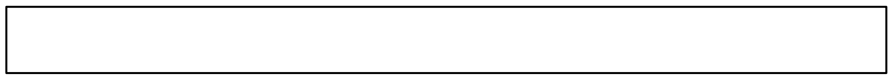
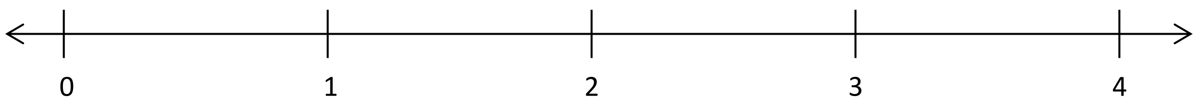
Name ______________________________________ Date ____________________

1. a. Partition the tape diagram to show $5 \times \frac{2}{3}$. Partition the number line to show $10 \times \frac{1}{3}$.





b. Use the models above to explain why $5 \times \frac{2}{3} = 10 \times \frac{1}{3}$.

2. Fill in the circles below with <, =, or > to make true number sentences. Use decomposition or multiplication to justify your answer.

a. $7 \bigcirc \frac{43}{6}$

b. $11\frac{1}{3} \bigcirc \frac{34}{3}$

c. $\frac{13}{6} \bigcirc \frac{38}{12}$

3. Generate a pattern of at least 13 fractions by adding $\frac{4}{3}$ to $\frac{1}{3}$ and then continuing to add $\frac{4}{3}$ to each fraction. Circle each fraction equal to a whole number. Write what you notice about the pattern of whole numbers. The first two fractions are written for you.

$\frac{1}{3}$, $\frac{5}{3}$,

4. Find each sum or difference.

a. $6\frac{4}{10} + 7\frac{7}{10}$

b. $3\frac{3}{8} + 6\frac{5}{8} + 1\frac{7}{8}$

c. $1\frac{9}{12} - 1\frac{4}{12}$

d. $5\frac{2}{5} - 1\frac{3}{5}$

5. a. Rewrite $3 \times \frac{6}{8}$ as the product of a unit fraction and a whole number. Solve.

   b. Rewrite $4 \times 6\frac{2}{3}$ as the product of a unit fraction and a whole number. Solve.

6. Determine if the following are true or false. Explain how you know using models or words. Make false problems true by rewriting the right side of the number sentence.

   a. $7\frac{1}{3} = 7 + \frac{1}{3}$

   b. $\frac{5}{3} = \frac{3}{3} + \frac{2}{3}$

   c. $\frac{13}{6} - \frac{5}{6} = \frac{13-5}{6}$

   d. $\frac{11}{3} = 11 + \frac{1}{3}$

   e. $\frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} = 4 \times \frac{7}{8}$

   f. $5 \times 3\frac{3}{4} = 15 + \frac{3}{4}$

7. The chart to the right shows data Amashi collected about butterfly wingspans.

Butterfly	Wingspan (inches)
Monarch	$3\frac{7}{8}$
Milbert's Tortoiseshell	$2\frac{5}{8}$
Zebra Swallowtail	$2\frac{1}{2}$
Viceroy	$2\frac{6}{8}$
Postman	$3\frac{3}{8}$
Purple Spotted Swallowtail	$2\frac{2}{8}$
Julia	$3\frac{2}{4}$
Southern Dogface	$2\frac{3}{8}$
Tiger Swallowtail	$3\frac{1}{2}$
Regal Fritillary	$3\frac{4}{8}$

a. At the bottom of this page, create a line plot to display the data in the table.

b. What is the difference in wingspan between the widest and narrowest butterflies on the chart?

c. Three butterflies have the same wingspan. Explain how you know the measurements are equal.

Solve each problem. Draw a model, write an equation, and write a statement for each.

d. Amashi wants to display a Postman and Viceroy side-by-side in a photo box with a width of 6 inches. Will these two butterflies fit? Explain how you know.

e. Compare the wingspan of the Milbert's Tortoiseshell and the Zebra Swallowtail using >, <, or =.

f. The Queen Alexandra Birdwing can have a wingspan that is 5 times as wide as the Southern Dogface's. How many inches can the Birdwing's wingspan be?

g. Amashi discovered a pattern. She started with $2\frac{2}{8}$ inches and added $\frac{1}{8}$ inch to each measurement. List the next four measurements in her pattern. Name the five butterflies whose wingspans match the measurements in her pattern.

