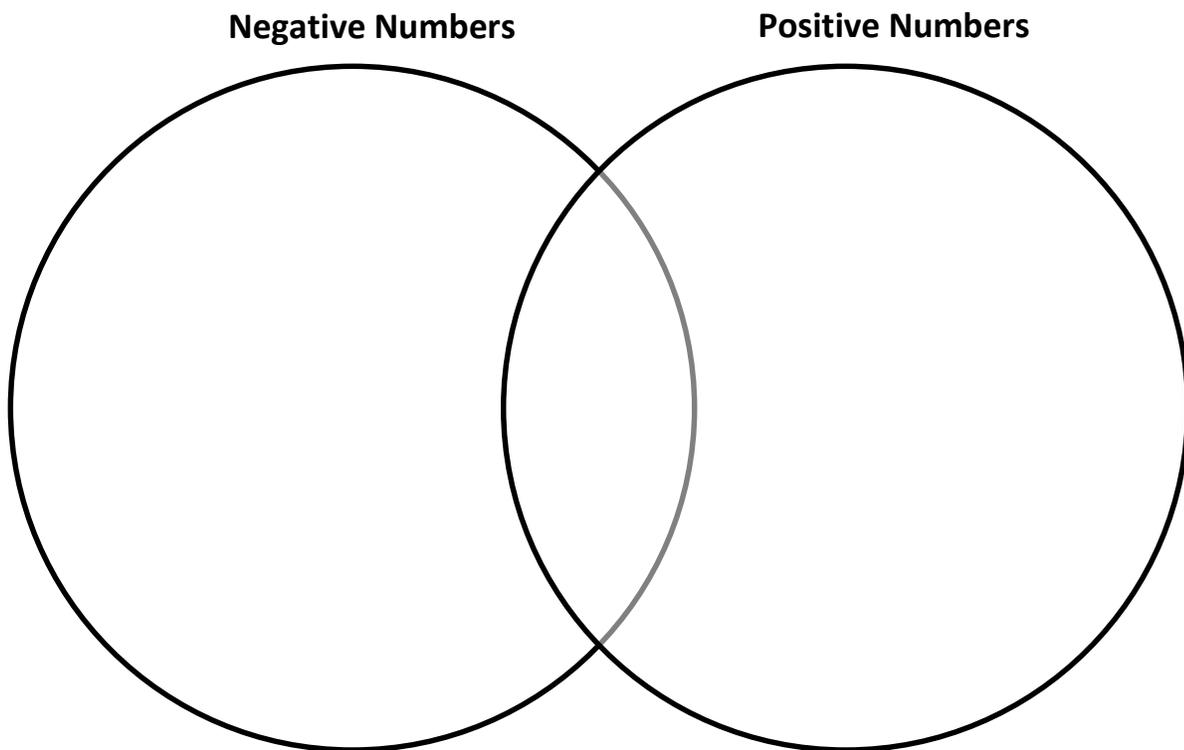


## Lesson 1: Opposite Quantities Combine to Make Zero

### Classwork

#### Exercise 1: Positive and Negative Numbers Review

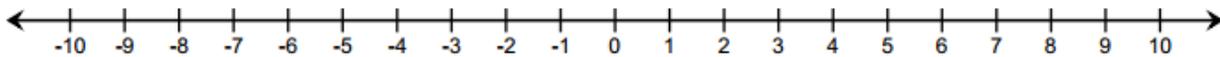
With your partner, use the graphic organizer below to record what you know about positive and negative numbers. Add or remove statements during the whole class discussion.



**Example 2: Counting Up and Counting Down on the Number Line**

Use the number line below to practice counting up and counting down.

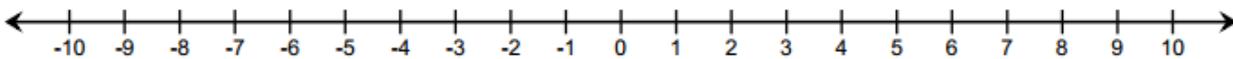
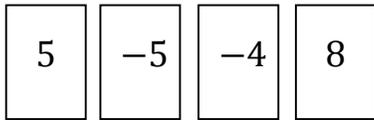
- *Counting up* corresponds to \_\_\_\_\_ numbers.
- *Counting down* corresponds to \_\_\_\_\_ numbers.



- Where do you begin when locating a number on the number line?
- What do you call the distance between a number and 0 on a number line?
- What is the relationship between 7 and  $-7$ ?

**Example 3: Using the Integer Game and the Number Line**

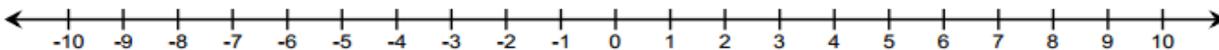
What is the sum of the card values shown? Use the counting on method on the provided number line to justify your answer.



- a. What is the final position on the number line? \_\_\_\_\_
- b. What card or combination of cards would you need to get back to 0? \_\_\_\_\_

**Exercise 2: The Additive Inverse**

Use the number line to answer each of the following questions.



- a. How far is 7 from 0 and in which direction? \_\_\_\_\_
- b. What is the opposite of 7? \_\_\_\_\_
- c. How far is -7 from 0 and in which direction? \_\_\_\_\_

- d. Thinking back to our previous work, how would you use the counting on method to represent the following: While playing the Integer Game, the first card selected is 7, and the second card selected is  $-7$ .
- e. What does this tell us about the sum of 7 and its opposite,  $-7$ ?
- f. Look at the curved arrows you drew for 7 and  $-7$ . What relationship exists between these two arrows that would support your claim about the sum of 7 and  $-7$ ?
- g. Do you think this will hold true for the sum of any number and its opposite?

For all numbers  $a$  there is a number  $-a$ , such that  $a + (-a) = 0$ .

The additive inverse of a real number is the opposite of that number on the real number line. For example, the opposite of  $-3$  is 3. A number and its additive inverse have a sum of 0. The sum of any number and its opposite is equal to zero.

### Exercise 3: Playing the Integer Game

Play the Integer Game with your group. Use a number line to practice counting on.

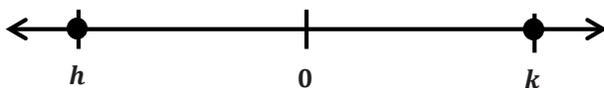
## Lesson Summary

- Add positive integers by counting up, and add negative integers by counting down.
- An integer plus its opposite sum to zero.
- The opposite of a number is called the additive inverse because the two numbers' sum is zero.

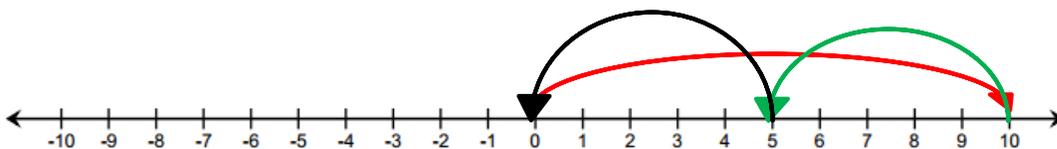
## Problem Set

For Problems 1 and 2, refer to the Integer Game.

1. You have two cards with a sum of  $(-12)$  in your hand.
  - a. What two cards could you have?
  - b. You add two more cards to your hand, but the total sum of the cards remains the same,  $(-12)$ . Give some different examples of two cards you could choose.
2. Choose one card value and its additive inverse. Choose from the list below to write a real-world story problem that would model their sum.
  - a. Elevation: above and below sea level
  - b. Money: credits and debits, deposits and withdrawals
  - c. Temperature: above and below 0 degrees
  - d. Football: loss and gain of yards
3. On the number line below, the numbers  $h$  and  $k$  are the same distance from 0. Write an equation to express the value of  $h + k$ . Explain.



4. During a football game, Kevin gained five yards on the first play. Then he lost seven yards on the second play. How many yards does Kevin need on the next play to get the team back to where they were when they started? Show your work.
5. Write an addition number sentence that corresponds to the arrows below.



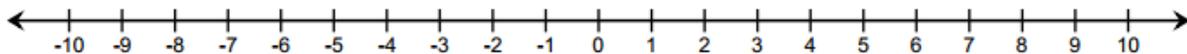
## Lesson 2: Using the Number Line to Model the Addition of Integers

### Classwork

#### Exercise 1: Real-World Introduction to Integer Addition

Answer the questions below.

- Suppose you received \$10 from your grandmother for your birthday. You spent \$4 on snacks. Using addition, how would you write a number sentence to represent this situation?
- How would you model your equation on a number line to show your answer?

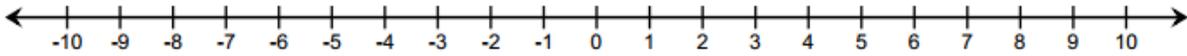


#### Example 1: Modeling Addition on the Number Line

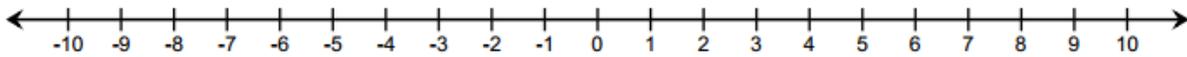
Complete the steps to finding the sum of  $-2 + 3$  by filling in the blanks. Model the number sentence using straight arrows called *vectors* on the number line below.

- Place the tail of the arrow on \_\_\_\_\_.
- Draw the arrow 2 units to the left of 0, and stop at \_\_\_\_\_. The direction of the arrow is to the \_\_\_\_\_ since you are counting down from 0.
- Start the next arrow at the end of the first arrow, or at \_\_\_\_\_.
- Draw the second arrow \_\_\_\_\_ units to the right since you are counting up from  $-2$ .
- Stop at \_\_\_\_\_.

- f. Circle the number at which the second arrow ends to indicate the ending value.



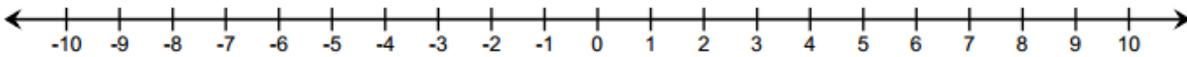
- g. Repeat the process from parts (a)–(f) for the expression  $3 + (-2)$ .



