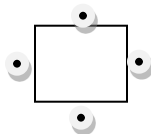


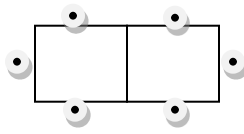
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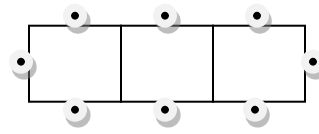
1. The diagram below shows how tables and chairs are arranged in the school cafeteria. One table can seat 4 people, and tables can be pushed together. When two tables are pushed together, 6 people can sit around the table.



1 Table



2 Tables



3 Tables

- a. Complete this table to show the relationship between the number of tables, n , and the number of students, S , that can be seated around the table.

n (tables)						
S (students)						

- b. If we make a sequence where the first term of the sequence is the number of students that can fit at one table, the second term of the sequence is the number of students that can fit at two tables, and so on, will the sequence be arithmetic, geometric, or neither? Explain your reasoning.
- c. Create an explicit formula for a sequence that models this situation. Use $n = 1$ as the first term representing how many students can sit at one table. How do the constants in your formula relate to the situation?
- d. Using this seating arrangement, how many students could fit around 15 tables pushed together in a row?

The cafeteria needs to provide seating for 189 students. They can fit up to 15 rows of tables in the cafeteria. Each row can contain at most 9 tables but could contain less than that. The tables on each row must be pushed together. Students will still be seated around the tables as described earlier.

- e. If they use exactly 9 tables pushed together to make each row, how many rows will they need to seat 189 students? What will be the total number of tables used to seat all of the students?
- f. Is it possible to seat the 189 students with fewer total tables? If so, what is the fewest number of tables needed? How many tables would be used in each row? (Remember that the tables on each row must be pushed together.) Explain your thinking.

2. Sydney was studying the following functions:

$$f(x) = 2x + 4 \text{ and } g(x) = 2(2)^x + 4$$

She said that linear functions and exponential functions are basically the same. She made her statement based on plotting points at $x = 0$ and $x = 1$ and graphing the functions.

Help Sydney understand the difference between linear functions and exponential functions by comparing and contrasting f and g . Support your answer with a written explanation that includes use of the average rate of change and supporting tables and/or graphs of these functions.

3. Dots can be arranged in rectangular shapes like the one shown below.



Figure 1

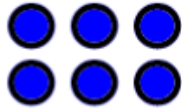


Figure 2

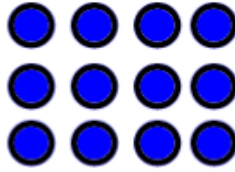


Figure 3

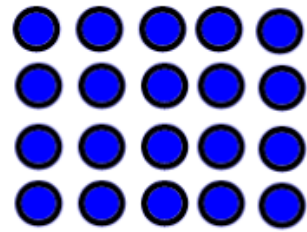


Figure 4

- a. Assuming the trend continues, draw the next three shapes in this particular sequence of rectangles. How many dots are in each of the shapes you drew?

The numbers that represent the number of dots in this sequence of rectangular shapes are called rectangular numbers. For example, 2 is the first rectangular number and 6 is the second rectangular number.

- b. What is the fiftieth rectangular number? Explain how you arrived at your answer.

- c. Write a recursive formula for the rectangular numbers.

- d. Write an explicit formula for the rectangular numbers.
- e. Could an explicit formula for the n^{th} rectangular number be considered a function? Explain why or why not. If yes, what would be the domain and range of the function?