End-of-Module Assessment Task Standards Addressed **Topics A–H**

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way*.

Extend understanding of fraction equivalence and ordering.

4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Represent and interpret data.

4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency.* In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for students is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the students CAN do now and what they need to work on next.

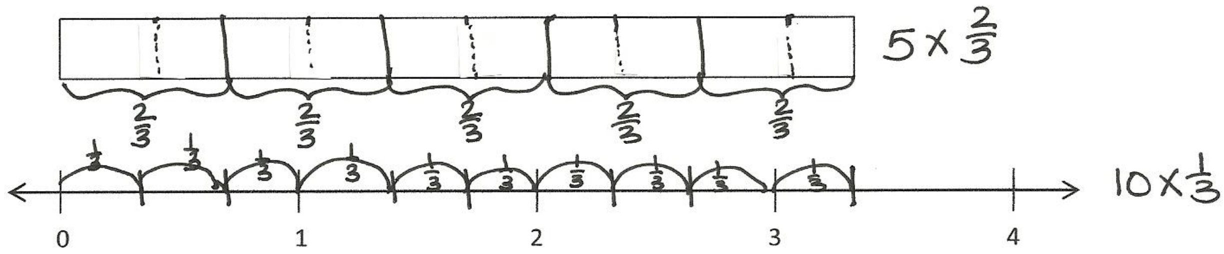
A Progression Toward Mastery				
Assessment Task Item and Standards Assessed	**STEP 1 Little evidence of reasoning without a correct answer. (1 Point)**	**STEP 2 Evidence of some reasoning without a correct answer. (2 Points)**	**STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)**	**STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)**
1 **4.NF.4ab**	The student incorrectly partitions the models and provides little to no reasoning.	The student incorrectly partitions the models but provides some reasoning for equivalence.	The student correctly partitions the models, providing some reasoning.	The student correctly does the following: a. Partitions the tape diagram and number line. b. Explains the equivalence using the models and number sentences.
2 **4.NF.1** **4.NF.2**	The student correctly answers fewer than two of the three parts with little to no reasoning.	The student is able to correctly compare one or two of the three number pairs with some reasoning.	The student is able to correctly compare two of the three number pairs and offers solid reasoning to support correct answers or correctly compares all three numbers and offers some reasoning.	The student correctly compares all three number pairs and offers appropriate modeling or reasoning to justify answers: a. < b. = c. <
3 **4.OA.5**	The student is unable to complete a majority of the problem.	The student is able to generate most of the pattern and find some whole numbers. The student provides little reasoning about the whole numbers.	The student is able to generate the pattern and find at least three whole numbers but cannot reason about the whole numbers.	The student correctly does the following: ▪ Generates the following pattern: ▪ $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \frac{17}{3}, \frac{21}{3}, \frac{25}{3}, \frac{29}{3}, \frac{33}{3}, \frac{37}{3}, \frac{41}{3}, \frac{45}{3}, \frac{49}{3}, \frac{53}{3}, \frac{57}{3}$ ▪ Circles $\frac{9}{3}, \frac{21}{3}, \frac{33}{3}, \frac{45}{3}, \frac{57}{3}$. ▪ Observes that whole numbers repeat every three fractions, determines that all whole numbers are

