Name _____ Date _____

1.

a. We define x as a year between 2008 and 2013, and y as the total number of smartphones sold that year, in millions. The table shows values of x and corresponding y values.

Year (x)	2008	2009	2010	2011	2012	2013
Number of smartphones in millions (y)	3.7	17.3	42.4	90	125	153.2

i. How many smartphones were sold in 2009?

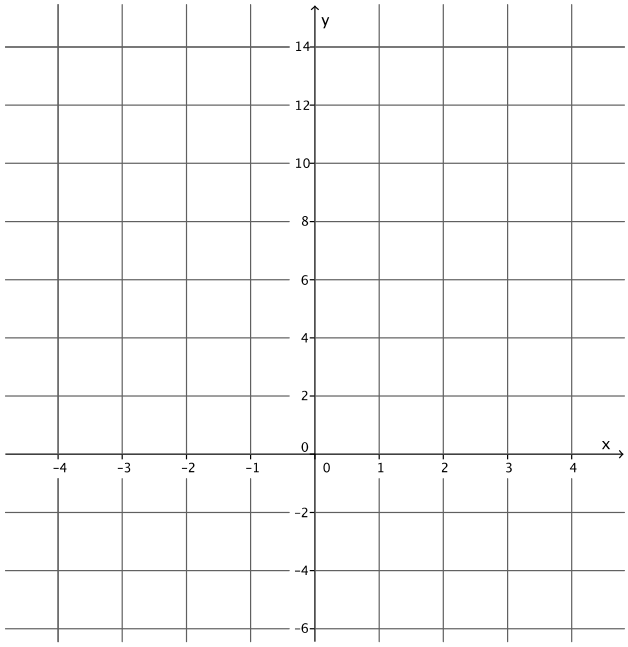
ii. In which year were 90 million smartphones sold?

iii. Is y a function of x? Explain why or why not.

b. Randy began completing the table below to represent a particular linear function. Write an equation to represent the function he used, and complete the table for him.

Input (x)	-3	-1	0	$\frac{1}{2}$	1	2	3
Output (y)	-5		4				13

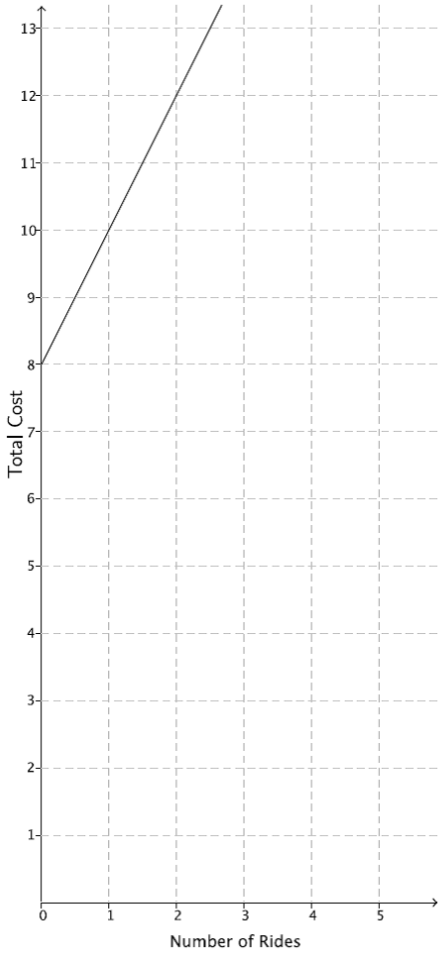
c. Create the graph of the function in part (b).



d. At NYU in 2013, the cost of the weekly meal plan options could be described as a function of the number of meals. Is the cost of the meal plan a linear or nonlinear function? Explain.

8 meals: $125/week
10 meals: $135/week
12 meals: $155/week
21 meals: $220/week

2. The cost to enter and go on rides at a local water park, Wally's Water World, is shown in the graph below.



A new water park, Tony's Tidal Takeover, just opened. You have not heard anything specific about how much it costs to go to this park, but some of your friends have told you what they spent. The information is organized in the table below.