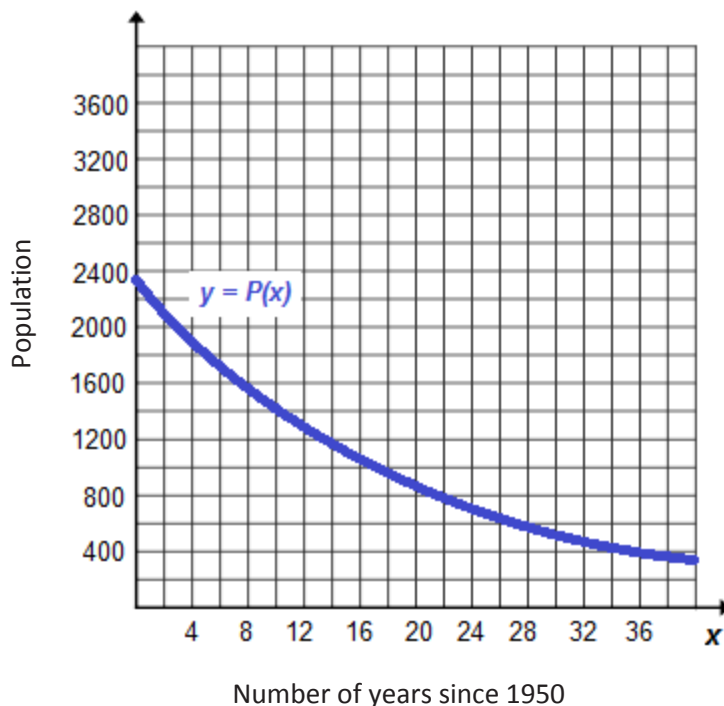
4. Stephen is assigning parts for the school musical.

- a. Suppose there are 20 students participating, and he has 20 roles available. If each of the 20 students will be assigned to exactly one role in the play, and each role will be played by only one student, is the assignment of the roles to the students in this way certain to be an example of a function? Explain why or why not. If yes, state the domain and range of the function.

The school musical also has a pit orchestra.

- b. Suppose there are 10 instrumental parts but only 7 musicians in the orchestra. The conductor assigns an instrumental part to each musician. Some musicians will have to cover two instrumental parts, but no two musicians will have the same instrumental part. If the instrumental parts are the domain and the musicians are the range, is the assignment of instrumental parts to musicians as described sure to be an example of a function? Explain why or why not. If so, what would be the meaning of $A(\text{Piano}) = \text{Scott}$?

- c. Suppose there are 10 instrumental parts but 13 musicians in the orchestra. The conductor assigns an instrumental part to each musician. Some instrumental parts will have two musicians assigned so that all the musicians have instrumental parts. When two musicians are assigned to one part, they alternate who plays at each performance of the play. If the instrumental parts are the domain, and the musicians are the range, is the assignment of instrumental parts to musicians as described sure to be an example of a function? Explain why or why not. If so, what would be the meaning of $A(\text{Piano}) = \text{Scott}$?
5. The population of a remote island has been experiencing a decline since the year 1950. Scientists used census data from 1950 and 1970 to model the declining population. In 1950 the population was 2,350. In 1962 the population was 1,270. They chose an exponential decay model and arrived at the function: $P(x) = 2350(0.95)^x$, $x \geq 0$, where x is the number of years since 1950. The graph of this function is given below.



- a. What is the y -intercept of the graph? Interpret its meaning in the context of the problem.
- b. Over what intervals is the function increasing? What does your answer mean within the context of the problem?
- c. Over what intervals is the function decreasing? What does your answer mean within the context of the problem?

Another group of scientists argues that the decline in population would be better modeled by a linear function. They use the same two data points to arrive at a linear function.

- d. Write the linear function that this second group of scientists uses.

- e. What is an appropriate domain for the function? Explain your choice within the context of the problem.
- f. Graph the function on the coordinate plane.
- g. What is the x -intercept of the function? Interpret its meaning in the context of the problem.

