

Name \_\_\_\_\_

Date \_\_\_\_\_

Round all decimal answers to the nearest hundredth.

1. Each student in a class of 38 students was asked to report how many siblings (brothers and sisters) he or she has. The data are summarized in the table below.

| Number of Siblings | 0 | 1  | 2  | 3 | 4 | 5 | 6 |
|--------------------|---|----|----|---|---|---|---|
| Count              | 8 | 13 | 12 | 3 | 1 | 0 | 1 |

- a. Based on the data, estimate the probability that a randomly selected student from this class is an only child.
- b. Based on the data, estimate the probability that a randomly selected student from this class has three or more siblings.

- c. Consider the following probability distribution for the number of siblings:

| Number of Siblings | 0    | 1    | 2    | 3    | 4    | 5    | 6    |
|--------------------|------|------|------|------|------|------|------|
| Probability        | 0.15 | 0.35 | 0.30 | 0.10 | 0.05 | 0.03 | 0.02 |

Explain how you could use simulation to estimate the probability that you will need to ask at least five students the question, “Are you an only child?” if each is an only child before you find the first one that is an only child.

2. A cell phone company wants to predict the probability of a seventh grader in your city, City A, owning a cell phone. Records from another city, City B, indicate that 201 of 1,000 seventh graders own a cell phone.
- a. Assuming the probability of a seventh grader owning a cell phone is similar for the two cities, estimate the probability that a randomly selected seventh grader from City A owns a cell phone.
- b. The company estimates the probability that a randomly selected seventh-grade male owns a cell phone is 0.25. Does this imply that the probability that a randomly selected seventh-grade female owns a cell phone is 0.75? Explain.

c. According to the data, which of the following is more likely?

- a seventh-grade male owning a cell phone
- a seventh grader owning a cell phone

Explain your choice.

Suppose the cell phone company sells three different plans to its customers:

- Pay-as-you-go: Customer is charged per minute for each call.
- Unlimited minutes: Customer pays a flat fee per month and can make unlimited calls with no additional charges.
- Basic plan: Customer not charged per minute unless the customer exceeds 500 minutes in the month; then, the customer is charged per minute for the extra minutes.

Consider the chance experiment of selecting a customer at random and recording which plan they purchased.

d. What outcomes are in the sample space for this chance experiment?

e. The company wants to assign probabilities to these three plans. Explain what is wrong with each of the following probability assignments.

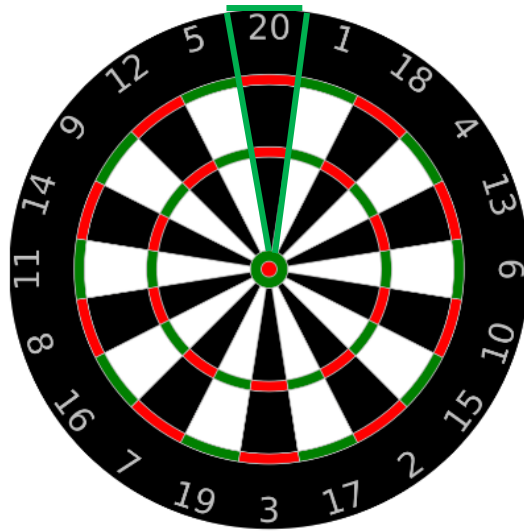
Case 1: Probability of pay-as-you-go = 0.40, probability of unlimited minutes = 0.40, and probability of basic plan = 0.30.

Case 2: Probability of pay-as-you-go = 0.40, probability of unlimited minutes = 0.70, and probability of basic plan =  $-0.10$ .

Now consider the chance experiment of randomly selecting a cell phone customer and recording both the cell phone plan for that customer and whether or not the customer exceeded 500 minutes last month.

- f. One possible outcome of this chance experiment is (pay-as-you-go, over 500). What are the other possible outcomes in this sample space?
- g. Assuming the outcomes of this chance experiment are equally likely, what is the probability that the selected cell phone customer had a basic plan and did not exceed 500 minutes last month?
- h. Suppose the company randomly selects 500 of its customers and finds that 140 of these customers purchased the basic plan and did not exceed 500 minutes. Would this cause you to question the claim that the outcomes of the chance experiment described in part (g) are equally likely? Explain why or why not.

