Explain how you know.
- d. Evaluate $h(0)$. What does this value tell you? Explain in the context of the problem.

- e. How long is the ball in the air? Explain your answer.
- f. State the domain of the function, and explain the restrictions on the domain based on the context of the problem.
- g. Graph the function indicating the vertex, axis of symmetry, intercepts, and the point representing the ball's maximum or minimum height. Label your axes using appropriate scales. Explain how your answer to part (d) is demonstrated in your graph.



- h. Does your graph illustrate the actual trajectory of the ball through the air as we see it?

A Progression Toward Mastery

Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	a–b A-APR.A.1	Student shows little evidence of understanding the properties of polynomial operations. <u>OR</u> Student shows little or no attempt to complete the problems, or the attempt is aborted before calculations are completed.	Student shows some evidence of understanding operations with polynomials, but there are errors in the calculations (e.g., side lengths are not doubled for perimeter, or the product is missing terms). The final answers are not given in standard polynomial form.	Student provides expressions that are treated accurately and appropriately with operations relating to perimeter and area that are carried out correctly. However, the final answers are not given in standard polynomial form.	Student provides expressions that are treated accurately and appropriately with operations relating to perimeter and area that are carried out correctly. Final answers are in simplest and standard polynomial form.
	c A-APR.A.1	Student makes no attempt to answer the question.	Student makes an attempt to explain, but the explanation shows little understanding of the definition of a polynomial and of the concept of closure for polynomial operations.	Student provides an explanation that shows some understanding of the definition of a polynomial or of the concept of closure for polynomial operations.	Student provides an explanation that is correct and includes understanding of the definition of a polynomial and of closure for polynomial operations.

	d A-SSE.B.3a A-CED.A.1 A-REI.D.11	<p>Student provides an answer (e.g., yes or no) with no supporting explanation.</p> <p><u>OR</u></p> <p>Student makes no attempt to answer the question.</p>	<p>Student creates and solves an equation and provides a logical explanation for the process and solution. However, the equation may be incorrect, or there are errors in calculation that lead to an incorrect final answer and incorrect values for x.</p>	<p>Student creates and solves a correct equation and provides a logical explanation for the process and solution. However, both values of x are given as the final solution with some explanation offered as justification.</p>	<p>Student creates and solves a correct equation and provides a logical explanation for the process and solution, which includes only the correct value of x that works in the equation, with the extraneous solution noted in the explanation.</p>
	e A-SSE.B.3a A-REI.D.11	<p>Student makes an attempt to answer this question, but the original equation representing the area is not factored correctly, and no correct results are found.</p> <p><u>OR</u></p> <p>Student makes no attempt to answer the question.</p>	<p>Student makes an attempt to factor the original form of the equation representing the area and sets it equal to zero. There is one (or no) correct result given.</p>	<p>Student makes an attempt to factor the original form of the equation representing the area and sets it equal to zero. However, only two correct results are given.</p>	<p>Student accurately factors the equation into its four linear factors and sets them equal to zero.</p> <p><u>AND</u></p> <p>Student correctly solves for the four values of x that make the product equal to zero.</p>
	f A-SSE.B.3a A-CED.A.1 A-REI.D.11	<p>Student makes no attempt to find the two values, or the attempt is aborted before a conclusion is reached.</p>	<p>Student only provides one correct value and checks it effectively.</p> <p><u>OR</u></p> <p>Student provides two values, but only one is checked effectively.</p> <p><u>OR</u></p> <p>Student provides two logically selected values, but the checks attempted are ineffective for both.</p>	<p>Student correctly selects two values and substitutes them into the equation. There are calculation errors in the check that do not affect the final outcome.</p>	<p>Student correctly selects two values and substitutes them into the equation to check whether the x-value produces a positive area.</p> <p>(Note: The zeros found in part (e) might be used as boundaries for the correct values in this part.)</p>