A Progression Toward Mastery

Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>OR</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem
1	a–d F-BF.A.1a F-LE.A.1 F-LE.A.2 F-LE.A.3 F-IF.A.2	Student is unable to identify the sequence as arithmetic and unable to create an explicit formula for finding the number of students that can be seated at n tables.	Student is able to recognize the sequence as arithmetic but displays a logic error in the explicit formula for the sequence. Student may also have errors in the table, the relating of constants in the formula to context, or in the computation of how many students can sit at 15 tables pushed together.	Student is able to recognize the sequence as arithmetic and create an explicit formula for the sequence. Student has one or more errors in the table, the relating of constants in the formula to context, or in the computation of how many students can sit at 15 tables pushed together.	Student provides correct entries for the table and explains that the sequence is arithmetic because each successive term is two more than the last term. Student writes an explicit formula for the sequence using an appropriate sequence notation, such as $f(n)$ or a_n , and correctly relates the constants of the formula to the context of the problem. Student correctly determines that 32 students can sit at 15 tables pushed together in a row.
	e F-IF.A.2	Student is not able to demonstrate how to determine the number of students that can sit at 9 tables pushed together.	Student makes calculation errors that lead to an incorrect answer but demonstrates some understanding of how to determine the number of students that can sit at 9 tables pushed together and some understanding of the need to divide 189 by 20 to see that it takes more than 9 rows.	Student articulates that 10 rows of 9 tables are needed to seat 189 students but fails to answer the question of how many tables that would be.	Student articulates that 10 rows of 9 tables are needed to seat 189 students and that 10 rows of 9 tables would be 90 tables.

	f F-IF.A.2	Student demonstrates very little reasoning skills in the attempted answer.	Student demonstrates some correct reasoning but is unable to arrive at one of the possible arrangements that use only 80 tables distributed across 15 rows.	Student chooses an arrangement of 80 tables distributed across 15 rows in one of any of the many possible configurations.	Student articulates that any arrangement involving 80 tables distributed among all 15 rows would use the minimum number of tables.
2	F-IF.B.5 F-IF.B.6 F-BF.A.1a F-LE.A.1 F-LE.A.2 F-LE.A.3	Student provides incorrect or insufficient tables, graphs, and written explanation.	Student provides a table and/or a graph that may have minor errors but does not provide a correct written explanation. <u>OR</u> Student provides a limited written explanation but does not support the answer with tables or graphs.	Student uses accurate tables or graphs to demonstrate that the functions are not the same and provides a limited written explanation that does not thoroughly describe the differences in the rates of change of the functions. <u>OR</u> Student has errors in his or her table or graphs but provides a thorough written explanation that references the rates of change of the functions.	Student uses accurate tables or graphs to demonstrate that the functions are not the same and provides a thorough written explanation that includes accurate references to the rates of change of the functions.
3	a F-IF.A.3 F-BF.A.1a	Student does not demonstrate any understanding of the pattern of the sequence described in the problem.	Student has a significant error or omission in the task but demonstrates some understanding of the pattern of the sequence described in the problem.	Student has a minor error or omission in the task but demonstrates clear understanding of the pattern of the sequence described in the problem.	Sequence is continued correctly three times, <u>AND</u> correct number of dots is given.
	b F-IF.A.3 F-BF.A.1a	Student demonstrates no understanding of using the pattern to find the fiftieth term.	Student attempts to use the pattern to find the fiftieth term but has a flaw in his reasoning leading to an incorrect answer.	Student correctly answers 2,550 but does not provide an explanation based on sound reasoning or makes a calculation error but provides an explanation based on sound reasoning.	Student correctly answers 2,550 and displays sound reasoning in the explanation, using the pattern of the sequence to arrive at the answer.
	c–d F-IF.A.3 F-BF.A.1a	Student work demonstrates no or very little understanding needed to write recursive or explicit formulas for this sequence.	Student is only able to provide either the recursive or the explicit formula and may have provided an explicit formula when asked for a recursive formula or vice versa.	Student provides correct formulas but uses an explicit formula for part (c) where a recursive one is required and a recursive one for part (d) where an explicit one is required.	Student provides a correct recursive formula for part (c) and a correct explicit formula for part (d), each using either function notation such as $f(n)$ or subscript notation such as a_n .