Number of rides	0	2	4	6
Dollars spent	\$12.00	\$13.50	\$15.00	\$16.50

Each park charges a different admission fee and a different fee per ride, but the cost of each ride remains the same.

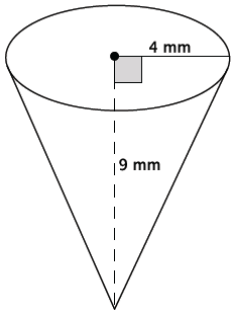
a. If you only have \$14 to spend, which park would you attend (assume the rides are the same quality)? Explain.

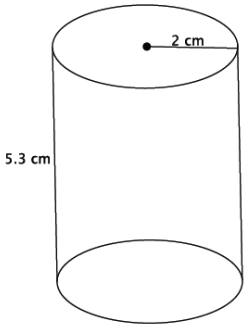
b. Another water park, Splash, opens, and they charge an admission fee of $30 with no additional fee for rides. At what number of rides does it become more expensive to go to Wally's Water World than Splash? At what number of rides does it become more expensive to go to Tony's Tidal Takeover than Splash?

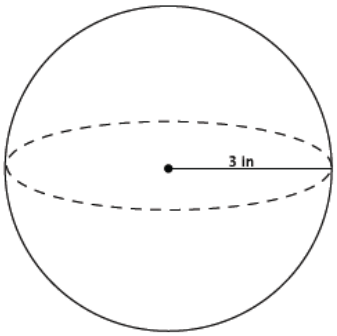
c. For all three water parks, the cost is a function of the number of rides. Compare the functions for all three water parks in terms of their rate of change. Describe the impact it has on the total cost of attending each park.

3. For each part below, leave your answers in terms of π.

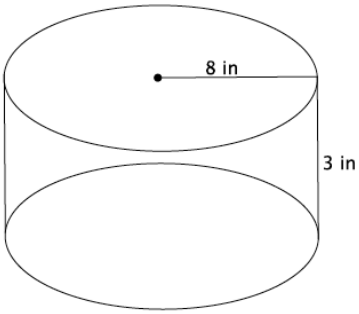
   a. Determine the volume for each three-dimensional figure shown below.



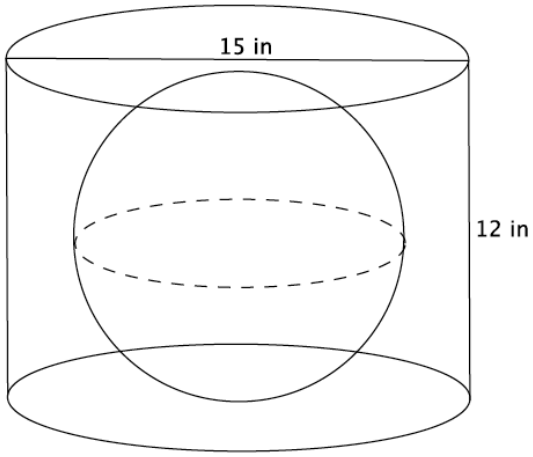




   b. You want to fill the cylinder shown below with water. All you have is a container shaped like a cone with a radius of 3 inches and a height of 5 inches; you can use this cone-shaped container to take water from a faucet and fill the cylinder. How many cones will it take to fill the cylinder?



c. You have a cylinder with a diameter of 15 inches and height of 12 inches. What is the volume of the largest sphere that will fit inside of it?