	g A-SSE.B.3a A-APR.A.1 A-REI.D.11	Student makes little or no attempt to answer the question.	Student attempts to answer the question; however, the explanation is missing important parts. For example, there are no references to the dimensions being positive or to the requirement that there must be an even number of negative factors for area. There might be specific examples of negative values that produce a positive area given, but they are without explanation.	Student correctly answers the question but only provides a partially correct explanation. For example, the explanation does not mention the need for an even number of negative factors in the area expression or that both dimensions must be positive (i.e., if two factors are negative, they must both represent the same dimension).	Student correctly answers the question, including the following requirements: there are references in the explanation to the need for positive dimensions and that if an x -value makes any of the factors negative, there must be an even number of negative factors. This means that both negative factors must be for the same dimension.
2	a–b A-SSE.A.1 A-SSE.A.2 A-SSE.B.3 A-APR.A.1	Student provides an answer with no evidence to indicate a connection is made between the information given in the prompt and the side lengths of squares 1 and 2.	Student provides an answer with evidence to indicate a connection is made between the information in the prompt and the side lengths of squares 1 and 2. However, there is no evidence that a connection is made to the side length of square 3 and the operations needed to answer the questions. Calculations contain errors, and the explanation is missing or inadequate.	Student provides an answer with evidence of understanding the connection between the information given in the prompt and the side length and area of square 3. Calculations are completed accurately, but the explanations are incomplete. <u>OR</u> Student calculations contain errors, but the explanation is adequate and is not dependent on errors in the calculations.	Student provides an answer with evidence of understanding the connection between the information given in the prompt and the side length and area of square 3. Calculations are completed accurately, and the explanations are complete. (Note: Equivalent forms of the solution are acceptable, e.g., $2(2x - 2) = 4x - 4$; $(4x - 4)^2 = 16x^2 - 32x + 16$.)
	c A-SSE.A.1 A-SSE.A.2 A-SSE.B.3 A-APR.A.1	Student makes little or no attempt to find the area using either method.	Student makes an attempt to find the total area by adding the three smaller areas, but there are errors and verification is impossible. Work is shown.	Student correctly determines the total area by adding the three smaller areas, but there is either no attempt to check by multiplying or there are errors in the attempt to check by multiplying. Work is shown and supports the correct results.	Student correctly determines the total area by adding the three smaller areas and correctly verifies the solution by multiplying the total length by total width. All work is shown and supports the results.

3	<p>a</p> <p>F-IF.B.4</p>	<p>Student makes no attempt to determine the function's value.</p>	<p>Student attempts to find the maximum value for the function but does not make a connection between the sign of the leading coefficient and the direction that the graph opens. Student also makes errors in the calculations.</p> <p><u>OR</u></p> <p>Student makes a connection between the sign of the leading coefficient and the direction the graph opens, but the graph is said to have a minimum because the leading coefficient is negative.</p>	<p>Student attempts to find the maximum value for the function, and makes a connection between the sign of the leading coefficient and the direction that the graph opens, but the explanation does not make it clear that the negative leading coefficient indicates that the graph opens down.</p>	<p>Student correctly finds the maximum value for the function and makes a clear connection between the sign of the leading coefficient and the direction that the graph opens. Student provides a clear and logical explanation.</p>
	<p>b–e</p> <p>A-APR.B.3 F-IF.B.4 F-IF.B.6</p>	<p>Student does not provide evidence of understanding the properties of the key features of the quadratic function. Calculations are ineffective or incorrect. Explanations are missing or ineffective.</p> <p><u>OR</u></p> <p>Student makes no attempt to answer the question.</p>	<p>Student provides some evidence of understanding the properties of the key features of the quadratic function. However, calculations are incorrect and explanations are missing or inadequate.</p>	<p>Student provides accurate interpretations of the key features of the quadratic function, but some calculations are incorrectly performed. Complete, logical explanations are supported by calculations.</p>	<p>Student provides accurate interpretations of the key features of the quadratic function, and all calculations are performed correctly and supported by complete, logical explanations.</p>
	<p>f</p> <p>A-APR.B.3 F-IF.B.4 F-IF.B.5</p>	<p>Student makes no attempt to state a domain of the function.</p>	<p>Student gives an incorrect domain as all real numbers (i.e., the domain of the function with no consideration of the context).</p>	<p>Student describes the domain only as positive or greater than zero, or as less than 5, with no consideration given to the context (i.e., partial consideration of the context).</p>	<p>Student provides an accurate description of domain or gives it as a set. Consideration is given to the beginning of the experiment (0 seconds) and to the end (5 seconds).</p>