			<p><u>OR</u></p> <p>Student demonstrates that he or she understands the difference between a recursive formula and an explicit one by providing formulas for both, but the formulas provided are incorrect. Student may or may not have provided the necessary declaration of the initial term number in each case and for the initial term value in the recursive case.</p>	<p><u>OR</u></p> <p>Student provides the correct formulas in each case but neglects to provide the necessary declaration of the initial term number in each case and for the initial term value in the recursive case.</p>	<p>Student also provides the necessary declaration of the initial term number in each case and for the initial term value in the recursive case.</p>
	<p>e</p> <p>F-IF.A.2 F-IF.A.3</p>	<p>Student leaves the question blank or answers that it is not a function.</p>	<p>Student answers that it is a function but gives insufficient reasoning. <u>OR</u> Student is unable to identify the domain and range.</p>	<p>Student answers that it is a function but makes minor errors or omissions in his or her reasoning or makes errors in naming the domain and range.</p>	<p>Student answers that it is a function and gives sufficient reasoning, stating the domain and the range accurately.</p>
4	<p>a</p> <p>F-IF.A.1 F-IF.A.2</p>	<p>Student indicates it is not a function and provides no reasoning or incorrect reasoning, indicating he or she does not have sufficient understanding of what is required for a relation to be a function.</p>	<p>Student reasoning indicates he or she understands what is required for a relation to be a function but is not able to discern that this relation is a function. <u>OR</u> Student indicates it is a function but does not provide sufficient explanation or omits the naming of the domain and range.</p>	<p>Student identifies the relation as a function but provides an explanation and/or identification of the domain and range that contains minor errors or omissions.</p>	<p>Student identifies the relation as a function and provides a thorough explanation of how this situation meets the criteria for a function: that every input is assigned to one and only one output. Student chooses either the list of students or the list of roles as the domain and the other list as the range.</p>
	<p>b</p> <p>F-IF.A.1 F-IF.A.2</p>	<p>Student indicates it is not a function and provides no reasoning or incorrect reasoning, indicating he or she does not have sufficient understanding of what is required for a relation to be a function.</p>	<p>Student reasoning indicates he or she understands what is required for a relation to be a function but is not able to discern that this relation is a function. <u>OR</u> Student indicates it is a function but does not provide sufficient explanation and/or omits or incorrectly</p>	<p>Student identifies the relation as a function but provides an explanation and/or interpretation of $A(\text{piano}) = \text{Scott}$ that contains minor errors or omissions.</p>	<p>Student identifies the relation as a function and provides a thorough explanation of how this situation meets the criteria for a function: that every input is assigned to one and only one output. Student interprets $A(\text{piano}) = \text{Scott}$ to mean that Scott is playing the piano.</p>

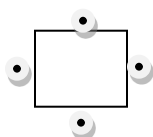
			interprets the meaning of $A(\text{piano}) = \text{Scott}$.		
	c F-IF.A.1 F-IF.A.2	Student indicates it is a function and provides no reasoning or incorrect reasoning, indicating he or she does not have sufficient understanding of what is required for a relation to be a function.	Student reasoning indicates he or she understands what is required for a relation to be a function but is not able to discern that this relation is not a function. <u>OR</u> Student indicates it is not a function but does not provide sufficient explanation.	Student identifies the relation is not a function but provides an explanation that contains minor errors or omissions.	Student identifies the relation is not a function and provides a thorough explanation of how this situation does not meet the criteria for a function: That every input is assigned to one and only one output.
5	a F-IF.B.4	Student is unable to correctly identify the y -intercept.	Student identifies that the y -intercept is the point $(0, 2350)$ but fails to correctly relate the point to the context of the problem.	Student identifies that the y -intercept is the point $(0, 2350)$ and relates 2,350 to the population when $x = 0$ but fails to relate $x = 0$ to the year 1950.	Student identifies that the y -intercept is the point $(0, 2350)$ and relates the point to the context that in 1950 the population was 2,350.
	b–c F-IF.B.4	Student is unable to correctly identify that the function is always decreasing, never increasing.	Student identifies that there are no intervals for which the function is increasing and that it is decreasing over its entire domain, but student does not correctly interpret this answer in the context of the problem.	Student identifies that there are no intervals for which the function is increasing that it is decreasing over its entire domain, and student interprets this answer in the context of the problem but has minor errors or omissions in the language used to answer the questions.	Student identifies that there are no intervals for which the function is increasing and that it is decreasing over its entire domain. Student interprets this answer in the context of the problem using mathematically correct language and sound reasoning.
	d, f, g F-BF.A.1a F-IF.B.4 F-IF.C.7a	Student does not use the two data points or does not create a linear equation using the two points; therefore, the graph or x -intercept of parts (f) and (g) are likely incorrect.	Student attempts to use the two data points to write a linear function but makes a significant error in arriving at the equation of the line. Student graphs the equation he or she created and attempts to identify an x -intercept but may have identified the y -intercept instead.	Student uses the two data points to write the linear function correctly using either function notation or an equation in two variables but may have made a computational error in finding the slope (yet it is evident that the student understands how to compute slope). Student graphs the function created but may have extended the graph beyond the domain identified in part (e).	Student uses the two data points to write the linear function correctly using either function notation or an equation in two variables. Student graphs the function correctly depicting only the domain values identified in part (e) and correctly identifies the x -intercept, relating it to the context of the problem.