A Progression Toward Mastery					
Assessment Task Item		**STEP 1** Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	**STEP 2** Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	**STEP 3** A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	**STEP 4** A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	a 8.F.A.1	Student makes little or no attempt to solve the problem.	Student answers at least one of the three questions correctly as 17.3 million, 2011, or yes. Student does not provide an explanation as to why y is a function of x.	Student answers all three questions correctly as 17.3 million, 2011, and yes. Student provides an explanation as to why y is a function of x. Student may not have used vocabulary related to functions.	Student answers all three questions correctly as 17.3 million, 2011, and yes. Student provides a *compelling* explanation as to why y is a function of x and uses appropriate vocabulary related to functions (e.g., assignment, input, and output).
	b 8.F.A.1	Student makes little or no attempt to solve the problem. Student does not write a function or equation. The outputs may or may not be calculated correctly.	Student does not correctly write the equation to describe the function. The outputs may be correct for the function described by the student. The outputs may or may not be calculated correctly. Student may have made calculation errors. Two or more of the outputs are calculated correctly.	Student correctly writes the equation to describe the function as $y = 3x + 4$. Three or more of the outputs are calculated correctly. Student may have made calculation errors.	Student correctly writes the equation to describe the function as $y = 3x + 4$. All four of the outputs are calculated correctly as when $x = -1$, $y = 1$; when $x = \frac{1}{2}$, $y = \frac{11}{2}$; when $x = 1$, $y = 7$; and when $x = 2$, $y = 10$.

	c **8.F.A.1**	Student makes little or no attempt to solve the problem. Student may have graphed some or all of the input/outputs given.	Student graphs the input/outputs incorrectly (e.g., $(4,0)$ instead of $(0,4)$). The input/outputs do not appear to be linear.	Student may or may not have graphed the input/outputs correctly (e.g., $(4,0)$ instead of $(0,4)$). The input/outputs appear to be linear.	Student graphs the input/outputs correctly as $(0,4)$. The input/outputs appear to be linear.
	d **8.F.A.3**	Student makes little or no attempt to solve the problem. Student may or may not have made a choice. Student does not give an explanation.	Student incorrectly determines that the meal plan is linear or correctly determines that it is nonlinear. Student does not give an explanation, or the explanation does not include any mathematical reasoning.	Student correctly determines that the meal plan is nonlinear. Explanation includes some mathematical reasoning. Explanation may or may not include reference to the graph.	Student correctly determines that the meal plan is nonlinear. Explanation includes substantial mathematical reasoning. Explanation includes reference to the graph.
2	**a** **8.F.A.2**	Student makes little or no attempt to solve the problem. Student may or may not have made a choice. Student does not give an explanation.	Student identifies either choice. Student makes significant calculation errors. Student gives little or no explanation.	Student identifies either choice. Student may have made calculation errors. Explanation may or may not have included the calculation errors.	Student identifies Wally's Water World as the better choice. Student references that for $\$14$ he can ride three rides at Wally's Water World but only two rides at Tony's Tidal Takeover.
	b **8.F.A.2**	Student makes little or no attempt to solve the problem. Student does not give an explanation.	Student identifies the number of rides at both parks incorrectly. Student may or may not identify functions to solve the problem. For example, student uses the table or counting method. Student makes some attempt to find the function for one or both of the parks. The functions used are incorrect.	Student identifies the number of rides at one of the parks correctly. Student makes some attempt to identify the function for one or both of the parks. Student may or may not identify functions to solve the problem. For example, student uses the table or counting method. One function used is correct.	Student identifies that the 25^{th} ride at Tony's Tidal Takeover makes it more expensive than Splash. Student may have stated that he could ride 24 rides for $\$30$ at Tony's. Student identifies that the 12^{th} ride at Wally's Water World makes it more expensive than Splash. Student may have stated that he could ride 11 rides for $\$30$ at Wally's. Student identifies functions to solve the problem (e.g., if x is the number of rides, $w = 2x + 8$ for the cost of Wally's, and $t = 0.75x + 12$ for the cost of Tony's).