A Progression Toward Mastery				
				odd numbers (3, 7, 11, 15, 19), or provides another acceptable response.
4 **4.NF.3c**	The student correctly evaluates one or fewer expressions.	The student correctly evaluates two expressions.	The student correctly evaluates three expressions.	The student correctly evaluates all four expressions: a. $14\frac{1}{10}$ b. $11\frac{7}{8}$ c. $\frac{5}{12}$ d. $3\frac{4}{5}$
5 **4.NF.4ab**	The student is unable to correctly complete either of the two parts.	The student is able to correctly complete one of the two parts.	The student correctly rewrites the expressions in Parts (a) and (b) but does not solve the expressions. Or the student correctly solves for an incorrect expression.	The student correctly rewrites and solves the expressions in both parts of the problem: a. $18 \times \frac{1}{8} = \frac{18}{8}$ or $2\frac{2}{8}$ b. $4 \times \frac{20}{3} = \frac{4 \times 20}{3} =$ $80 \times \frac{1}{3} = \frac{80}{3}$ or $26\frac{2}{3}$
6 **4.NF.3a** **4.NF.4b**	The student correctly solves less than four expressions with little to no reasoning.	The student correctly solves three or four of the six expressions with some reasoning.	The student correctly solves five of the six expressions with solid reasoning.	The student correctly analyzes all six expressions, revises the incorrect number sentences (answers may vary), and provides solid reasoning using models or words: a. True b. True c. True d. False, $\frac{11}{3} = 11 \times \frac{1}{3}$ e. True f. False, $5 \times 3\frac{3}{4} =$ $5 \times (3 + \frac{3}{4})$

A Progression Toward Mastery				
7 **4.NF.1** **4.NF.2** **4.NF.3** **4.NF.4** **4.OA.5** **4.MD.4**	The student correctly completes three or fewer parts with little to no reasoning.	The student correctly completes at least three parts of the question with some reasoning.	The student correctly completes five or six parts of the question, providing solid reasoning. Or the student correctly answers all parts with only some reasoning.	The student correctly completes all seven parts of the question: a. Creates an accurate line plot with 10 data points from the table. b. $1\frac{5}{8}$ inches. c. Shows $3\frac{1}{2} = 3\frac{2}{4} = 3\frac{4}{8}$ using a model or showing eqivalences through expressions. d. No. The combined wingspan is $6\frac{1}{8}$, which is longer than the box. e. $2\frac{5}{8} > 2\frac{1}{2}$. f. $11\ \frac{7}{8}$ inches. g. $2\frac{3}{8}, 2\frac{1}{2}\left(2\frac{4}{8}\right), 2\frac{5}{8}, 2\frac{6}{8}$. Purple Spotted Swallowtail, Southern Dogface, Zebra Swallowtail, Milbert's Tortoiseshell, and Viceroy.

Name Jack Date ____________

1. a. Partition the tape diagram to show $5 \times \frac{2}{3}$. Partition the number line to show $10 \times \frac{1}{3}$.



b. Use the models above to explain why $5 \times \frac{2}{3} = 10 \times \frac{1}{3}$.

When you double the size of the piece, you only need half as many to be the same length.

2. Fill in the circles below with <, =, or > to make true number sentences. Use decomposition or multiplication to justify your answer.

a. $7 < \frac{43}{6}$ $\frac{43}{6} = 7 \times \frac{6}{6} + \frac{1}{6} = 7\frac{1}{6}$

b. $11\frac{1}{3} = \frac{34}{3}$ $\frac{34}{3} = 11 \times \frac{3}{3} + \frac{1}{3} = 11\frac{1}{3}$

c. $\frac{13}{6} < \frac{38}{12}$ $\frac{13 \times 2}{6 \times 2} = \frac{26}{12}$ $\frac{26}{12} < \frac{38}{12}$

3. Generate a pattern of at least 13 fractions by adding $\frac{4}{3}$ to $\frac{1}{3}$ and then continuing to add $\frac{4}{3}$ to each fraction. Circle each fraction equal to a whole number. Write what you notice about the pattern of whole numbers. The first two fractions are written for you.

$\frac{1}{3}$, $\frac{5}{3}$, $\frac{9}{3}$, $\frac{13}{3}$, $\frac{17}{3}$, $\frac{21}{3}$, $\frac{25}{3}$, $\frac{29}{3}$, $\frac{33}{3}$, $\frac{37}{3}$, $\frac{41}{3}$, $\frac{45}{3}$, $\frac{49}{3}$, $\frac{53}{3}$, $\frac{57}{3}$

3 7 11 15 19

I noticed that the pattern of the whole numbers increases by 4 each time and they are all odd numbers.