				Student identifies the x -intercept, relating it to the context of the problem.	
	e F-IF.B.5	Student does not demonstrate sound reasoning in restricting the domain given the context of the problem.	Student restricts the domain and explains the answer by either considering that the domain should start in the year 1950 or considering that it does not make sense to continue the model when the population has fallen below zero but does not consider both factors.	Student restricts the domain and explains the answer by considering that the domain should start in the year 1950 or considering that it does not make sense to continue the model when the population has fallen below zero, but student makes an error in calculating the end points of the interval for the domain.	Student restricts the domain and explains the answer by considering that the domain should start in the year 1950 or considering that it does not make sense to continue the model when the population has fallen below zero. Student correctly calculates the end points of the interval for the domain.

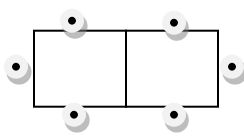
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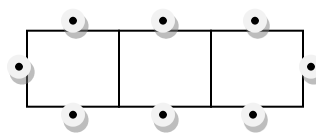
1. The diagram below shows how tables and chairs are arranged in the school cafeteria. One table can seat 4 people, and tables can be pushed together. When two tables are pushed together, 6 people can sit around the table.



1 Table



2 Tables



3 Tables

- a. Complete this table to show the relationship between the number of tables, n , and the number of students, S , that can be seated around the table.

n (tables)	1	2	3	4	5	6
S (students)	4	6	8	10	12	14

- b. If we make a sequence where the first term of the sequence is the number of students that can fit at one table, the second term of the sequence is the number of students that can fit at two tables, and so on, will the sequence be arithmetic, geometric, or neither? Explain your reasoning.

It would be an arithmetic sequence because every term is 2 more than the previous term.

- c. Create an explicit formula for a sequence that models this situation. Use $n = 1$ as the first term representing how many students can sit at one table. How do the constants in your formula relate to the situation?

$$f(n) = 4 + 2(n - 1)$$

4 is the number of students that can be seated at one table by itself.

2 is the number of additional students that can be seated each time a table is added.

- d. Using this seating arrangement, how many students could fit around 15 tables pushed together in a row?

$$f(15) = 4 + 2(15 - 1) = 32$$

The cafeteria needs to provide seating for 189 students. They can fit up to 15 rows of tables in the cafeteria. Each row can contain at most 9 tables but could contain less than that. The tables on each row must be pushed together. Students will still be seated around the tables as described earlier.

- e. If they use exactly 9 tables pushed together to make each row, how many rows will they need to seat 189 students? What will be the total number of tables used to seat all of the students? ?

$$f(9) = 4 + 2(9 - 1) = 20$$

9 tables pushed together seats 20 students.

It will take 10 rows to get enough rows to seat 189 students.

10 rows of 9 tables each is 90 tables.

- f. Is it possible to seat the 189 students with fewer total tables? If so, what is the fewest number of tables needed? How many tables would be used in each row? (Remember that the tables on each row must be pushed together.) Explain your thinking.

Yes, they would use the fewest tables to seat the 189 students if they used all of the 15 rows, because with each new row, you get the added benefit of the 2 students that sit on each end of the row.