	c **8.F.A.2**	Student makes little or no attempt to solve the problem.	Student may have identified the rate of change for each park, but does so incorrectly. Student may not have compared the rate of change for each park. Student may have described the impact of the rate of change on total cost for one or two of the parks, but draws incorrect conclusions.	Student correctly identifies the rate of change for each park. Student may or may not have compared the rate of change for each park. Student may have described the impact of the rate of change on total cost for all parks, but makes minor mistakes in the description.	Student correctly identifies the rate of change for each park: Wally's is 2, Tony's is 0.75, and Splash is 0. Student compares the rate of change for each park and identifies which park has the greatest rate of change (or least rate of change) as part of the comparison. Student describes the impact of the rate of change on the total cost for each park.
3	**a** **8.G.C.9**	Student makes little or no attempt to solve the problem. Student finds none or one of the volumes correctly. Student may or may not have included correct units. Student may have omitted π from one or more of the volumes (i.e., the volume of the cone is 48).	Student finds two out of three volumes correctly. Student may or may not have included correct units. Student may have omitted π from one or more of the volumes (i.e., the volume of the cone is 48).	Student finds all three of the volumes correctly. Student does not include the correct units. Student may have omitted π from one or more of the volumes (i.e., the volume of the cone is 48).	Student finds all three of the volumes correctly, that is, the volume of the cone is 48π mm^3, the volume of the cylinder is 21.2π cm^3, and the volume of the sphere is 36π in^3. Student includes the correct units.
	b **8.G.C.9**	Student makes little or no attempt to solve the problem.	Student does not correctly calculate the number of cones. Student makes significant calculation errors. Student may have used the wrong formula for volume of the cylinder or the cone. Student may not have answered in a complete sentence.	Student may have correctly calculated the number of cones, but does not correctly calculate the volume of the cylinder or cone (e.g., volume of the cone is 192, omitting the π). Student correctly calculates the volume of the cone at 15π in^3 or the volume of the cylinder at 192π in^3, but not both. Student may have used incorrect units. Student may have made minor calculation errors. Student may not answer in a complete sentence.	Student correctly calculates that it will take 12.8 cones to fill the cylinder. Student correctly calculates the volume of the cone at 15π in^3 and the volume of the cylinder at 192π in^3. Student answers in a complete sentence.

	c **8.G.C.9**	Student makes little or no attempt to solve the problem.	Student does not correctly calculate the volume. Student may have used the diameter instead of the radius for calculations. Student may have made calculation errors. Student may or may not have omitted π. Student may or may not have included the units.	Student correctly calculates the volume, but does not include the units or includes incorrect units (e.g., in^2). Student uses the radius of 6 to calculate the volume. Student may have calculated the volume as 288 (π is omitted).	Student correctly calculates the volume as 288π in^3. Student uses the radius of 6 to calculate the volume. Student includes correct units.

Name _____ Date _____

1.

a. We define x as a year between 2008 and 2013 and y as the total number of smartphones sold that year, in millions. The table shows values of x and corresponding y values.

Year (x)	2008	2009	2010	2011	2012	2013
Number of smartphones in millions (y)	3.7	17.3	42.4	90	125	153.2

How many smartphones were sold in 2009?

17.3 MILLION SMARTPHONES WERE SOLD IN 2009.

In which year were 90 million smartphones sold?

90 MILLION SMARTPHONES WERE SOLD IN 2011.

Is y a function of x? Explain why or why not.

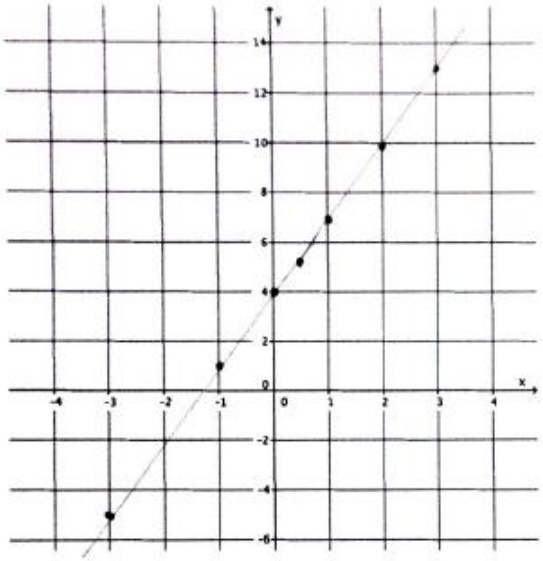
YES IT IS A FUNCTION BECAUSE FOR EACH INPUT THERE IS EXACTLY ONE OUTPUT. SPECIFICALLY, ONLY ONE NUMBER WILL BE ASSIGNED TO REPRESENT THE NUMBER OF SMARTPHONES SOLD IN THE GIVEN YEAR.

b. Randy began completing the table below to represent a particular linear function. Write an equation to represent the function he used, and complete the table for him.