3. In the game of Darts, players throw darts at a circle divided into 20 wedges. In one variation of the game, the score for a throw is equal to the wedge number that the dart hits. So, if the dart hits anywhere in the 20 wedge, you earn 20 points for that throw.



- a. If you are equally likely to land in any wedge, what is the probability you will score 20 points?
- b. If you are equally likely to land in any wedge, what is the probability you will land in the upper right and score 20, 1, 18, 4, 13, or 6 points?
- c. Below are the results of 100 throws for one player. Does this player appear to have a tendency to land in the upper right more often than we would expect if the player was equally likely to land in any wedge?

| Points | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|--------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| Count  | 7 | 9 | 2 | 6 | 6 | 3 | 5 | 2 | 4 | 7  | 2  | 6  | 4  | 6  | 5  | 7  | 4  | 6  | 5  | 4  |

| A Progression Toward Mastery |                   |  |   |   |  |
|------------------------------|-------------------|--|---|---|--|
| Assessment Task Item         |                   | STEP 1<br>Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem. | STEP 2<br>Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.                            | STEP 3<br>A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem. | STEP 4<br>A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.  |
| 1                            | a<br><br>7.SP.C.7 | Student does not make use of the table. There is no evidence of reading the table.   | Student incorrectly reads the table.  | Student only reports 8.   | Student determines the total class size (38) and answers $\frac{8}{38}$ .  |
|                              | b<br><br>7.SP.C.7 | Student does not make use of the table. Student gives no evidence of reading the table.                                    | Student incorrectly reads the table.  | Student reports $\frac{3}{38}$ or $3 + 1 + 1 = 5$ .   | Student answers $3 + 1 + 1 = 5$ , so $\frac{5}{38}$ .  |
|                              | c<br><br>7.SP.C.7 | Student does not use simulation in answering this question.  | Student discusses simulation but does not provide a valid way of representing the outcomes of zero siblings or in estimating the final probability. | Student indicates how to represent the probability distribution (or just zero siblings) with simulation but does not address the question of estimating the probability of five or more students.   | Student makes a reasonable assignment to represent probabilities or just probability of zero siblings and then describes a simulation method, including finding the number of repetitions with five or more students. Student may not be perfectly clear on four trials vs. five trials. |
| 2                            | a<br><br>7.SP.C.6 | Student provides no understanding of probability.  | Student bases answer solely on intuition (or assumes 50/50) and does not use information from stem.   | Student uses the information about 201 out of 1,000 cell phone owners but does not report a probability in decimal or fraction format.  | Student uses $\frac{201}{1000} = 0.201$ or approximately $\frac{1}{5}$ .   |

|                             |   |   |   |  |
|-----------------------------|---|---|---|--|
| <b>b</b><br><b>7.SP.C.5</b> | Student does not address probability rules.                         | Student claims that the probabilities should sum to one.  | Student recognizes that the probabilities need not sum to one but does not provide complete justification. For example, student only states that these are not all the possible outcomes.                 | Student correctly identifies these probabilities as belonging to different sample spaces and/or identifies the correct “complement” events (a male <i>not</i> owning a cell phone).  |
| <b>c</b><br><b>7.SP.C.5</b> | Student demonstrates no understanding of probability or likelihood. | Student bases answer solely on intuition or focuses only on the fact that the sizes of the two groups will not be the same.     | Student uses the information from the problem but does not provide complete justification. For example, student only states “0.25 is larger” without clearly comparing to 0.201.                          | Student indicates that the outcome with probability 0.25 is more likely than the outcome with probability 0.201. (Note: Student may correctly use an incorrect value from part (a).) |
| <b>d</b><br><b>7.SP.C.7</b> | Student demonstrates no understanding of the term sample space.     | Student lists variables related to cell phone plans but does not address the cell phone company definitions of the three plans. | Student addresses the cell phone plan categorizations but does not have three outcomes or focuses on probabilities rather than outcomes.  | Student lists three cell phone plans (basic, pay-as-you-go, and unlimited).  |
| <b>e</b><br><b>7.SP.C.5</b> | Student identifies no errors.                                       | Student claims the three probabilities should be equal.   | Student only states that the probability for one of the categories must be larger or smaller, without complete justification based on needing nonnegative probabilities and probabilities summing to one. | Student cites the sum larger than one in Case 1 and cites the negative probability in Case 2.  |
| <b>f</b><br><b>7.SP.C.8</b> | Student provides no understanding of the term sample space.         | Student cites multiple outcomes but not compound outcomes for this chance experiment.   | Student cites compound outcomes but not consistently (some are compound but not all) or completely (not all six).   | Student cites the six compound outcomes (pay-as-you-go, over 500); (pay-as-you-go, under 500); (unlimited, under 500); (unlimited, over 500); (basic, under 500); (basic, over 500). |