Any arrangement that uses 80 total tables spread among all 15 rows will be the best. There will be 1 extra seat, but no extra tables.

One solution that evens out the rows pretty well but still uses as few tables as possible would be 5 rows of 6 tables and 10 rows of 5 tables.

Another example that has very uneven rows would be 8 rows of 9 tables, 1 row of 2 tables, and 6 rows of 1 table.

2. Sydney was studying the following functions:

$$f(x) = 2x + 4 \text{ and } g(x) = 2(2)^x + 4$$

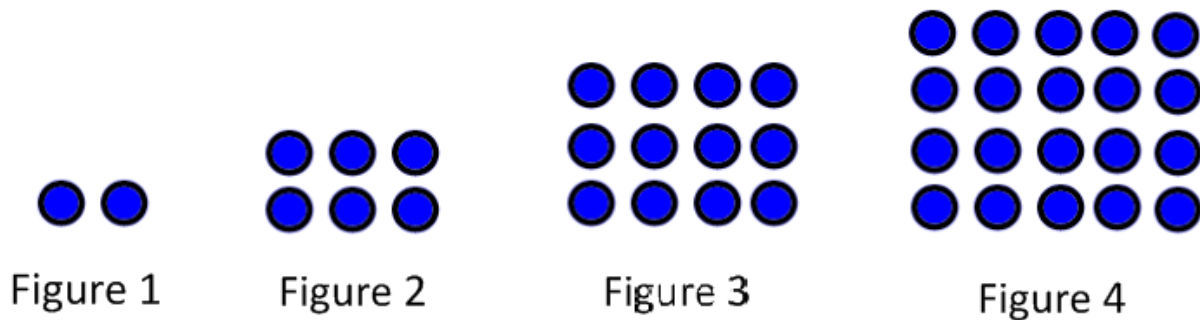
She said that linear functions and exponential functions are basically the same. She made her statement based on plotting points at $x = 0$ and $x = 1$ and graphing the functions.

Help Sydney understand the difference between linear functions and exponential functions by comparing and contrasting f and g . Support your answer with a written explanation that includes use of the average rate of change and supporting tables and/or graphs of these functions.

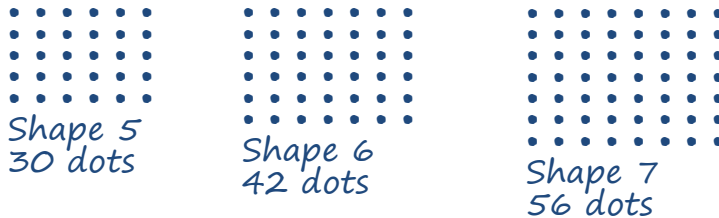
x	$f(x)$	Avg rate of change of $f(x)$ from previous x -value to this one	$g(x)$	Avg rate of change of $g(x)$ from previous x -value to this one
0	4		6	
1	6	2	8	2
2	8	2	12	4
3	10	2	20	8
4	12	2	36	16
5	14	2	68	32

Linear functions have a constant rate of change. $f(x)$ increases by 2 units for every 1 unit that x increases. Exponential functions do not have a constant rate of change. The rate of change of $g(x)$ is increasing as x increases. The average rate of change across an x interval of length 1 doubles for each successive x interval of length 1. No matter how large the rate of change is for the linear function, there is a x -value at which the rate of change for the exponential function will exceed the rate of change for the linear function.

3. Dots can be arranged in rectangular shapes like the one shown below.



- a. Assuming the trend continues, draw the next three shapes in this particular sequence of rectangles. How many dots are in each of the shapes you drew?



The numbers that represent the number of dots in this sequence of rectangular shapes are called rectangular numbers. For example, 2 is the first rectangular number and 6 is the second rectangular number.

- b. What is the fiftieth rectangular number? Explain how you arrived at your answer.

$$50(51) = 2550$$

The 1st figure had 1 row and 2 columns, giving 1(2) dots. The 2nd figure had 2 rows and 3 columns, giving 2(3) dots. The pattern for the n^{th} figure is n rows and $n+1$ columns. So, the 50th figure will have 50(51) dots.