Input (x)	-3	-1	0	$\frac{1}{2}$	1	2	3
Output (y)	-5	1	4	$\frac{11}{2}$	7	1	13

$y = 3x + 4$

c. Create the graph of the function in part (b).

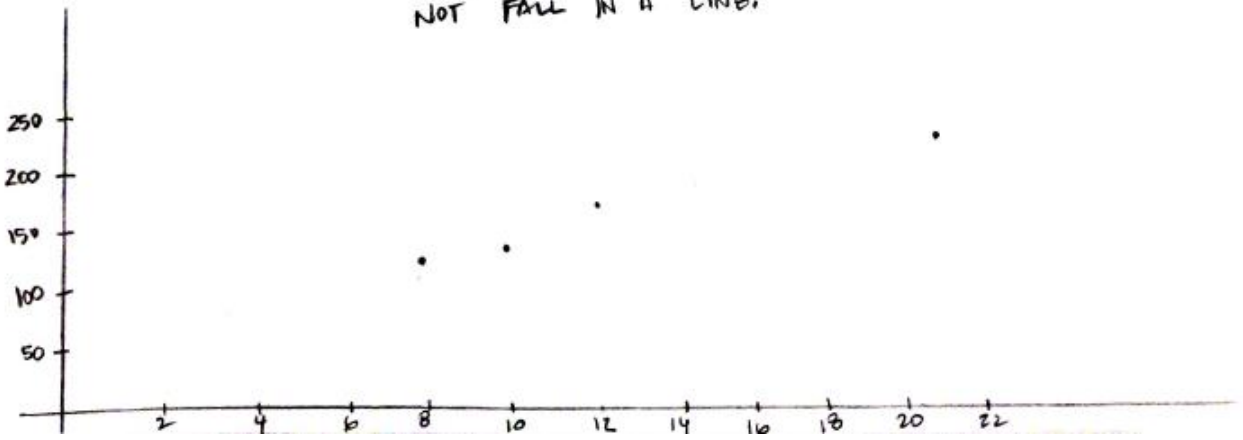


d. At NYU in 2013, the cost of the weekly meal plan options could be described as a function of the number of meals. Is the cost of the meal plan a linear or non-linear function? Explain.

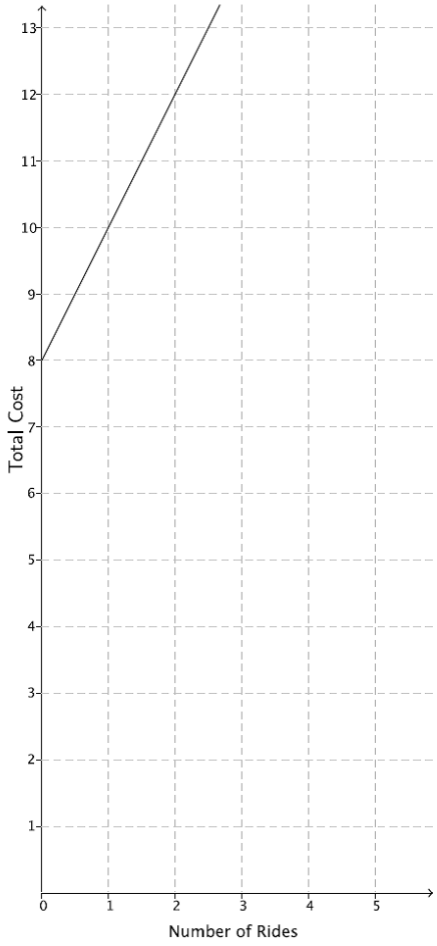
8 meals: \$125/week
10 meals: \$135/week
12 meals: \$155/week
21 meals: \$220/week