4. Find each sum or difference.

a. $6\frac{4}{10} + 7\frac{7}{10} = 14\frac{1}{10}$

$6+7=13$

$\frac{4}{10}+\frac{7}{10}=\frac{11}{10}=1\frac{1}{10}$

$13+1\frac{1}{10}=14\frac{1}{10}$

b. $3\frac{3}{8} + 6\frac{5}{8} + 1\frac{7}{8} = 11\frac{7}{8}$

$3+6+1=10$

$\frac{3}{8}+\frac{5}{8}+\frac{7}{8}=\frac{15}{8}=1\frac{7}{8}$

$10+1\frac{7}{8}=11\frac{7}{8}$

c. $1\frac{9}{12} - 1\frac{4}{12} = \frac{5}{12}$

$1-1=0$

$\frac{9}{12}-\frac{4}{12}=\frac{5}{12}$

d. $5\frac{2}{5} - 1\frac{3}{5} = 3\frac{4}{5}$

$5\frac{2}{5}=4\frac{7}{5}$

$$\left.\begin{array}{l} 4-1=3 \\ \frac{7}{5}-\frac{3}{5}=\frac{4}{5} \end{array}\right\} 3\frac{4}{5}$$

5. a. Rewrite $3 \times \frac{6}{8}$ as the product of a unit fraction and a whole number. Solve.

$$3 \times \frac{6}{8} = 18 \times \frac{1}{8} = \frac{18}{8} = \frac{8}{8} + \frac{8}{8} + \frac{2}{8} = 1 + 1 + \frac{2}{8} = 2\frac{2}{8}$$

b. Rewrite $4 \times 6\frac{2}{3}$ as the product of a unit fraction and a whole number. Solve

$$4 \times \underbrace{6\frac{2}{3}}_{\frac{18}{3}\ \ \frac{2}{3}} = 4 \times \frac{20}{3} = \frac{4 \times 20}{3} = 80 \times \frac{1}{3} = \frac{80}{3} = 26\frac{2}{3}$$

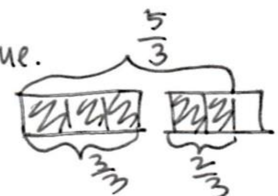
6. Determine if the following are true or false. Explain how you know using models or words. Make false problems true by rewriting the right side of the number sentence.

a. $7\frac{1}{3} = \underbrace{7 + \frac{1}{3}}_{7\frac{1}{3}}$

True

b. $\frac{5}{3} = \frac{3}{3} + \frac{2}{3}$

True.



c. $\frac{13}{6} - \frac{5}{6} = \frac{13-5}{6}$

True.

$\frac{13-5}{6}$ is the same as $\frac{13}{6} - \frac{5}{6}$

d. $\frac{11}{3} = 11 + \frac{1}{3}$ → $\frac{11}{3} = 11 \times \frac{1}{3}$

False.

$11 + \frac{1}{3} = 11\frac{1}{3}$

$\frac{11}{3} \neq 11\frac{1}{3}$

e. $\underbrace{\frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8}}_{\frac{28}{8}} = \underbrace{4 \times \frac{7}{8}}_{\frac{28}{8}}$

True.

f. $5 \times 3\frac{3}{4} = 15 + \frac{3}{4}$ → $5 \times 3\frac{3}{4} = 5 \times \left(3 + \frac{3}{4}\right)$

$5 \times \frac{15}{4} = 75 \times \frac{1}{4} = \frac{75}{4} = 18\frac{3}{4}$

False. $18\frac{3}{4} \neq 15\frac{3}{4}$

7. The chart to the right shows data Amashi collected about butterfly wingspans.

Butterfly	**Wingspan (inches)**
Monarch	$3\frac{7}{8}$
Milbert's Tortoiseshell	$2\frac{5}{8}$
Zebra Swallowtail	$2\frac{1}{2}$
Viceroy	$2\frac{6}{8}$
Postman	$3\frac{3}{8}$
Purple Spotted Swallowtail	$2\frac{2}{8}$
Julia	$3\frac{2}{4}$
Southern Dogface	$2\frac{3}{8}$
Tiger Swallowtail	$3\frac{1}{2}$
Regal Fritillary	$3\frac{4}{8}$

a. At the bottom of this page, create a line plot to display the data in the table.

b. What is the difference in wingspan between the widest and narrowest butterflies on the chart?