	g A-APR.B.3 F-IF.B.4 F-IF.B.5 F-IF.C.7a	<p>Student provides little indication of understanding the graphic representation of the function. The graph is incorrectly drawn, and the key features are missing or incorrectly identified.</p> <p><u>OR</u></p> <p>Student makes little or no attempt to graph the function.</p>	<p>Student attempts to graph the function, but key features are not indicated on the graph. The axes are not labeled clearly, but the scale fits the graph or allows for visual verification of the key features (i.e., y-intercept $(0, 80)$, vertex $(2, 144)$, and x-intercept $(5, 0)$).</p>	<p>Student graphs the function clearly and correctly but does not indicate key features on the graph. The axes are labeled clearly with a scale that fits the graph and allows for visual verification of the key features (even though they are not marked).</p>	<p>Student graphs the function clearly and correctly with the y-intercept $(0, 80)$, the vertex $(2, 144)$, and the x-intercept $(5, 0)$ identified correctly. The axes are labeled clearly with a scale that fits the graph.</p>
	h F-IF.B.4 F-IF.C.7a	<p>Student provides an answer, but there is no explanation provided.</p> <p><u>OR</u></p> <p>Student makes no attempt to answer the question.</p>	<p>Student attempts to use the laws of physics in the explanation for this question (i.e., the horizontal axis represents the change in time rather than forward motion). However, the answer to the question is given incorrectly.</p>	<p>Student answers the question correctly and provides an explanation. However, the explanation is based only partially on the physics addressed in this problem (i.e., the horizontal axis represents the change in time rather than forward motion).</p>	<p>Student answers the question correctly and provides an explanation that shows an understanding of the physics addressed in this problem (i.e., the horizontal axis represents the change in time rather than forward motion).</p>

Name _____

Date _____

1. A rectangle with positive area has length represented by the expression $3x^2 + 5x - 8$ and width by $2x^2 + 6x$. Write expressions in terms of x for the perimeter and area of the rectangle. Give your answers in standard polynomial form and show your work.

- a. Perimeter:

$$\begin{aligned} & 2(3x^2 + 5x - 8) + 2(2x^2 + 6x) \\ &= 6x^2 + 10x - 16 + 4x^2 + 12x \\ &= 10x^2 + 22x - 16 \end{aligned}$$

- b. Area:

$$\begin{aligned} & (3x^2 + 5x - 8)(2x^2 + 6x) \\ &= 6x^4 + 18x^3 + 10x^3 + 30x^2 - 16x^2 - 48x \\ &= 6x^4 + 28x^3 + 14x^2 - 48x \end{aligned}$$

- c. Are both your answers polynomials? Explain why or why not for each.

Yes, both have terms with only whole number exponents (greater than or equal to 0), coefficients that are real numbers, and a leading coefficient that is not 0.

- d. Is it possible for the perimeter of the rectangle to be 16 units? If so, what value(s) of x will work? Use mathematical reasoning to explain how you know you are correct.

$$10x^2 + 22x - 16 = 16$$

$$10x^2 + 22x - 32 = 0$$

$$2(5x^2 + 11x - 16) = 0$$

$$2(5x + 16)(x - 1) = 0$$

$$\text{So } x = -\frac{16}{5} \text{ or } 1 \text{ OR } -3.2 \text{ or } 1$$

$$\begin{aligned} \text{Check: } 2(\text{length}) + 2(\text{width}) &= 2(6.72) + 2(1.28) = \\ 13.44 + 2.56 &= 16 \end{aligned}$$

Yes, the perimeter could be 16 units with length 6.72 and width 1.28.

If $x = 1$, the length would be $3(1) + 5(1) - 8 = 0$; therefore, $x \neq 1$.