$\frac{125}{8} = 15.625$ $\frac{135}{10} = 13.5$ $\frac{155}{12} = 12.917$ $\frac{220}{21} = 10.476$

THE COST OF THE MEAL PLAN IS A NON-LINEAR FUNCTION. THE COST OF EACH MEAL IS DIFFERENT BASED ON THE PLAN. FOR EXAMPLE, ONE PLAN CHARGES ABOUT \$16 PER MEAL, ANOTHER PLAN CHARGES JUST \$10. ALSO, WHEN THE DATA IS GRAPHED, THE POINTS DO NOT FALL IN A LINE.



2. The cost to enter and go on rides at a local water park, Wally's Water World, is shown in the graph below.



LET x REPRESENT THE # OF RIDES
LET W REPRESENT THE TOTAL COST AT WALLY'S WATER WORLD
$W = 2x + 8$

A new water park just opened named Tony's Tidal Takeover. You haven't heard anything specific about how much it costs to go to this park but some of your friends have told you what they spent. The information is organized in the table below; your friends told you they paid an admission fee to get in and then the same amount for each ride.

# of rides	0	2	4	6
$ spent	12	13.50	15	16.50

1.50 1.50

LET x REPRESENT THE # OF RIDES
LET T REPRESENT TOTAL COST AT TONY'S TIDAL TAKEOVER
$T = 0.75x + 12$

a. If you only have $14 to spend, which park would you attend (assume the rides are the same quality)? Explain.

WALLY'S
$W = 2x + 8$
$14 = 2x + 8$
$6 = 2x$
$3 = x$

TONY'S
$T = 0.75x + 12$
$14 = 0.75x + 12$
$2 = 0.75x$
$2.67 \approx x$

AT WALLY'S, YOU CAN GO ON 3 RIDES WITH $14, AT TONY'S JUST 2 RIDES. THEREFORE I WOULD GO TO WALLY'S BECAUSE YOU CAN GO ON MORE RIDES.

b. Another water park, Splash, opens and they charge an admission fee of $30 with no additional fee for rides. At what number of rides does it become more expensive to go to Wally's Water Park than Splash? At what number of rides does it become more expensive to go to Tony's Tidal Takeover than Splash?

LET S REPRESENT TOTAL COST AT SPLASH, $S = 30$.

WALLY'S
$30 = 2x + 8$
$22 = 2x$
$11 = x$

TONY'S
$30 = 0.75x + 12$
$18 = 0.75x$
$24 = x$

AT WALLY'S YOU CAN GO ON 11 RIDES WITH $30. THE 12th RIDE MAKES WALLY'S MORE EXPENSIVE THAN SPLASH.

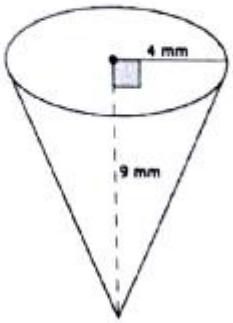
AT TONY'S YOU CAN GO ON 24 RIDES WITH $30. THE 25th RIDES MAKES TONY'S MORE EXPENSNE THAN SPLASH.

c. For all three water parks, the cost is a function of the number of rides. Compare the functions for all three water parks in terms of their rate of change. Describe the impact it has on the total cost of attending each park.

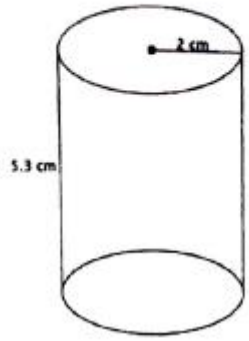
WALLY'S RATE OF CHANGE IS 2, $2 PER RIDE.
TONY'S RATE OF CHANGE IS 0.75, $0.75 PER RIDE.
SPLASH'S RATE OF CHANGE IS 0, $0 EXTRA PER RIDE.
WALLY'S HAS THE GREATEST RATE OF CHANGE. THAT MEANS THAT THE TOTAL COST AT WALLY'S WILL INCREASE THE FASTEST AS WE GO ON MORE RIDES. AT TONY'S, THE RATE OF CHANGE IS JUST 0.75 SO THE TOTAL COST INCREASES WITH THE NUMBER OF RIDES WE GO ON, BUT NOT AS QUICKLY AS WALLY'S. SPLASH HAS A RATE OF CHANGE OF ZERO. THE NUMBER OF RIDES WE GO ON DOES NOT IMPACT THE TOTAL COST AT ALL.