|          |                             |  |  |   |  |
|----------|-----------------------------|--|--|---|--|
|          | <b>g</b><br><b>7.SP.C.7</b> | Student does not provide a viable probability.   | Student assigns an incorrect probability based on faulty assumptions (e.g., 0.5 because equally likely) or focuses on only one variable ( $\frac{1}{3}$ because three categories). | Student uses the answer to part (f) but incorrectly or inconsistently (e.g., only focuses on pay-as-you-go).  | Student correctly considers the six outcomes and which one corresponds to this compound event ( $\frac{1}{6}$ ). Answer may be incorrect but may follow from answer in part (f).   |
|          | <b>h</b><br><b>7.SP.C.7</b> | Student does not make use of $\frac{140}{500}$ . | Student computes relative frequency and comments on its value but does not address equally likelihood of outcomes in sample space.   | Student computes relative frequency, $\frac{140}{500} = 0.280$ , and makes a judgment about equally likely but does not give a clear comparison value (e.g., compares to 0.250 but not how 0.250 was obtained). | Student compares observed relative frequency, $\frac{140}{500} = 0.280$ , to value corresponding to his or her answer to (f) ( $\frac{1}{6}$ ) or compares 140 to $\frac{500}{6} = 83.33$ .  |
| <b>3</b> | <b>a</b><br><b>7.SP.C.7</b> | Student is not able to calculate a probability.  | Student attempts to estimate a probability but does not see connection to number of wedges and equal probability.  | Student claims the outcomes are equally likely with no justification or value provided.   | Student specifies $\frac{1}{20}$ .   |
|          | <b>b</b><br><b>7.SP.C.8</b> | Student is not able to calculate a probability.  | Student attempts to estimate a probability but does not see connection to number of wedges in upper right region.  | Student answer uses either equal probability condition or number of regions in upper right but does not connect the two (e.g., $\frac{1}{6}$ ).   | Student specifies $\frac{6}{20}$ or $\frac{3}{10}$ or 0.30.  |
|          | <b>c</b><br><b>7.SP.C.6</b> | Student does not answer the question.            | Student makes errors in total counts and fails to compare the result to the answer from part (b).  | Student makes errors in totaling the counts but is focused on the counts in those six regions or fails to compare the result to the answer in part (b).   | Student determines the number of hits in the upper right and divides by the number of throws:<br>$\frac{4 + 7 + 6 + 6 + 4 + 3}{100} = 0.30$ .<br>This answer is then compared to $\frac{6}{20} = 0.30$ , and student concludes there is no evidence that this player is landing in that region more often than expected. |



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Date \_\_\_\_\_

Round all decimal answers to the nearest hundredth.

1. Each student in a class of 38 students was asked to report how many siblings (brothers and sisters) he or she has. The data are summarized in the table below.

| Number of Siblings | 0 | 1  | 2  | 3 | 4 | 5 | 6 |
|--------------------|---|----|----|---|---|---|---|
| Count              | 8 | 13 | 12 | 3 | 1 | 0 | 1 |

- a. Based on the data, estimate the probability that a randomly selected student from this class is an only child.

$$\frac{8}{38} = 0.21$$

- b. Based on the data, estimate the probability that a randomly selected student from this class has three or more siblings.

$$\frac{3+0+1+1}{38} = \frac{5}{38} = 0.13$$

- c. Consider the following probability distribution for the number of siblings:

| Number of Siblings | 0    | 1    | 2    | 3    | 4    | 5    | 6    |
|--------------------|------|------|------|------|------|------|------|
| Probability        | 0.15 | 0.35 | 0.30 | 0.10 | 0.05 | 0.03 | 0.02 |

Explain how you could use simulation to estimate the probability that you will need to ask at least five students the question, “Are you an only child?” if each is an only child before you find the first one that is an only child.