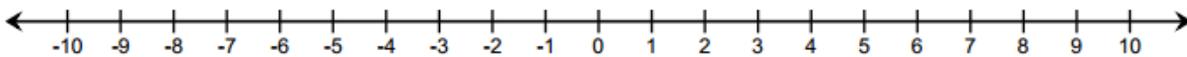
- h. What can you say about the sum of  $-2 + 3$  and  $3 + (-2)$ ? Does order matter when adding numbers? Why or why not?

**Example 2: Expressing Absolute Value as the Length of an Arrow on the Real Number Line**

- a. How does absolute value determine the arrow length for  $-2$ ?



- b. How does the absolute value determine the arrow length for 3?

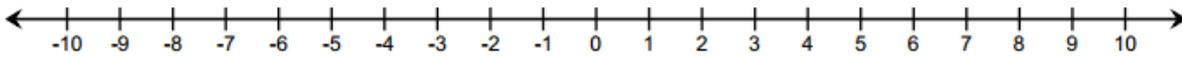


- c. How does absolute value help you to represent  $-10$  on a number line?

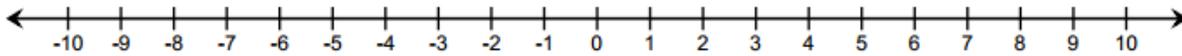
**Exercise 2**

Create a number line model to represent each of the expressions below.

- a.  $-6 + 4$

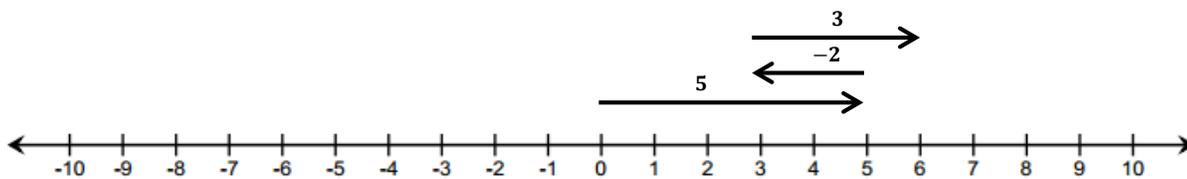


- b.  $3 + (-8)$



**Example 3: Finding Sums on a Real Number Line Model**

Find the sum of the integers represented in the diagram below.



- a. Write an equation to express the sum.
- b. What three cards are represented in this model? How did you know?

- c. In what ways does this model differ from the ones we used in Lesson 1?
- d. Can you make a connection between the sum of 6 and where the third arrow ends on the number line?
- e. Would the sum change if we changed the order in which we add the numbers, for example,  $(-2) + 3 + 5$ ?
- f. Would the diagram change? If so, how?

### Exercise 3

Play the Integer Game with your group. Use a number line to practice “counting on”.

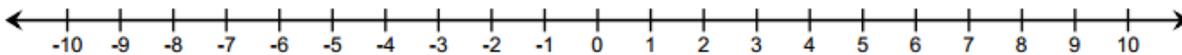
**Lesson Summary**

- On a number line, arrows are used to represent integers; they show length and direction.
- The length of an arrow on the number line is the absolute value of the integer.
- Adding several arrows is the same as combining integers in the Integer Game.
- The sum of several arrows is the final position of the last arrow.

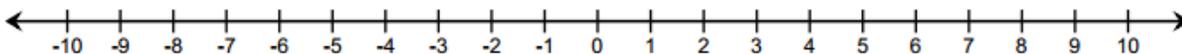
**Problem Set**

For Questions 1–3, represent each of the following problems using both a number line diagram and an equation.

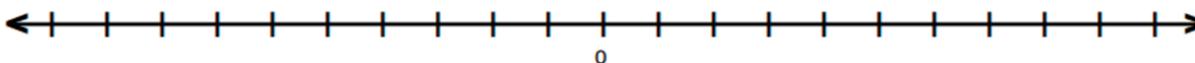
1. David and Victoria are playing the Integer Card Game. David drew three cards,  $-6$ ,  $12$ , and  $-4$ . What is the sum of the cards in his hand? Model your answer on the number line below.



2. In the Integer Card Game, you drew the cards,  $2$ ,  $8$ , and  $-11$ . Your partner gave you a  $7$  from his hand.  
 a. What is your total? Model your answer on the number line below.



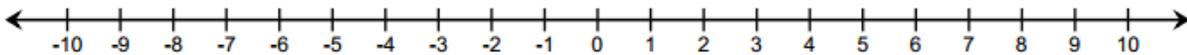
- b. What card(s) would you need to get your score back to zero? Explain. Use and explain the term “additive inverse” in your answer.
3. If a football player gains  $40$  yards on a play, but on the next play, he loses  $10$  yards, what would his total yards be for the game if he ran for another  $60$  yards? What did you count by to label the units on your number line?



4. Find the sums.

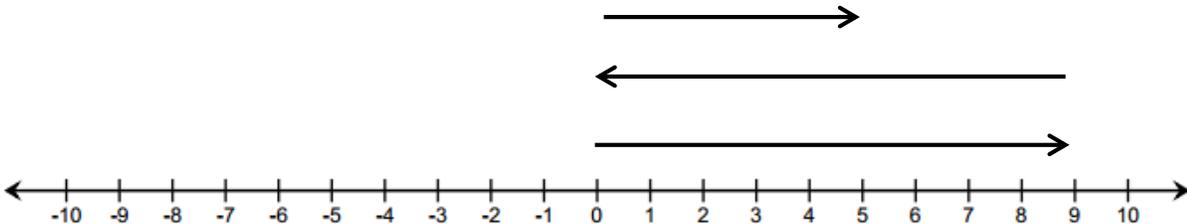
- a.  $-2 + 9$
- b.  $-8 + -8$
- c.  $-4 + (-6) + 10$
- d.  $5 + 7 + (-11)$

5. Mark an integer between 1 and 5 on a number line, and label it point  $Z$ . Then, locate and label each of the following points by finding the sums.

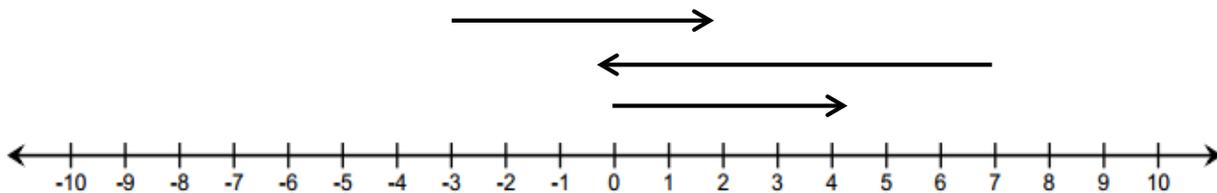


- a. Point  $A$ :  $Z + 5$
- b. Point  $B$ :  $Z + (-3)$
- c. Point  $C$ :  $(-4) + (-2) + Z$
- d. Point  $D$ :  $-3 + Z + 1$

6. Write a story problem that would model the sum of the arrows in the number diagram below.



7. Do the arrows correctly represent the equation  $4 + (-7) + 5 = 2$ ? If not, draw a correct model below.



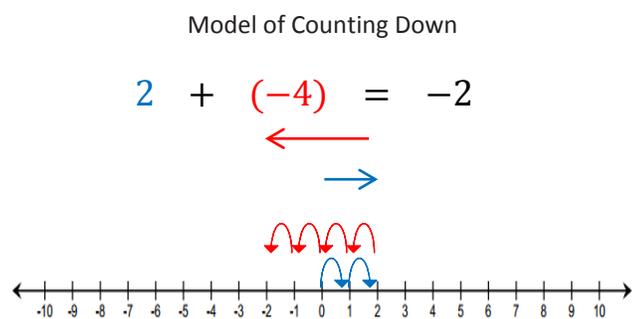
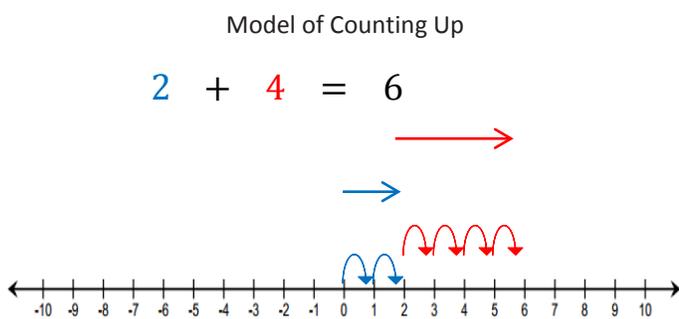
## Lesson 3: Understanding Addition of Integers

### Classwork

#### Exercise 1: Addition Using the Integer Game

Play the Integer Game with your group without using a number line.

#### Example 1: “Counting On” to Express the Sum as Absolute Value on a Number Line



Counting up  $-4$  is the same as “the opposite of counting up 4” and also means counting down 4.

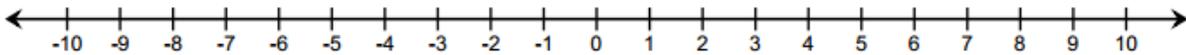
- For each example above, what is the distance between 2 and the sum?
- Does the sum lie to the right or left of 2 on a horizontal number line? Above or below on a vertical number line?
- Given the expression  $54 + 81$ , determine, without finding the sum, the distance between 54 and the sum. Explain.

- d. Is the sum to the right or left of 54 on the horizontal number line? Above or below on a vertical number line?
- e. Given the expression  $14 + (-3)$ , determine, without finding the sum, the distance between 14 and the sum? Why?
- f. Is the sum to the right or left of 14 on the number line? Above or below on a vertical number line?