- c. Write a recursive formula for the rectangular numbers.

$$f(1) = 2 = 1 \cdot 2 \quad f(n) = f(n-1) + 2n; \text{ natural number } n > 1, \text{ and } f(1) = 2$$

$$f(2) = 6 = 2 \cdot 3 = f(1) + 4$$

$$f(3) = 12 = 3 \cdot 4 = f(2) + 6$$

$$f(4) = 20 = 4 \cdot 5 = f(3) + 8$$

- d. Write an explicit formula for the rectangular numbers.

$$f(n) = n(n+1); \text{ natural number } n > 0.$$

- e. Could an explicit formula for the n^{th} rectangular number be considered a function? Explain why or why not. If yes, what would be the domain and range of the function?

Yes, consider the domain to be all the integers greater than or equal to 1, and the range to all the rectangular numbers. Then every element in the domain corresponds to exactly one element in the range.

4. Stephen is assigning parts for the school musical.

- a. Suppose there are 20 students participating, and he has 20 roles available. If each of the 20 students will be assigned to exactly one role in the play, and each role will be played by only one student, is the assignment of the roles to the students in this way certain to be an example of a function? Explain why or why not. If yes, state the domain and range of the function.

Yes, since every student gets a role and every role gets a student, and there are exactly 20 roles and 20 students, there is no possibility that a student is given more than one role, or that a role is given to more than one student. Therefore, the domain could be the list of students with the range being the list of roles, or we could consider the domain to be the list of roles and the range to be the list of students. Either way you would have an example of a function.

The school musical also has a pit orchestra.

- b. Suppose there are 10 instrumental parts but only 7 musicians in the orchestra. The conductor assigns an instrumental part to each musician. Some musicians will have to cover two instrumental parts, but no two musicians will have the same instrumental part. If the instrumental parts are the domain and the musicians are the range, is the assignment of instrumental parts to musicians as described sure to be an example of a function? Explain why or why not. If so, what would be the meaning of $A(\text{Piano}) = \text{Scott}$?

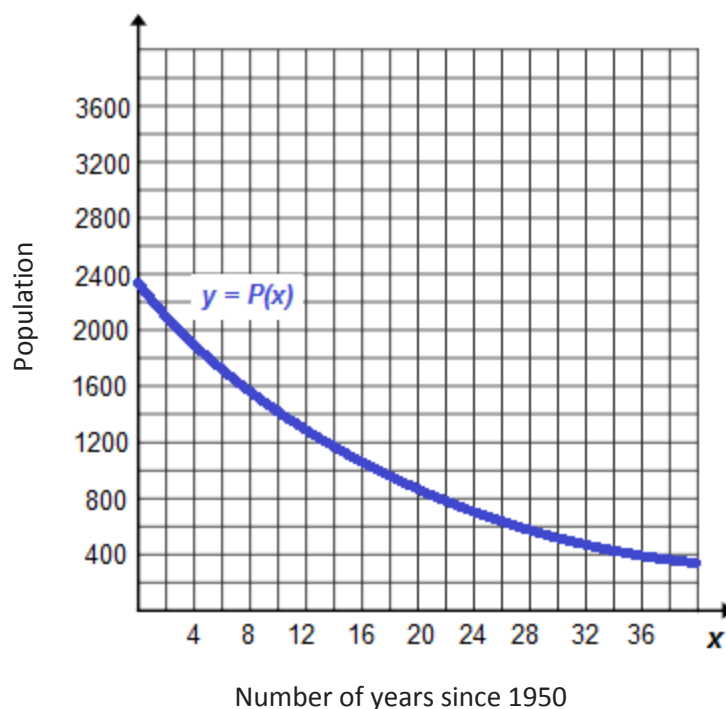
Yes, each element of the domain (the instrumental parts) are assigned to one and only one element in the range (the musicians).

$A(\text{Piano}) = \text{Scott}$ means that the part of the piano is being played by Scott.