*Put 100 chips in a bag with 15 red and 85 blue. Red represents the portion of students who don't have a sibling and blue represents the portion of students who have one or more siblings. Pull out chips until you get a red one (putting the chips back each time) and record the number of chip it took to pull out a red chip. Repeat 1000 times and see how often you need 5 or more chips.*

2. A cell phone company wants to predict the probability of a seventh grader in your city, City A, owning a cell phone. Records from another city, City B, indicate that 201 of 1,000 seventh graders own a cell phone.

- a. Assuming the probability of a seventh grader owning a cell phone is similar for the two cities, estimate the probability that a randomly selected seventh grader from City A owns a cell phone.

$$\frac{201}{1000} = 0.201$$

- b. The company estimates the probability that a randomly selected seventh-grade male owns a cell phone is 0.25. Does this imply that the probability that a randomly selected seventh-grade female owns a cell phone is 0.75? Explain.

*No 0.75 would be the probability a male does not own a cell phone. The probabilities for females could be very different.*

c. According to the data, which of the following is more likely?

- a seventh-grade male owning a cell phone
- a seventh grader owning a cell phone

Explain your choice.

*male = 0.25 > overall = 0.201. This implies the probability is a little higher for male seventh-grade students than for all seventh-grade students so males are more likely.*

Suppose the cell phone company sells three different plans to its customers:

- Pay-as-you-go: Customer is charged per minute for each call.
- Unlimited minutes: Customer pays a flat fee per month and can make unlimited calls with no additional charges.
- Basic plan: Customer not charged per minute unless the customer exceeds 500 minutes in the month; then, the customer is charged per minute for the extra minutes.

Consider the chance experiment of selecting a customer at random and recording which plan they purchased.

d. What outcomes are in the sample space for this chance experiment?

- (1) *pay-as-you-go*
- (2) *unlimited minutes*
- (3) *basic plan*

e. The company wants to assign probabilities to these three plans. Explain what is wrong with each of the following probability assignments.

Case 1: Probability of pay-as-you-go = 0.40, probability of unlimited minutes = 0.40, and probability of basic plan = 0.30.

*The sum of the three probabilities in the sample space is larger than 1 which cannot happen.*

Case 2: Probability of pay-as-you-go = 0.40, probability of unlimited minutes = 0.70, and probability of basic plan = -0.10.

*Cannot have negative probabilities*

Now consider the chance experiment of randomly selecting a cell phone customer and recording both the cell phone plan for that customer and whether or not the customer exceeded 500 minutes last month.

- f. One possible outcome of this chance experiment is (pay-as-you-go, <500). What are the other possible outcomes in this sample space?

*(pay-as-you-go, <500)*

*(basic plan, ≥500)*

*(basic plan, <500)*

*(unlimited, >500)*

*(unlimited, <500)*

*(put exactly 500 in <500)*

- g. Assuming the outcomes of this chance experiment are equally likely, what is the probability that the selected cell phone customer had a basic plan and did not exceed 500 minutes last month?

$$\frac{1}{6} \approx 0.167$$

- h. Suppose the company randomly selects 500 of its customers and finds that 140 of these customers purchased the basic plan and did not exceed 500 minutes. Would this cause you to question the claim that the outcomes of the chance experiment described in part (g) are equally likely? Explain why or why not.

$$\frac{140}{500} = 0.280$$

*Yes, 0.280 is a lot higher than 0.167, which seems to indicate this outcome is more likely than some of the others.*

3. In the game of Darts, players throw darts at a circle divided into 20 wedges. In one variation of the game, the score for a throw is equal to the wedge number that the dart hits. So, if the dart hits anywhere in the 20 wedge, you earn 20 points for that throw.



- a. If you are equally likely to land in any wedge, what is the probability you will score 20 points?

$$\frac{1}{20} = 0.05$$

- b. If you are equally likely to land in any wedge, what is the probability you will land in the upper right and score 20, 1, 18, 4, 13, or 6 points?

$$\frac{6}{20} = 0.30$$

- c. Below are the results of 100 throws for one player. Does this player appear to have a tendency to land in the upper right more often than we would expect if the player was equally likely to land in any wedge?

| Points | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|--------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| Count  | 7 | 9 | 2 | 6 | 6 | 3 | 5 | 2 | 4 | 7  | 2  | 6  | 4  | 6  | 5  | 7  | 4  | 6  | 5  | 4  |

$$\frac{7+6+3+4+6+4}{100} = \frac{30}{100}$$

*This is exactly how often we would predict this to happen.*