$3\frac{7}{8} - 2\frac{2}{8} = 1\frac{5}{8}$

$3 - 2 = 1$

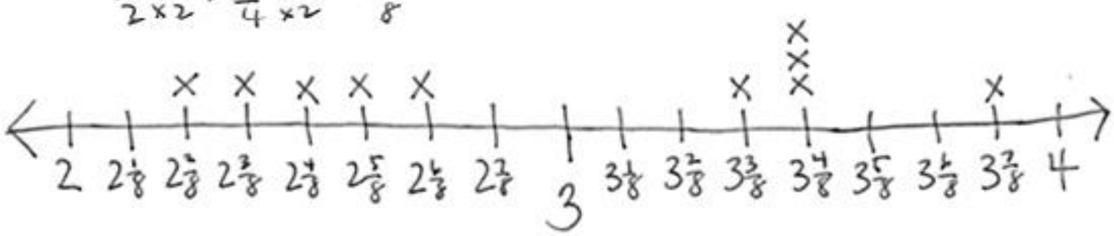
$\frac{7}{8} - \frac{2}{8} = \frac{5}{8}$

c. Three butterflies have the same wingspan. Explain how you know the measurements are equal.

The Julia, Tiger Swallowtail, and Regal Fritillary all have the same wingspan. I know because:

$3\frac{2}{4} = 3\frac{1}{2} = 3\frac{4}{8}$

$\frac{1 \times 2}{2 \times 2} = \frac{2 \times 2}{4 \times 2} = \frac{4}{8}$



Solve each problem. Draw a model, write an equation, and write a statement for each.

d. Amashi wants to display a Postman and Viceroy side-by-side in a photo box with a width of 6 inches. Will these two butterflies fit? Explain how you know.

Postman $3\frac{3}{8}$

Viceroy $2\frac{6}{8}$

$3\frac{3}{8} + 2\frac{6}{8} = 6\frac{1}{8}$

$\left.\begin{array}{l} 3+2=5 \\ \frac{3}{8}+\frac{6}{8}=\frac{9}{8}=1\frac{1}{8} \end{array}\right\} 6\frac{1}{8}$

The two butterflies will <u>not</u> fit because when I add their wingspans together I get $6\frac{1}{8}$ which is greater than 6 inches.

e. Compare the wingspan of the Milbert's Tortoiseshell and the Zebra Swallowtail using >, < or =.

$2\frac{5}{8} > 2\frac{1}{2} = 2\frac{4}{8}$

$\frac{1 \times 4}{2 \times 4} = \frac{4}{8}$

Milbert Tortoise shell Zebra Swallowtail

f. The Queen Alexandra Birdwing can have a wingspan that is 5 times as wide as the Southern Dogface's. How many inches can the Birdwing's wingspan be?

$5 \times 2\frac{3}{8} = (5 \times 2) + (5 \times \frac{3}{8}) =$

$10 + \frac{5 \times 3}{8} = 10 + \frac{15}{8} = 10 + 1\frac{7}{8} = 11\frac{7}{8}$

The Queen Alexandra Birdwing's wingspan could be $11\frac{7}{8}$ inches.

g. Amashi discovered a pattern. She started with $2\frac{2}{8}$ inches and added $\frac{1}{8}$ inch to each measurement. List the next four measurements in her pattern. Name the five butterflies whose wingspans match the measurements in her pattern.

$2\frac{2}{8}$, $2\frac{3}{8}$, $2\frac{4}{8}$, $2\frac{5}{8}$, $2\frac{6}{8}$

Purple Spotted Swallowtail, Southern Dogface, Zebra Swallowtail, Milbert's Tortoiseshell, Viceroy.