- c. Suppose there are 10 instrumental parts but 13 musicians in the orchestra. The conductor assigns an instrumental part to each musician. Some instrumental parts will have two musicians assigned so that all the musicians have instrumental parts. When two musicians are assigned to one part, they alternate who plays at each performance of the play. If the instrumental parts are the domain, and the musicians are the range, is the assignment of instrumental parts to musicians as described sure to be an example of a function? Explain why or why not. If so, what would be the meaning of $A(\text{Piano}) = \text{Scott}$?

No, if the instrumental parts are the domain, then it cannot be an example of a function because there are 3 cases where one element in the domain (the instrumental parts) will be assigned to more than one element of the range (the musicians).

5. The population of a remote island has been experiencing a decline since the year 1950. Scientists used census data from 1950 and 1970 to model the declining population. In 1950 the population was 2,350. In 1962 the population was 1,270. They chose an exponential decay model and arrived at the function: $P(x) = 2350(0.95)^x$, $x \geq 0$, where x is the number of years since 1950. The graph of this function is given below.



- a. What is the y -intercept of the graph? Interpret its meaning in the context of the problem.

The y -intercept is the point $(0, 2350)$. When x is 0, there have been 0 years since 1950, so in the year 1950, the population was 2350.

- b. Over what intervals is the function increasing? What does your answer mean within the context of the problem?

There are no intervals in the domain where it is increasing. This means that the population is always decreasing, never increasing.

- c. Over what intervals is the function decreasing? What does your answer mean within the context of the problem?

The function is decreasing over its entire domain: $[0, \infty)$. This means that the population will continue to decline, except eventually when the function value is close to zero; then essentially the population will be zero from that point forward.

Another group of scientists argues that the decline in population would be better modeled by a linear function. They use the same two data points to arrive at a linear function.

- d. Write the linear function that this second group of scientists uses.

$$L(x) = \frac{(1270-2350)}{12} x + 2350$$

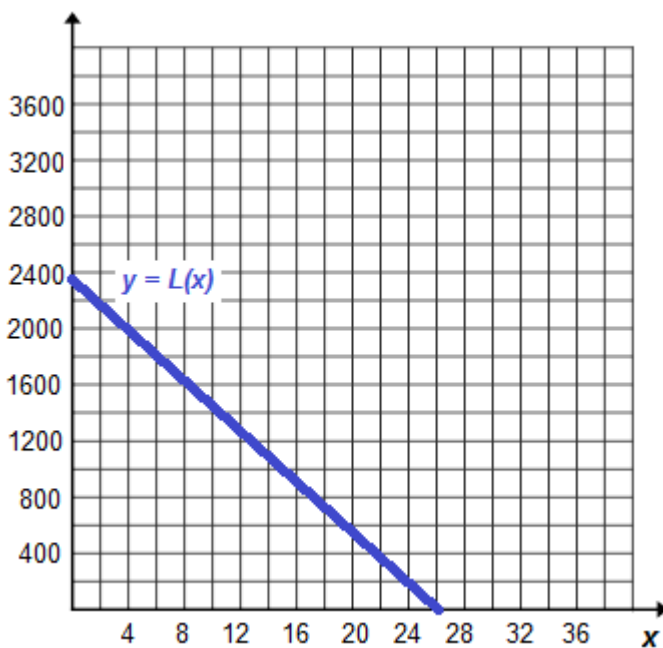
$$L(x) = -90x + 2350$$

- e. What is an appropriate domain for the function? Explain your choice within the context of the problem.

$$D: 0 \leq x \leq 26\frac{1}{9}$$

We are only modeling the decline of the population which scientists say started in 1950, so that means x starts at 0 years past 1950. Once the population hits zero, which occurs $26\frac{1}{9}$ years past 1950, the model no longer makes sense because population cannot be a negative number.

- f. Graph the function on the coordinate plane.



- g. What is the x -intercept of the function? Interpret its meaning in the context of the problem.

$$\left(26\frac{1}{9}, 0\right)$$

At $26\frac{1}{9}$ years past 1950, in the year 1976, the population will be zero.