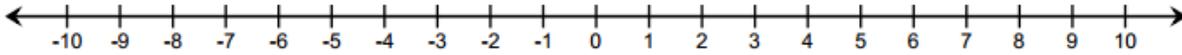
### Exercise 2

Work with a partner to create a horizontal number line model to represent each of the following expressions. Describe the sum using distance from the  $p$ -value along the number line.

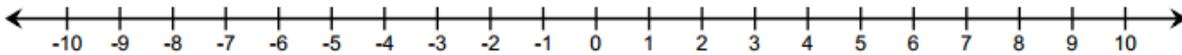
- a.  $-5 + 3$



b.  $-6 + (-2)$



c.  $7 + (-8)$

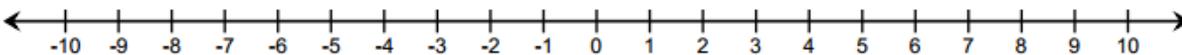


**Exercise 3: Writing an Equation Using Verbal Descriptions**

Write an equation, and using the number line, create an “arrow diagram” given the following information:

“The  $p$ -value is 6, and the sum lies 15 units to the left of the  $p$ -value.”

**Equation:**



**Lesson Summary**

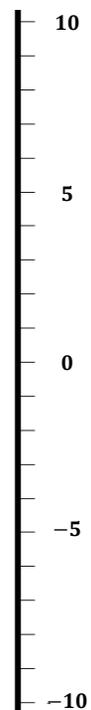
- Addition of integers is represented on a number line as “counting up,” where counting up a negative number of times is the same as “counting down.”
- Arrows show the sum of two integers on a number line.
- The sum is the distance  $|q|$  from the  $p$ -value (the first addend) to the right if  $q$  is positive and to the left if  $q$  is negative.

**Problem Set**

1. Below is a table showing the change in temperature from morning to afternoon for one week.
  - a. Use the vertical number line to help you complete the table. As an example, the first row is completed for you.

**Change in Temperatures from Morning to Afternoon**

Morning Temperature	Change	Afternoon Temperature	Number Sentence
1°C	Rise of 3°C	4°C	$1 + 3 = 4$
2°C	Rise of 8°C		
-2°C	Fall of 6°C		
-4°C	Rise of 7°C		
6°C	Fall of 9°C		
-5°C	Fall of 5°C		
7°C	Fall of 7°C		



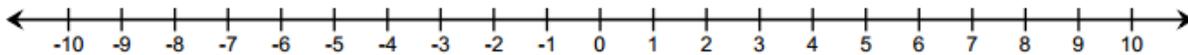
- b. Do you agree or disagree with the statement: “A rise of  $-7^{\circ}\text{C}$ ” means “a fall of  $7^{\circ}\text{C}$ ”? Explain. (Note: No one would ever say, “A rise of  $-7$  degrees;” however, mathematically speaking, it is an equivalent phrase.)

For Questions 2–3, refer to the Integer Game.

2. Terry selected two cards. The sum of her cards is  $-10$ .
  - a. Can both cards be positive? Explain why or why not.
  - b. Can one of the cards be positive and the other be negative? Explain why or why not.
  - c. Can both cards be negative? Explain why or why not.

3. When playing the Integer Game, the first two cards you selected were  $-8$  and  $-10$ .
- What is the value of your hand? Write an equation to justify your answer.
  - For part (a), what is the distance of the sum from  $-8$ ? Does the sum lie to the right or left of  $-8$  on the number line?
  - If you discarded the  $-10$  and then selected a  $10$ , what would be the value of your hand? Write an equation to justify your answer.
4. Given the expression  $67 + (-35)$ , can you determine, without finding the sum, the distance between  $67$  and the sum? Is the sum to the right or left of  $67$  on the number line?
5. Use the information given below to write an equation. Then create an “arrow diagram” of this equation on the number line provided below.

“The  $p$ -value is  $-4$ , and the sum lies 12 units to the right of the  $p$ -value.”

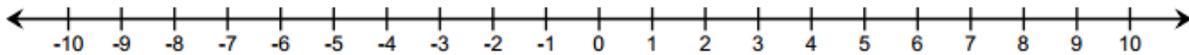


## Lesson 4: Efficiently Adding Integers and Other Rational Numbers

### Classwork

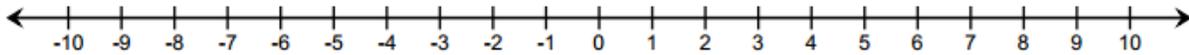
#### Example 1: Rule for Adding Integers with Same Signs

- a. Represent the sum of  $3 + 5$  using arrows on the number line.



- How long is the arrow that represents 3?
- What direction does it point?
- How long is the arrow that represents 5?
- What direction does it point?
- What is the sum?
- If you were to represent the sum using an arrow, how long would the arrow be and what direction would it point?

- vii. What is the relationship between the arrow representing the number on the number line and the absolute value of the number?
- viii. Do you think that adding two positive numbers will always give you a greater positive number? Why?
- b. Represent the sum of  $-3 + (-5)$  using arrows that represent  $-3$  and  $-5$  on the number line. From part (a), use the same questions to elicit feedback. In the Integer Game, I would combine  $-3$  and  $-5$  to give me  $-8$ .



- i. How long is the arrow that represents  $-3$ ?
- ii. What direction does it point?
- iii. How long is the arrow that represents  $-5$ ?
- iv. What direction does it point?
- v. What is the sum?

- vi. If you were to represent the sum using an arrow, how long would the arrow be and what direction would it point?
- vii. Do you think that adding two negative numbers will always give you a smaller negative number? Why?
- c. What do both examples have in common?

*RULE: Add rational numbers with the same sign by adding the absolute values and using the common sign.*

### Exercise 2

- a. Decide whether the sum will be positive or negative without actually calculating the sum.
- i.  $-4 + (-2)$  \_\_\_\_\_
- ii.  $5 + 9$  \_\_\_\_\_
- iii.  $-6 + (-3)$  \_\_\_\_\_
- iv.  $-1 + (-11)$  \_\_\_\_\_
- v.  $3 + 5 + 7$  \_\_\_\_\_
- vi.  $-20 + (-15)$  \_\_\_\_\_

b. Find the sum.

i.  $15 + 7$

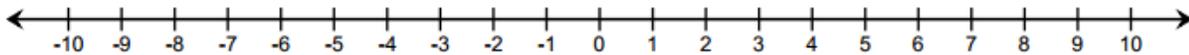
ii.  $-4 + (-16)$

iii.  $-18 + (-64)$

iv.  $-205 + (-123)$

### Example 2: Rule for Adding Opposite Signs

a. Represent the  $5 + (-3)$  using arrows on the number line.



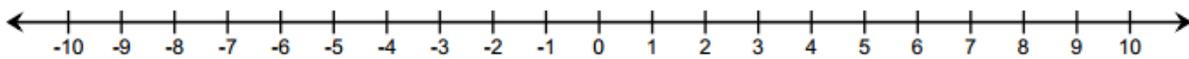
i. How long is the arrow that represents 5?

ii. What direction does it point?

iii. How long is the arrow that represents  $-3$ ?

iv. What direction does it point?

- v. Which arrow is longer?
- vi. What is the sum? If you were to represent the sum using an arrow, how long would the arrow be and what direction would it point?
- b. Represent the  $4 + (-7)$  using arrows on the number line.



- i. In the two examples above, what is the relationship between length of the arrow representing the sum and the lengths of the arrows representing the  $p$ -value and  $q$ -value?
- ii. What is the relationship between the direction of the arrow representing the sum and the direction of arrows representing the  $p$ -value and  $q$ -value?
- iii. Write a rule that will give the length and direction of the arrow representing the sum of two values that have opposite signs.

*RULE: Add rational numbers with opposite signs by subtracting the absolute values and using the sign of the integer with the greater absolute value.*

**Exercise 3**

- a. Circle the integer with the greater absolute value. Decide whether the sum will be positive or negative without actually calculating the sum.

i.  $-1 + 2$

ii.  $5 + (-9)$

iii.  $-6 + 3$

iv.  $-11 + 1$

- b. Find the sum.

i.  $-10 + 7$

ii.  $8 + (-16)$

iii.  $-12 + (65)$

iv.  $105 + (-126)$

**Example 3: Applying Integer Addition Rules to Rational Numbers**

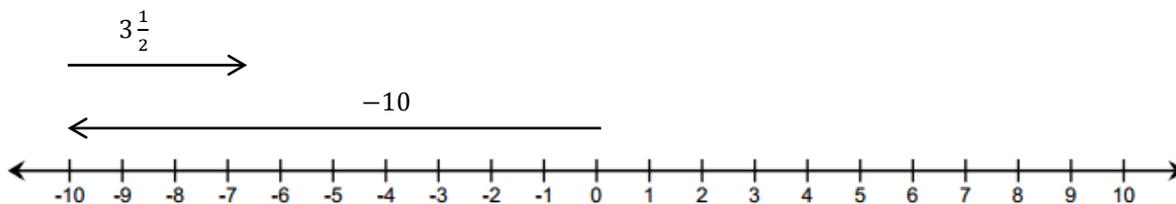
Find the sum of  $6 + \left(-2\frac{1}{4}\right)$ . The addition of rational numbers follows the same rules of addition for integers.

- Find the absolute values of the numbers.
- Subtract the absolute values.
- The answer will take the sign of the number that has the greater absolute value.

**Exercise 4**

Solve the following problems. Show your work.

- Find the sum of  $-18 + 7$ .
- If the temperature outside was 73 degrees at 5:00 p.m., but it fell 19 degrees by 10:00 p.m., what is the temperature at 10:00 p.m.? Write an equation and solve.
- Write an addition sentence, and find the sum using the diagram below.



**Lesson Summary**

- Add integers with the same sign by adding the absolute values and using the common sign.
- Steps to adding numbers with opposite signs:
  1. Find the absolute values of the numbers.
  2. Subtract the absolute values.
  3. The answer will take the sign of the integer that has the greater absolute value.
- To add rational numbers, follow the same rules used to add integers.