3.

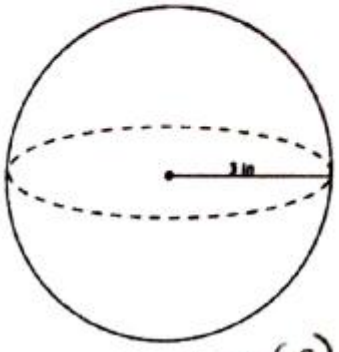
a. Determine the volume for each of the three-dimensional figures shown below.



$V = \frac{1}{3}\pi(16)(9)$
$= (16)(3)(\pi)$
$= 48\pi \text{ mm}^3$

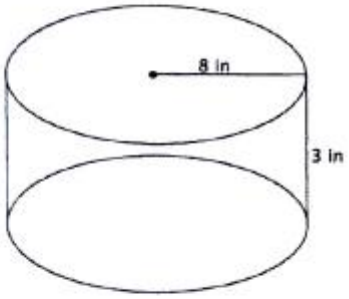


$V = \pi(4)(5.3)$
$= \pi(21.2)$
$= 21.2\pi \text{ cm}^3$



$V = \frac{4}{3}\pi(3^3)$
$= 4(9)\pi$
$= 36\pi \text{ in}^3$

b. You want to fill the cylinder shown below with water. All you have is a container shaped like a cone with a radius of 3 inches and a height of 5 inches; you can use this cone-shaped container to take water from a faucet and fill the cylinder. How many cones will it take to fill the cylinder?



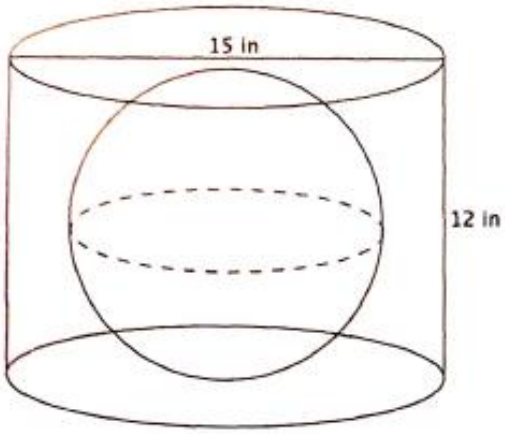
VOLUME OF CYLINDER $= \pi(64)(3)$
$= 192\pi \text{ in}^3$

VOLUME OF CONE $= \frac{1}{3}\pi(9)(5)$
$= \frac{45}{3}\pi$
$= 15\pi \text{ in}^3$

$\frac{192\pi}{15\pi} = \frac{192}{15} = 12.8$

IT TAKES 12.8 CONES OF THE GIVEN SIZE TO FILL THE CYLINDER.

c. You have a cylinder with a diameter of 15 cm and height of 12 cm. What is the volume of the largest sphere that will fit inside of it?



THE CYLINDER HAS RADIUS OF 7.5 cm, BUT THE HEIGHT IS JUST 12cm. THAT MEANS THE MAXIMUM RADIUS FOR THE SPHERE IS 6 cm. ANYTHING LARGER WOULD NOT FIT IN THE CYLINDER. THEN THE VOLUME OF THE LARGEST SPHERE THAT WILL FIT IN THE CYLINDER IS

$$V = \frac{4}{3}\pi(6^3)$$
$$= \frac{4}{3}\pi(216)$$
$$= 288\pi \text{ cm}^3.$$