If $x = -3.2$, the length would be $3(-3.2)^2 + 5(-3.2) - 8 = 3(10.24) - 16 - 8 = 30.72 - 24 = 6.72$, and the width would be $2(-3.2)^2 + 6(-3.2) = 20.48 - 19.2 = 1.28$.

- e. For what value(s) of the domain will the area equal zero?

In factored form: $(3x^2 + 5x - 8)(2x^2 + 6x) = (3x + 8)(x - 1)(2x)(x + 3) = 0$

The Area = 0 when $x = -\frac{8}{3}, 1, 0$, or -3 .

- f. The problem states that the area of the rectangle is positive. Find and check two positive domain values that will produce a positive area.

Check the values around those we found in part (e), since on either side of the zeros there is likely to be either positive or negative values.

Try substituting $x = 2$ into the factored form. The factors will then be $(+)(+)(+)(+)>0$.

So, all numbers greater than 1 will give positive results ($x = 3$, etc.).

Note: If there are any, there must be an even number of negative factors, and any pair of negative factors must be for the same dimension.

- g. Is it possible that negative domain values could produce a positive function value (area)? Explain why or why not in the context of the problem.

As long as the dimensions are positive, it is possible that the value of x is negative. That means that either two of the four factors must be negative, and the negative factors must both be from the same dimension (length or width), or all four of the factors must be negative. Using the logic in part (f), it is possible that numbers less than $-\frac{8}{3}$ or possibly between 0 and $-\frac{8}{3}$ might work.

Let's try $x = -4$: The factors would be $(-)(-)(-)(-)$. This one works since both dimensions will be positive.

Let's try $x = -1$: The factors would be $(+)(-)$... I can stop now because the length is negative, which is impossible in the context of the problem.

So, the answer is YES. There are negative values for x that produce positive area. They are less than $-\frac{8}{3}$, and they result in both positive dimensions and positive area.

2. A father divided his land so that he could give each of his two sons a plot of his own and keep a larger plot for himself. The sons' plots are represented by squares 1 and 2 in the figure below. All three shapes are squares. The area of square 1 equals that of square 2, and each can be represented by the expression $4x^2 - 8x + 4$.

- a. Find the side length of the father's plot, which is square 3, and show or explain how you found it.

$4x^2 - 8x + 4$ is a perfect square that factors to

$$(2x - 2)^2.$$

The side length is the square root of $(2x-2)^2$, which is $(2x-2)$.

The father's plot is twice the length of one of the smaller squares, or the sum of the two.

The side length for plot 3 is $2(2x - 2) = 4x - 4$.



- b. Find the area of the father's plot, and show or explain how you found it.

The area of the father's plot is the square of the side length:

$$(4x - 4)^2 = 16x^2 - 32x + 16.$$

- c. Find the total area of all three plots by adding the three areas, and verify your answer by multiplying the outside dimensions. Show your work.

By adding the areas of the three squares:

$$(4x^2 - 8x + 4) + (4x^2 - 8x + 4) + (16x^2 - 32x + 16) = 24x^2 - 48x + 24.$$

By multiplying total length by total width:

$$\text{Total length} = (2x - 2) + (4x - 4) = 6x - 6;$$

$$\text{Total width} = (2x - 2) + (2x - 2) = 4x - 4;$$

$$\text{Area} = (6x - 6)(4x - 4) = 24x^2 - 48x + 24.$$

3. The baseball team pitcher was asked to participate in a demonstration for his math class. He took a baseball to the edge of the roof of the school building and threw it up into the air at a slight angle so that the ball eventually fell all the way to the ground. The class determined that the motion of the ball from the time it was thrown could be modeled closely by the function

$$h(t) = -16t^2 + 64t + 80,$$

where $h(t)$ represents the height of the ball in feet after t seconds.

- a. Determine whether the function has a maximum value or a minimum value. Explain your answer mathematically.

The function has a maximum because the leading coefficient is negative, making the graph of the function open down.

- b. Find the maximum or minimum value of the function. After how many seconds did the ball reach this value? Show how you found your answers.