**Problem Set**

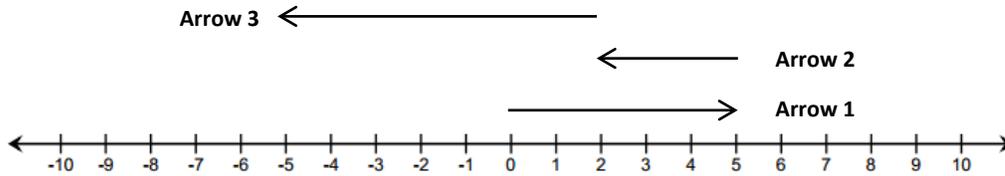
1. Find the sum. Show your work to justify your answer.
  - a.  $4 + 17$
  - b.  $-6 + (-12)$
  - c.  $2.2 + (-3.7)$
  - d.  $-3 + (-5) + 8$
  - e.  $\frac{1}{3} + \left(-2\frac{1}{4}\right)$
2. Which of these story problems describes the sum  $19 + (-12)$ ? Check all that apply. Show your work to justify your answer.

\_\_\_\_\_ Jared's dad paid him \$19 for raking the leaves from the yard on Wednesday. Jared spent \$12 at the movie theater on Friday. How much money does Jared have left?

\_\_\_\_\_ Jared owed his brother \$19 for raking the leaves while Jared was sick. Jared's dad gave him \$12 for doing his chores for the week. How much money does Jared have now?

\_\_\_\_\_ Jared's grandmother gave him \$19 for his birthday. He bought \$8 worth of candy and spent another \$4 on a new comic book. How much money does Jared have left over?

3. Use the diagram below to complete each part.



- Label each arrow with the number the arrow represents.
- How long is each arrow? What direction does each arrow point?

Arrow	Length	Direction
1		
2		
3		

c. Write an equation that represents the sum of the numbers. Find the sum.

4. Jennifer and Katie were playing the Integer Game in class. Their hands are represented below.



- What is the value of each of their hands? Show your work to support your answer.
- If Jennifer drew two more cards, is it possible for the value of her hand not to change? Explain why or why not.
- If Katie wanted to win the game by getting a score of 0, what card would she need? Explain.
- If Jennifer drew a  $-1$  and a  $-2$ , what would be her new score? Show your work to support your answer.

# Lesson 5: Understanding Subtraction of Integers and Other Rational Numbers

## Classwork

### Example 1: Exploring Subtraction with the Integer Game

Play the Integer Game in your group. Start Round 1 by selecting four cards. Follow the steps for each round of play.

1. Write the value of your hand in the Total column.
2. Then, record what card values you select in the Action 1 column and discard from your hand in the Action 2 column.
3. After each action, calculate your new total, and record it under the appropriate Results column.
4. Based on the results, describe what happens to the value of your hand under the appropriate Descriptions column. For example, "Score increased by 3."

Round	Total	Action 1	Result 1	Description	Action 2	Result 2	Description
1							
2							
3							
4							
5							

**Discussion: Making Connections to Integer Subtraction**

1. How did selecting positive value cards change the value of your hand?
  
2. How did selecting negative value cards change the value of your hand?
  
3. How did discarding positive value cards change the value of your hand?
  
4. How did discarding negative value cards change the value of your hand?
  
5. What operation reflects selecting a card?
  
6. What operation reflects discarding or removing a card?
  
7. Based on the game, can you make a prediction about what happens to the result when
  - a. Subtracting a positive integer?
  
  
  - b. Subtracting a negative integer?

At the end of the lesson, the class will review its predictions.

**Example 2: Subtracting a Positive Number**

Follow along with your teacher to complete the diagrams below.

4

2

$4 + 2 = \square$

Show that discarding (subtracting) a positive card, which is the same as subtracting a positive number, decreases the value of the hand.

4

2

$4 + 2 - 2 = \square$

or

4

2

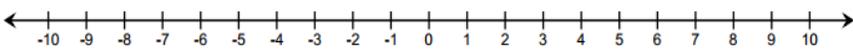
-2

$4 + 2 + (-2) = \square$

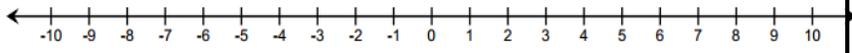
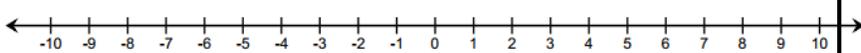
Removing ( \_\_\_\_\_ ) a positive card changes the score in the same way as \_\_\_\_\_ a card whose value is the \_\_\_\_\_ (or opposite). In this case, adding the corresponding \_\_\_\_\_.

**Example 3: Subtracting a Negative Number**

Follow along with your teacher to complete the diagrams below.

4	-2	
$4 + (-2) = \square$		

How does removing a negative card change the score, or value, of the hand?

4	<del>-2</del>	
$4 + (-2) - (-2) = \square$		
or		
4	-2	2
$4 + (-2) + 2 = \square$		

Removing ( \_\_\_\_\_ ) a negative card changes the score in the same way as \_\_\_\_\_ a card whose value is the \_\_\_\_\_ (or opposite). In this case, adding the corresponding \_\_\_\_\_.

**The Rule of Subtraction:** Subtracting a number is the same as adding its additive inverse (or opposite).

### Exercises 1–3: Subtracting Positive and Negative Integers

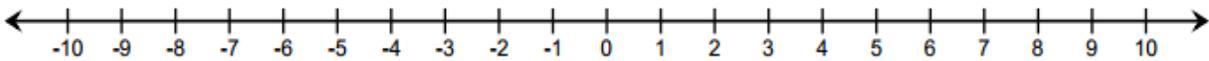
1. Using the rule of subtraction, rewrite the following subtraction sentences as addition sentences and solve. Use the number line below if needed.

a.  $8 - 2$

b.  $4 - 9$

c.  $-3 - 7$

d.  $-9 - (-2)$



2. Find the differences.

a.  $-2 - (-5)$

b.  $11 - (-8)$

c.  $-10 - (-4)$

3. Write two equivalent expressions that represent the situation. What is the difference in their elevations?  
“An airplane flies at an altitude of 25,000 feet. A submarine dives to a depth of 600 feet below sea level.”

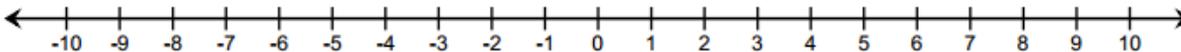
**Lesson Summary**

- **The Rule for Subtraction:** Subtracting a number is the same as adding its opposite.
- Removing (subtracting) a positive card changes the score in the same way as adding a corresponding negative card.
- Removing (subtracting) a negative card makes the same change as adding the corresponding positive card.
- For all rational numbers, subtracting a number and adding it back gets you back to where you started:  
 $(m - n) + n = m$ .

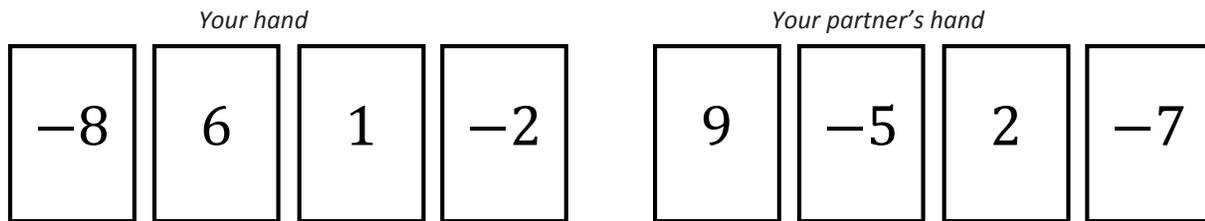
**Problem Set**

1. On a number line, find the difference of each number and 4? Complete the table to support your answers. The first example is provided.

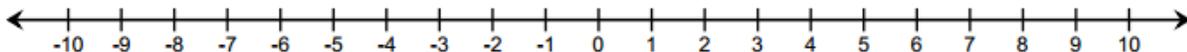
Number	Subtraction Expression	Addition Expression	Answer
10	$10 - 4$	$10 + (-4) = 6$	6
2			
-4			
-6			
1			



2. You and your partner were playing the Integer Game in class. Here are the cards in both hands.



- a. Find the value of each hand. Who would win based on the current scores? (The score closest to 0 wins.)
  - b. Find the value of each hand if you discarded the  $-2$  and selected a  $5$ , and your partner discarded the  $-5$  and selected a  $5$ . Show your work to support your answer.
  - c. Use your score values from part (b) to determine who would win the game now.
3. Write the following expressions as a single integer.
- a.  $-2 + 16$
  - b.  $-2 - (-16)$
  - c.  $18 - 26$
  - d.  $-14 - 23$
  - e.  $30 - (-45)$
4. Explain what is meant by the following and illustrate with an example:  
 “For any real numbers,  $p$  and  $q$ ,  $p - q = p + (-q)$ .”
5. Choose an integer between  $-1$  and  $-5$  on the number line, and label it point  $P$ . Locate and label the following points on the number line. Show your work.



- a. Point  $A$ :  $P - 5$
- b. Point  $B$ :  $(P - 4) + 4$
- c. Point  $C$ :  $-P - (-7)$

Challenge Problem:

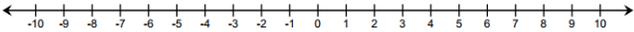
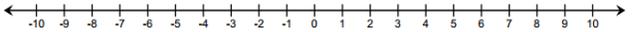
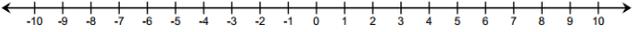
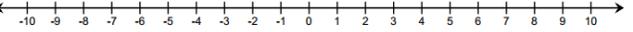
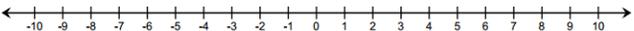
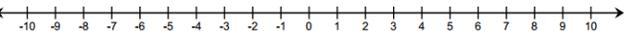
6. Write two equivalent expressions that represent the situation. What is the difference in their elevations?  
 “An airplane flies at an altitude of 26,000 feet. A submarine dives to a depth of 700 feet below sea level.”

## Lesson 6: The Distance Between Two Rational Numbers

### Classwork

#### Exercise 1

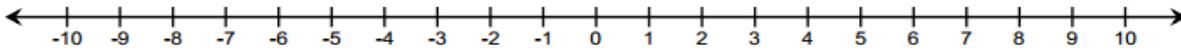
Use the number line to answer each of the following.

Person A	Person B
<p>What is the distance between <math>-4</math> and <math>5</math>?</p> 	<p>What is the distance between <math>5</math> and <math>-4</math>?</p> 
<p>What is the distance between <math>-5</math> and <math>-3</math>?</p> 	<p>What is the distance between <math>-3</math> and <math>-5</math>?</p> 
<p>What is the distance between <math>7</math> and <math>-1</math>?</p> 	<p>What is the distance between <math>-1</math> and <math>7</math>?</p> 

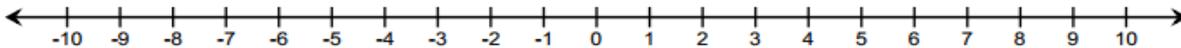
**Exercise 2**

Use the number line to answer each of the following questions.

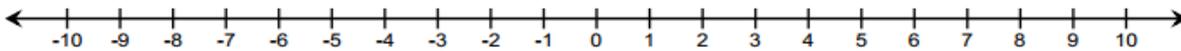
- a. What is the distance between 0 and  $-8$ ?



- b. What is the distance between  $-2$  and  $-1\frac{1}{2}$ ?



- c. What is the distance between  $-6$  and  $-10$ ?

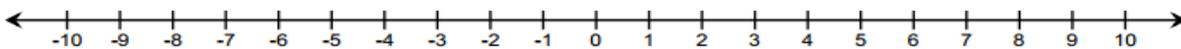


**Example 1: Formula for the Distance Between Two Rational Numbers**

Find the distance between  $-3$  and  $2$ .

Step 1: Start on an endpoint.

Step 2: Count the number of units from the endpoint you started on to the other endpoint.



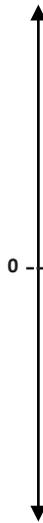
Using a formula, \_\_\_\_\_

For two rational numbers  $p$  and  $q$ , the distance between  $p$  and  $q$  is  $|p - q|$ .

**Example 2: Change in Elevation vs. Distance**

Distance is positive. Change in elevation or temperature may be positive or negative depending on whether it is increasing or decreasing (going up or down).

- a. A hiker starts hiking at the beginning of a trail at a point which is 200 feet below sea level. He hikes to a location on the trail that is 580 feet above sea level and stops for lunch.
- What is the vertical distance between 200 feet below sea level and 580 feet above sea level?
  - How should we interpret 780 feet in the context of this problem?
- b. After lunch, the hiker hiked back down the trail from the point of elevation, which is 580 feet above sea level, to the beginning of the trail which is 200 feet below sea level.



- What is the vertical distance between 580 feet above sea level and 200 feet below sea level?

- ii. What is the change in elevation?

### Exercise 3

The distance between a negative number and a positive number is  $12\frac{1}{2}$ . What are the numbers?

### Exercise 4

Use the distance formula to find each answer. Support your answer using a number line diagram.

- a. Find the distance between  $-7$  and  $-4$ .
- b. Find the change in temperature if the temperature rises from  $-18^{\circ}\text{F}$  to  $15^{\circ}\text{F}$  (use a vertical number line).

- c. Would your answer for part (b) be different if the temperature dropped from  $15^{\circ}\text{F}$  to  $-18^{\circ}\text{F}$ ? Explain.
- d. Beryl is the first person to finish a 5K race and is standing 15 feet beyond the finish line. Another runner, Jeremy, is currently trying to finish the race and has approximately 14 feet before he reaches the finish line. What is the minimum possible distance between Beryl and Jeremy?
- e. What is the change in elevation from 140 feet above sea level to 40 feet below sea level? Explain.

**Lesson Summary**

- To find the distance between two rational numbers on a number line, you can count the number of units between the numbers.
- Using a formula, the distance between rational numbers,  $p$  and  $q$ , is  $|p - q|$ .
- Distance is always positive.
- Change may be positive or negative. For instance, there is a  $-4^\circ$  change when the temperature goes from  $7^\circ$  to  $3^\circ$ .

**Problem Set**

1.  $|-19 - 12|$

2.  $|19 - (-12)|$

3.  $|10 - (-43)|$

4.  $|-10 - 43|$

5.  $|-1 - (-16)|$

6.  $|1 - 16|$

7.  $|0 - (-9)|$

8.  $|0 - 9|$

9.  $|-14.5 - 13|$

10.  $|14.5 - (-13)|$

11. Describe any patterns you see in the answers to the problems in the left- and right-hand columns. Why do you think this pattern exists?

## Lesson 7: Addition and Subtraction of Rational Numbers

### Classwork

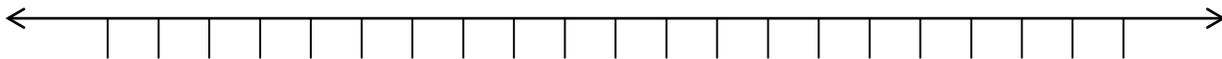
#### Exercise 1: Real-World Connection to Adding and Subtracting Rational Numbers

Suppose a 7<sup>th</sup> grader's birthday is today, and she is 12 years old. How old was she  $3\frac{1}{2}$  years ago? Write an equation and use a number line to model your answer.

#### Example 1: Representing Sums of Rational Numbers on a Number Line

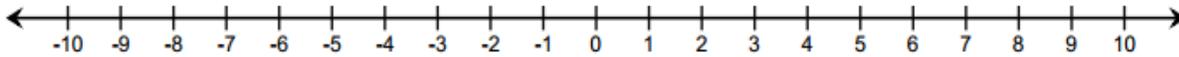
- Place the tail of the arrow on 12.
- The length of the arrow is the absolute value of  $-3\frac{1}{2}$ ,  $|-3\frac{1}{2}| = 3\frac{1}{2}$ .
- The direction of the arrow is to the *left* since you are adding a negative number to 12.

Draw the number line model in the space below.



**Exercise 2**

Find the following sum using a number line diagram.  $-2\frac{1}{2} + 5$ .

**Example 2: Representing Differences of Rational Numbers on a Number Line**

a. Rewrite the difference  $1 - 2\frac{1}{4}$  as a sum:  $1 + (-2\frac{1}{4})$ .

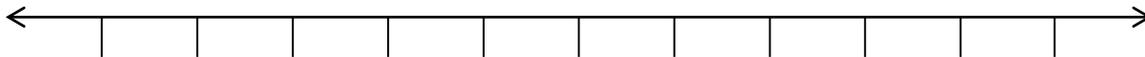
Now follow the steps to represent the sum:

b. Place the tail of the arrow on 1.

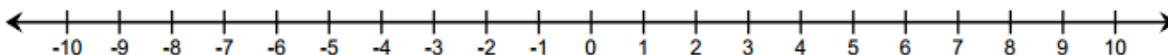
c. The length of the arrow is the absolute value of  $-2\frac{1}{4}$ ;  $|-2\frac{1}{4}| = 2\frac{1}{4}$ .

d. The direction of the arrow is to the *left* since you are adding a negative number to 1.

Draw the number line model in the space below.

**Exercise 3**

Find the following difference, and represent it on a number line.  $-5\frac{1}{2} - (-8)$ .



**Exercise 4**

Find the following sums and differences using a number line model.

a.  $-6 + 5\frac{1}{4}$

b.  $7 - (-0.9)$

c.  $2.5 + \left(-\frac{1}{2}\right)$

d.  $-\frac{1}{4} + 4$

e.  $\frac{1}{2} - (-3)$

**Exercise 5**

Create an equation and number line diagram to model each answer.

- a. Samantha owes her father \$7. She just got paid \$5.50 for babysitting. If she gives that money to her dad, how much will she still owe him?
- b. At the start of a trip, a car's gas tank contains 12 gallons of gasoline. During the trip, the car consumes  $10\frac{1}{8}$  gallons of gasoline. How much gasoline is left in the tank?
- c. A fish was swimming  $3\frac{1}{2}$  feet below the water's surface at 7:00 a.m. Four hours later, the fish was at a depth that is  $5\frac{1}{4}$  feet below where it was at 7:00 a.m. What rational number represents the position of the fish with respect to the water's surface at 11:00 a.m.?

**Lesson Summary**

The rules for adding and subtracting integers apply to all rational numbers.

The sum of two rational numbers (e.g.,  $-1 + 4.3$ ) can be found on the number line by placing the tail of an arrow at  $-1$  and locating the head of the arrow 4.3 units to the right to arrive at the sum, which is 3.3.

To model the difference of two rational numbers on a number line (e.g.,  $-5.7 - 3$ ), first rewrite the difference as a sum,  $-5.7 + (-3)$ , and then follow the steps for locating a sum. Place a single arrow with its tail at  $-5.7$  and the head of the arrow 3 units to the left to arrive at  $-8.7$ .

**Problem Set**

Represent each of the following problems using both a number line diagram and an equation.