To find the zeros of the function, we factor as follows:

$$-16(t^2 - 4t - 5) = 0 = -16(t - 5)(t + 1).$$

So, $t = -1$ or 5 . There are 6 units between -1 and 5 , so using symmetry, we find the vertex at half that distance between the two. $-1 + 3 = 2$; therefore, the t -coordinate of the vertex is $t = 2$.

If we substitute 2 for t into the original function, we find that the vertex is at $(2, 144)$; this tells us that the maximum height is 144 ft., which occurs after 2 seconds.

- c. For what interval of the domain is the function increasing (i.e., ball going up)? For what interval of the domain is the function decreasing (i.e., ball going down)? Explain how you know.

The function is increasing from 0 to 2 seconds and decreasing from 2 to 5 seconds. The rate of change over $[0, 2]$ is positive. The rate of change over $[2, 5]$ is negative. For an answer based on the graph: The graph has positive slope from 0 to 2 seconds and negative slope from 2 to 5 seconds.

- d. Evaluate $h(0)$. What does this value tell you? Explain in the context of the problem.

$h(0) = 80$. This is the initial height, the height at which the ball was when it was thrown upward. The roof was 80 ft. high.

- e. How long is the ball in the air? Explain your answer.

The ball is in the air for 5 seconds. When $t = 0$, the ball is released. When $t = 5$, the height is 0, which means the ball hits the ground 5 seconds after it is thrown.

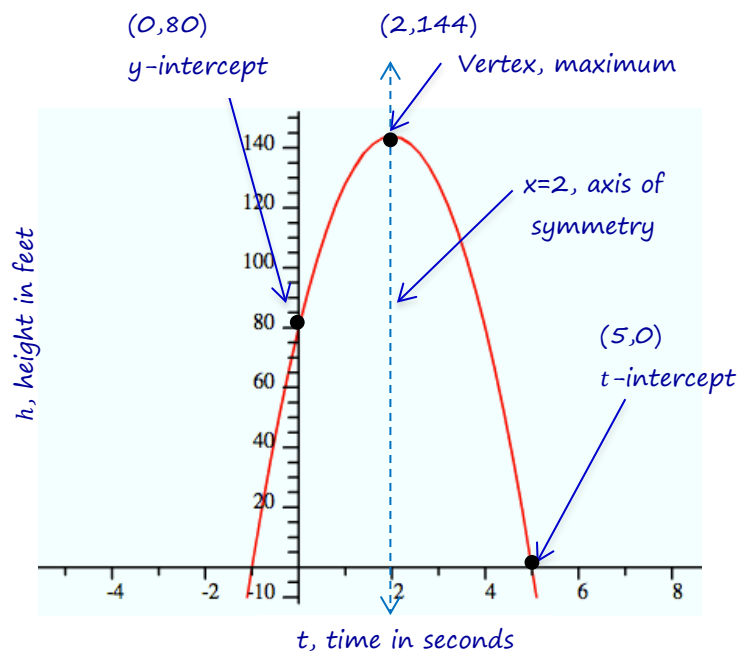
- f. State the domain of the function, and explain the restrictions on the domain based on the context of the problem.

We consider the experiment over at the time the ball reaches the ground, so it must be less than or equal to 5. Additionally, the values for t as described in this context must be greater than or equal to 0 because time began when the ball was thrown.

$$t: \{0 \leq t \leq 5\}$$

- g. Graph the function indicating the vertex, axis of symmetry, intercepts, and the point representing the ball's maximum or minimum height. Label your axes using appropriate scales. Explain how your answer to part (d) is demonstrated in your graph.

The graph shows the function crossing the y -axis at $(0, 80)$, the height at which the ball was thrown. Then, it travels to a height of 144 ft. after 2 seconds, and hits the ground at 5 seconds.



- h. Does your graph illustrate the actual trajectory of the ball through the air as we see it?

No, the graph does not illustrate the actual trajectory of the ball through the air as we see it because movement along the horizontal axis represents changes in time, not horizontal distance. The ball could be going straight up and then straight down with very little change in horizontal position, and the graph would be the same.