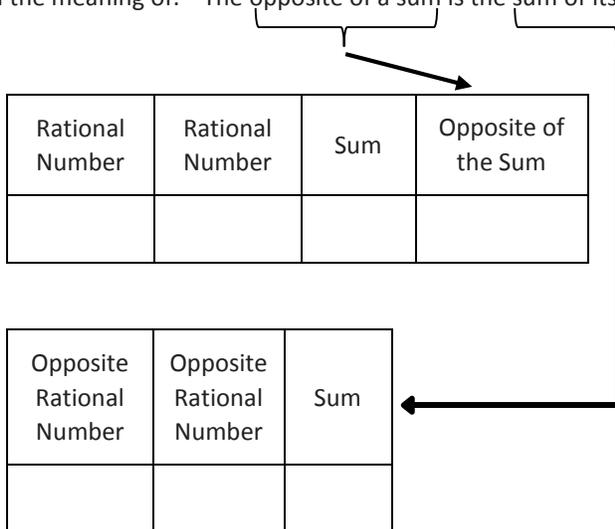
1. A bird that was perched atop a  $15\frac{1}{2}$ -foot tree dives down six feet to a branch below. How far above the ground is the bird's new location?
2. Mariah had owed her grandfather \$2.25 but was recently able to pay him back \$1.50. How much does Mariah currently owe her grandfather?
3. Jake is hiking a trail that leads to the top of a canyon. The trail is 4.2 miles long, and Jake plans to stop for lunch after he completes 1.6 miles. How far from the top of the canyon will Jake be when he stops for lunch?
4. Sonji and her friend Rachel are competing in a running race. When Sonji is 0.4 miles from the finish line, she notices that her friend Rachel has fallen. If Sonji runs one tenth of a mile back to help her friend, how far will she be from the finish line?
5. Mr. Henderson did not realize his checking account had a balance of \$200 when he used his debit card for a \$317.25 purchase. What is his checking account balance after the purchase?
6. If the temperature is  $-3^{\circ}\text{F}$  at 10:00 p.m., and the temperature falls four degrees overnight, what is the resulting temperature?

# Lesson 8: Applying the Properties of Operations to Add and Subtract Rational Numbers

## Classwork

### Example 1: The Opposite of a Sum is the Sum of its Opposites

Explain the meaning of: “The opposite of a sum is the sum of its opposites.” Use a specific math example.



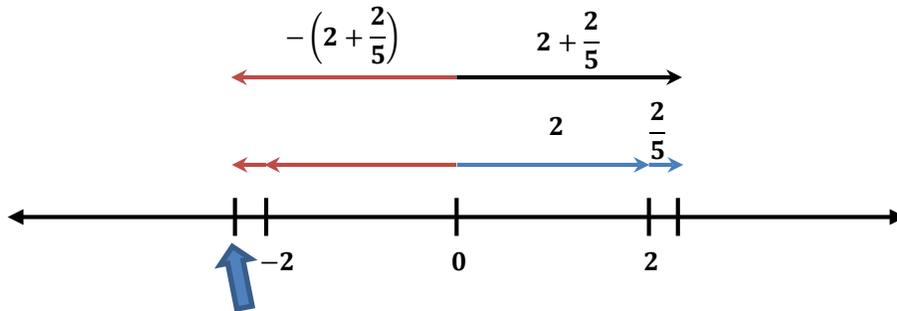
### Exercise 1

Represent the following expression with a single rational number.

$$-2\frac{2}{5} + 3\frac{1}{4} - \frac{3}{5}$$

**Example 2: A Mixed Number is a Sum**

Use the number line model shown below to explain and write the opposite of  $2\frac{2}{5}$  as a sum of two rational numbers.



The opposite of a sum (top single arrow pointing left) and the sum of the opposites correspond to the same point on the number line.

**Exercise 2**

Rewrite each mixed number as the sum of two signed numbers.

a.  $-9\frac{5}{8}$

b.  $-2\frac{1}{2}$

c.  $8\frac{11}{12}$

**Exercise 3**

Represent each sum as a mixed number.

a.  $-1 + (-\frac{5}{12})$

b.  $30 + \frac{1}{8}$

c.  $-17 + (-\frac{1}{9})$

**Exercise 4**

Mr. Mitchell lost 10 pounds over the summer by jogging each week. By winter time, he had gained  $5\frac{1}{8}$  pounds. Represent this situation with an expression involving signed numbers. What is the overall change in Mr. Mitchell's weight?

**Exercise 5**

Jamal is completing a math problem and represents the expression  $-5\frac{5}{7} + 8 - 3\frac{2}{7}$  with a single rational number as shown in the steps below. Justify each of Jamal's steps. Then, show another way to solve the problem.

$$\begin{aligned} &= -5\frac{5}{7} + 8 + \left(-3\frac{2}{7}\right) \\ &= -5\frac{5}{7} + \left(-3\frac{2}{7}\right) + 8 \\ &= -5 + \left(-\frac{5}{7}\right) + (-3) + \left(-\frac{2}{7}\right) + 8 \\ &= -5 + \left(-\frac{5}{7}\right) + \left(-\frac{2}{7}\right) + (-3) + 8 \\ &= -5 + (-1) + (-3) + 8 \\ &= -6 + (-3) + 8 \\ &= (-9) + 8 \\ &= -1 \end{aligned}$$

## Lesson Summary

- Use the properties of operations to add and subtract rational numbers more efficiently. For instance,

$$-5\frac{2}{9} + 3.7 + 5\frac{2}{9} = \left(-5\frac{2}{9} + 5\frac{2}{9}\right) + 3.7 = 0 + 3.7 = 3.7.$$

- The opposite of a sum is the sum of its opposites as shown in the examples that follow:

$$-4\frac{4}{7} = -4 + \left(-\frac{4}{7}\right)$$

$$-(5 + 3) = -5 + (-3)$$

## Problem Set

1. Represent each sum as a single rational number.

a.  $-14 + \left(-\frac{8}{9}\right)$

b.  $7 + \frac{1}{9}$

c.  $-3 + \left(-\frac{1}{6}\right)$

Rewrite each of the following to show that *the opposite of a sum is the sum of the opposites*. Problem 2 has been completed as an example.

2.  $-(9 + 8) = -9 + (-8)$   
 $-17 = -17$

3.  $-\left(\frac{1}{4} + 6\right)$

4.  $-(10 + (-6))$

5.  $-\left((-55) + \frac{1}{2}\right)$

Use your knowledge of rational numbers to answer the following questions.

6. Meghan said the opposite of the sum of  $-12$  and  $4$  is  $8$ . Do you agree? Why or why not?
7. Jolene lost her wallet at the mall. It had  $\$10$  in it. When she got home her brother felt sorry for her and gave her  $\$5.75$ . Represent this situation with an expression involving rational numbers. What is the overall change in the amount of money Jolene has?
8. Isaiah is completing a math problem and is at the last step:  $25 - 28\frac{1}{5}$ . What is the answer? Show your work.

9. A number added to its opposite equals zero. What do you suppose is true about *a sum added to its opposite*? Use the following examples to reach a conclusion. Express the answer to each example as a single rational number.

a.  $(3 + 4) + (-3 + -4)$

b.  $(-8 + 1) + (8 + (-1))$

c.  $\left(-\frac{1}{2} + \left(-\frac{1}{4}\right)\right) + \left(\frac{1}{2} + \frac{1}{4}\right)$

## Lesson 9: Applying the Properties of Operations to Add and Subtract Rational Numbers

### Classwork

#### Exercise 1

Unscramble the cards, and show the steps in the correct order to arrive at the solution to  $5\frac{2}{9} - (8.1 + 5\frac{2}{9})$ .

$$0 + (-8.1)$$

$$\left(5\frac{2}{9} + \left(-5\frac{2}{9}\right)\right) + (-8.1)$$

$$-8.1$$

$$5\frac{2}{9} + \left(-8.1 + \left(-5\frac{2}{9}\right)\right)$$

$$5\frac{2}{9} + \left(-5\frac{2}{9} + (-8.1)\right)$$

**Examples 1–2**

Represent each of the following expressions as one rational number. Show and explain your steps.

1.  $4\frac{4}{7} - (4\frac{4}{7} - 10)$

2.  $5 + (-4\frac{4}{7})$

**Exercise 2: Team Work!**

a.  $-5.2 - (-3.1) + 5.2$

b.  $32 + \left(-12\frac{7}{8}\right)$

c.  $3\frac{1}{6} + 20.3 - \left(-5\frac{5}{6}\right)$

d.  $\frac{16}{20} - (-1.8) - \frac{4}{5}$

**Exercise 3**

Explain step by step, how to arrive at a single rational number to represent the following expression. Show both a written explanation and the related math work for each step.

$$-24 - \left(-\frac{1}{2}\right) - 12.5$$

**Lesson Summary**

- Use the properties of operations to add and subtract rational numbers more efficiently. For instance,

$$-5\frac{2}{9} + 3.7 + 5\frac{2}{9} = \left(-5\frac{2}{9} + 5\frac{2}{9}\right) + 3.7 = 0 + 3.7 = 3.7.$$

- The opposite of a sum is the sum of its opposites as shown in the examples that follow:

$$-4\frac{4}{7} = -4 + \left(-\frac{4}{7}\right).$$

$$-(5 + 3) = -5 + (-3).$$

**Problem Set**

Show all steps taken to rewrite each of the following as a single rational number.

1.  $80 + \left(-22\frac{4}{15}\right)$

2.  $10 + \left(-3\frac{3}{8}\right)$

3.  $\frac{1}{5} + 20.3 - \left(-5\frac{3}{5}\right)$

4.  $\frac{11}{12} - (-10) - \frac{5}{6}$

5. Explain, step by step, how to arrive at a single rational number to represent the following expression. Show both a written explanation and the related math work for each step.

$$1 - \frac{3}{4} + \left(-12\frac{1}{4}\right)$$

## Lesson 10: Understanding Multiplication of Integers

### Classwork

#### Exercise 1: Integer Game Revisited

In groups of four, play one round of the Integer Game (see Integer Game outline for directions).

#### Example 1: Product of a Positive Integer and a Negative Integer

Part A:



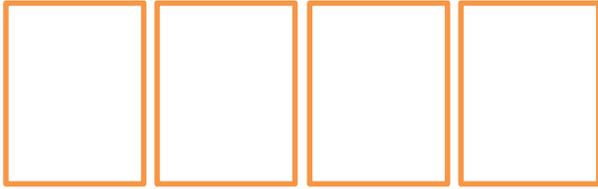
Part B:



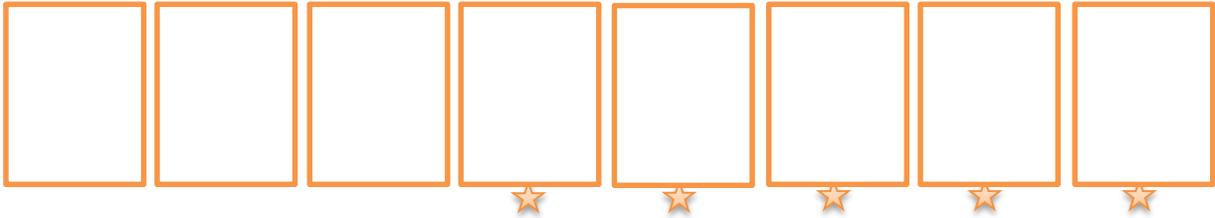
Use your cards from Part B to answer the questions below.

- Write a product that describes the three matching cards.
- Write an expression that represents how each of the  $\star$  cards changes your score.
- Write an equation that relates these two expressions.
- Write an integer that represents the total change to your score by the three  $\star$  cards.
- Write an equation that relates the product and how it affects your score.

Part C:



Part D:

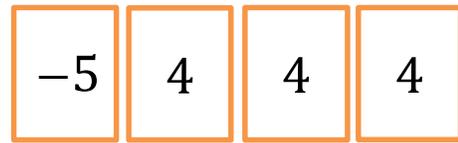


Use your cards from Part D to answer the questions below.

- f. Write a product that describes the five matching cards.
- g. Write an expression that represents how each of the ★ cards changes your score.
- h. Write an equation that relates these two expressions.
- i. Write an integer that represents the total change to your score by the three ★ cards.
- j. Write an equation that relates the product and how it affects your score.
- k. Use the expression  $5 \times 4$  to relate the multiplication of a positive valued card to addition.
- l. Use the expression  $3 \times (-5)$  to relate the multiplication of a negative valued card to addition.

**Example 2: Product of a Negative Integer and a Positive Integer**

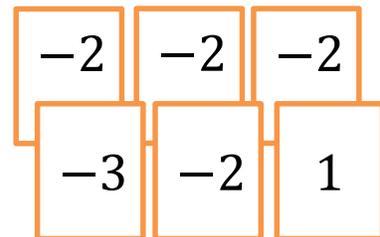
- a. If all of the 4's from the playing hand on the right are discarded, how will the score be affected? Model this using a product in an equation.



- b. What three matching cards could be added to those pictured to get the same change in score? Model this using a product in an equation.
- c. Seeing how each play affects the score, relate the products that you used to model them. What do you conclude about multiplying integers with opposite signs?

**Example 3: Product of Two Negative Integers**

- a. If the matching cards from the playing hand on the right are discarded, how will this hand's score be affected? Model this using a product in an equation.



- b. What four matching cards could be added to those pictured to get the same change in score? Model this using a product in an equation.
- c. Seeing how each play affects the score, relate the products that you used to model them. What do you conclude about multiplying integers with the same sign?
- d. Using the conclusions from Examples 2 and 3, what can we conclude about multiplying integers? Write a few examples.

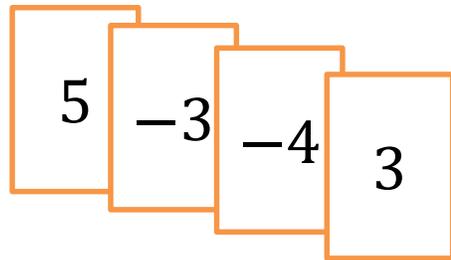
**Lesson Summary**

Multiplying integers is repeated addition and can be modeled with the Integer Game. If  $3 \times a$  corresponds to what happens to your score if you get three cards of value  $a$ , then  $(-3) \times a$  corresponds to what happens to your score if you lose three cards of value  $a$ . Adding a number multiple times has the same effect as removing the opposite value the same number of times (e.g.,  $a \times b = (-a) \times (-b)$  and  $a \times (-b) = (-a) \times b$ .)

**Problem Set**

- Describe sets of two or more matching integer cards that satisfy the criteria in each part below:
  - Cards increase the score by eight points.
  - Cards decrease the score by 9 points.
  - Removing cards that increase the score by 10 points.
  - Positive cards that decrease the score by 18 points.

- You have the integer cards shown at the right when your teacher tells you to choose a card to multiply four times. If your goal is to get your score as close to zero as possible, which card would you choose? Explain how your choice changes your score.



- Sherry is playing the Integer Game and is given a chance to discard a set of matching cards. Sherry determines that if she discards one set of cards her score will increase by 12. If she discards another set, then her score will decrease by eight. If her matching cards make up all six cards in her hand, what cards are in Sherry's hand? Are there any other possibilities?

# Lesson 11: Develop Rules for Multiplying Signed Numbers

## Classwork

### Example 1: Extending Whole Number Multiplication to the Integers

Part A: Complete quadrants *I* and *IV* of the table below to show how sets of matching integer cards will affect a player’s score in the Integer Game. For example, three 2’s would increase a player’s score by  $0 + 2 + 2 + 2 = 6$  points.

	Quadrant <i>II</i>					Quadrant <i>I</i>						
What does this quadrant represent?					5						What does this quadrant represent?	
_____					4						_____	
_____					3						_____	
_____					2			6			_____	
_____					1						_____	
					1	1	2	3	4	5	← Number of matching cards	
What does this quadrant represent?					-1						What does this quadrant represent?	
_____					-2						_____	
_____					-3						_____	
_____					-4						_____	
_____					-5						_____	
	Quadrant <i>III</i>					↑	Quadrant <i>IV</i>					
						Integer card values						

- a. What patterns do you see in the right half of the table?
  
- b. Enter the missing integers in the left side of the middle row and describe what they represent.

Part B: Complete quadrant *II* of the table.

- c. What relationships or patterns do you notice between the products (values) in quadrant *II* and the products (values) in quadrant *I*?
  
  
  
  
  
  
  
  
  
  
- d. What relationships or patterns do you notice between the products (values) in quadrant *II* and the products (values) in quadrant *IV*?
  
  
  
  
  
  
  
  
  
  
- e. Use what you know about the products (values) in quadrants *I*, *II*, and *IV* to describe what quadrant *III* will look like when its products (values) are entered.

Part C: Complete the quadrant *III* of the table.

Refer to the completed table to help you answer the following questions:

- f. Is it possible to know the sign of a product of two integers just by knowing in which quadrant each integer is located? Explain.
  
  
  
  
  
  
  
  
  
  
- g. Which quadrants contain which values? Describe an integer game scenario represented in each quadrant.

**Exercise 1: Multiplication of Integers in the Real World**

Generate real-world situations that can be modeled by each of the following multiplication problems. Use the Integer Game as a resource.

a.  $-3 \times 5$

b.  $-6 \times (-3)$

c.  $4 \times (-7)$

**Lesson Summary**

To multiply signed numbers, multiply the absolute values to get the absolute value of the product. The sign of the product is positive if the factors have the same sign and negative if they have opposite signs.

**Problem Set**

1. Complete the problems below. Then, answer the question that follows.

$3 \times 3 =$	$3 \times 2 =$	$3 \times 1 =$	$3 \times 0 =$	$3 \times (-1) =$	$3 \times (-2) =$
$2 \times 3 =$	$2 \times 2 =$	$2 \times 1 =$	$2 \times 0 =$	$2 \times (-1) =$	$2 \times (-2) =$
$1 \times 3 =$	$1 \times 2 =$	$1 \times 1 =$	$1 \times 0 =$	$1 \times (-1) =$	$1 \times (-2) =$
$0 \times 3 =$	$0 \times 2 =$	$0 \times 1 =$	$0 \times 0 =$	$0 \times (-1) =$	$0 \times (-2) =$
$-1 \times 3 =$	$-1 \times 2 =$	$-1 \times 1 =$	$-1 \times 0 =$	$-1 \times (-1) =$	$-1 \times (-2) =$
$-2 \times 3 =$	$-2 \times 2 =$	$-2 \times 1 =$	$-2 \times 0 =$	$-2 \times (-1) =$	$-2 \times (-2) =$
$-3 \times 3 =$	$-3 \times 2 =$	$-3 \times 1 =$	$-3 \times 0 =$	$-3 \times (-1) =$	$-3 \times (-2) =$

Which row shows the same pattern as the outlined column? Are the problems similar or different? Explain.

- Explain why  $(-4) \times (-5) = 20$ . Use patterns, an example from the Integer Game, or the properties of operations to support your reasoning.
- Each time that Samantha rides the commuter train, she spends \$4 for her fare. Write an integer that represents the change in Samantha’s money from riding the commuter train to and from work for 13 days. Explain your reasoning.
- Write a real-world problem that can be modeled by  $4 \times (-7)$ .

**Challenge:**

- Use properties to explain why for each integer  $a$ ,  $-a = -1 \times a$ . (Hint: What does  $(1 + (-1)) \times a$  equal? What is the additive inverse of  $a$ ?)

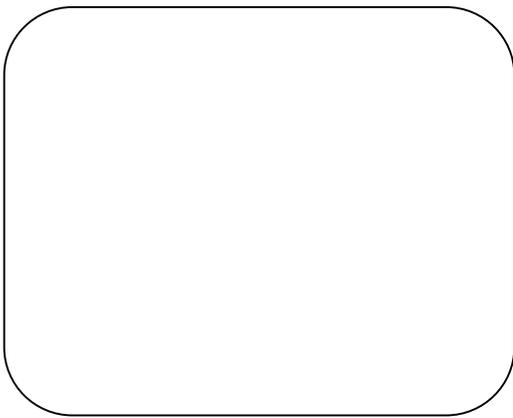
## Lesson 12: Division of Integers

### Classwork

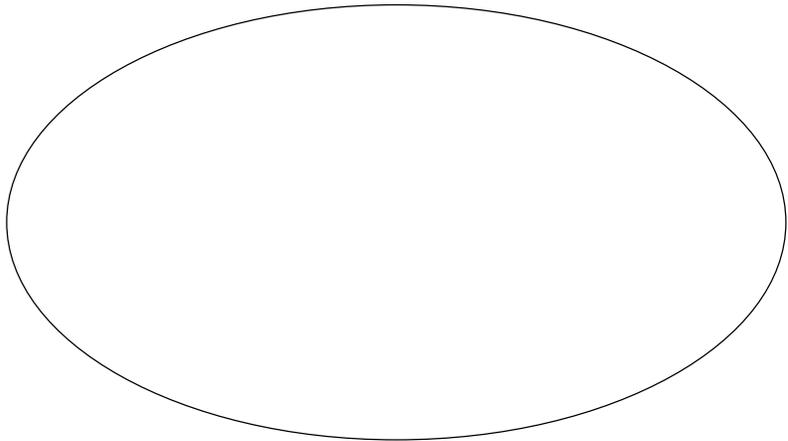
#### Exercise 1: Recalling the Relationship Between Multiplication and Division

Record equations from Exercise 1 on the left.

Equations



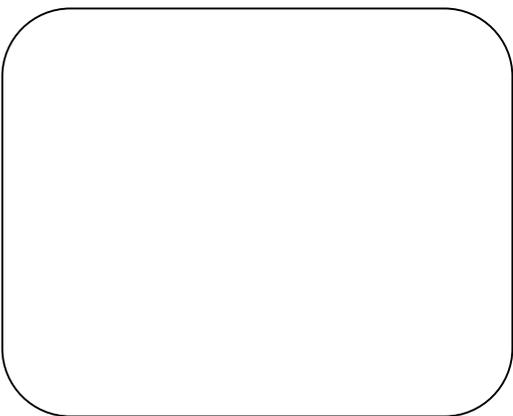
Integers



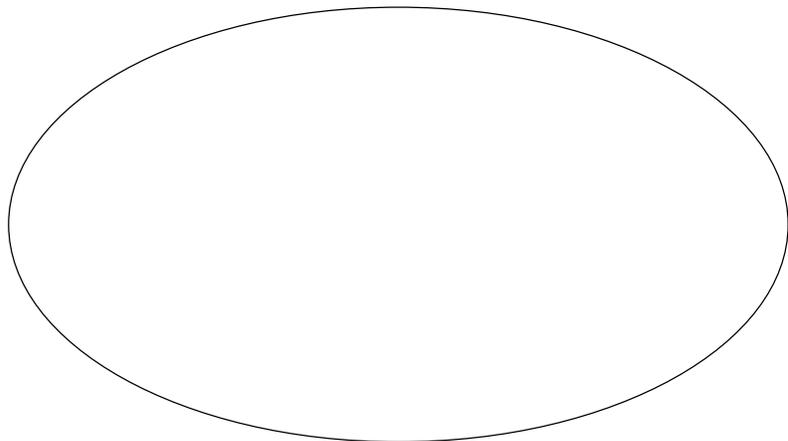
#### Example 1: Transitioning from Integer Multiplication Rules to Integer Division Rules

Record your group's number sentences in the space on the left below.

Equations



Integers



- a. List examples of division problems that produced a quotient that is a negative number.
- b. If the quotient is a negative number, what must be true about the signs of the dividend and divisor?
- c. List your examples of division problems that produced a quotient that is a positive number.
- d. If the quotient is a positive number, what must be true about the signs of the dividend and divisor?

Rules for Dividing Two Integers:

- A quotient is negative if the divisor and the dividend have \_\_\_\_\_ signs.
- A quotient is positive if the divisor and the dividend have \_\_\_\_\_ signs.

**Exercise 2: Is the Quotient of Two Integers Always an Integer?**

Is the quotient of two integers always an integer? Use the work space below to create quotients of integers. Answer the question and use examples or a counterexample to support your claim.

Work Space:

Answer:

**Exercise 3: Different Representation of the Same Quotient**

Are the answers to the three quotients below the same or different? Why or why not?

a.  $-14 \div 7$

b.  $14 \div (-7)$

c.  $-(14 \div 7)$

**Lesson Summary**

The rules for dividing integers are similar to the rules for multiplying integers (when the divisor is not zero). The quotient is positive if the divisor and dividend have the same signs and negative if they have opposite signs.

The quotient of any two integers (with a non-zero divisor) will be a rational number. If  $p$  and  $q$  are integers, then.

$$-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$$

**Problem Set**

1. Find the missing values in each column.

Column A	Column B	Column C	Column D
$48 \div 4 =$	$24 \div 4 =$	$63 \div 7 =$	$21 \div 7 =$
$-48 \div (-4) =$	$-24 \div (-4) =$	$-63 \div (-7) =$	$-21 \div (-7) =$
$-48 \div 4 =$	$-24 \div 4 =$	$-63 \div 7 =$	$-21 \div 7 =$
$48 \div (-4) =$	$24 \div (-4) =$	$63 \div (-7) =$	$21 \div (-7) =$

2. Describe the pattern you see in each column's answers in Problem 1, relating it to the problems' divisors and dividends. Why is this so?
3. Describe the pattern you see between the answers for Columns A and B in Problem 1 (e.g., compare the first answer in Column A to the first answer in Column B; compare the second answer in Column A to the second answer in Column B). Why is this so?
4. Describe the pattern you see between the answers for Columns C and D in Problem 1. Why is this so?

## Lesson 13: Converting Between Fractions and Decimals Using Equivalent Fractions

### Classwork

#### Example 1: Representations of Rational Numbers in the Real World

Following the Opening Exercise and class discussion, describe why we need to know how to represent rational numbers in different ways.

#### Example 2: Using Place Values to Write (Terminating) Decimals as Equivalent Fractions

- What is the value of the number 2.25? How can this number be written as a fraction or mixed number?
- Rewrite the fraction in its simplest form showing all steps that you use.
- What is the value of the number 2.025? How can this number be written as a mixed number?
- Rewrite the fraction in its simplest form showing all steps that you use.

**Exercise 1**

Use place value to convert each terminating decimal to a fraction. Then rewrite each fraction in its simplest form.

a. 0.218

b. 0.16

c. 2.72

d. 0.0005

**Example 3: Converting Fractions to Decimals—Fractions with Denominators Having Factors of only 2 or 5**

a. What are *decimals*?

b. Use the meaning of *decimal* to relate decimal place values.

c. Write the number  $\frac{3}{100}$  as a decimal. Describe your process.

d. Write the number  $\frac{3}{20}$  as a decimal. Describe your process.

e. Write the number  $\frac{10}{25}$  as a decimal. Describe your process.

f. Write the number  $\frac{8}{40}$  as a decimal. Describe your process.

**Exercise 2**

Convert each fraction to a decimal using an equivalent fraction.

a.  $\frac{3}{16} =$

b.  $\frac{7}{5} =$

c.  $\frac{11}{32} =$

d.  $\frac{35}{50} =$

**Lesson Summary**

Any terminating decimal can be converted to a fraction using place value (e.g., 0.35 is thirty-five hundredths or  $\frac{35}{100}$ ). A fraction whose denominator includes only factors of 2 and 5 can be converted to a decimal by writing the denominator as a power of ten.

**Problem Set**

- Convert each terminating decimal to a fraction in its simplest form.
  - 0.4
  - 0.16
  - 0.625
  - 0.08
  - 0.012
- Convert each fraction or mixed number to a decimal using an equivalent fraction.
  - $\frac{4}{5}$
  - $\frac{3}{40}$
  - $\frac{8}{200}$
  - $3\frac{5}{16}$
- Tanja is converting a fraction into a decimal by finding an equivalent fraction that has a power of 10 in the denominator. Sara looks at the last step in Tanja's work (shown below) and says that she cannot go any further. Is Sara correct? If she is, explain why. If Sara is incorrect, complete the remaining steps.

$$\frac{72}{480} = \frac{2^3 \cdot 3^2}{2^5 \cdot 3 \cdot 5}$$

## Lesson 14: Converting Rational Numbers to Decimals Using Long Division

### Classwork

#### Example 1: Can All Rational Numbers Be Written as Decimals?

- a. Using the division button on your calculator, explore various quotients of integers 1 through 11. Record your fraction representations and their corresponding decimal representations in the space below.
- b. What two types of decimals do you see?

#### Example 2: Decimal Representations of Rational Numbers

In the chart below, organize the fractions and their corresponding decimal representation listed in Example 1 according to their type of decimal.

What do these fractions have in common?	What do these fractions have in common?
---	---

**Example 3: Converting Rational Numbers to Decimals Using Long Division**

Use the long division algorithm to find the decimal value of  $-\frac{3}{4}$ .

**Exercise 1**

Students convert each rational number to its decimal form using long division.

a.  $-\frac{7}{8} =$

b.  $\frac{3}{16} =$

**Example 4: Converting Rational Numbers to Decimals Using Long Division**

Use long division to find the decimal representation of  $\frac{1}{3}$ .

**Exercise 2**

Calculate the decimal values of the fraction below using long division. Express your answers using bars over the shortest sequence of repeating digits.

a.  $-\frac{4}{9}$

b.  $-\frac{1}{11}$

c.  $\frac{1}{7}$

d.  $-\frac{5}{6}$



### Lesson Summary

The real world requires that we represent rational numbers in different ways depending on the context of a situation. All rational numbers can be represented as either terminating decimals or repeating decimals using the long division algorithm. We represent repeating decimals by placing a bar over the shortest sequence of repeating digits.

### Problem Set

1. Convert each rational number into its decimal form.

$$\frac{1}{3} = \underline{\hspace{2cm}}$$

$$\frac{1}{6} = \underline{\hspace{2cm}}$$

$$\frac{2}{6} = \underline{\hspace{2cm}}$$

$$\frac{3}{6} = \underline{\hspace{2cm}}$$

$$\frac{2}{3} = \underline{\hspace{2cm}}$$

$$\frac{4}{6} = \underline{\hspace{2cm}}$$

$$\frac{5}{6} = \underline{\hspace{2cm}}$$

$$\frac{1}{9} = \underline{\hspace{2cm}}$$

$$\frac{2}{9} = \underline{\hspace{2cm}}$$

$$\frac{3}{9} = \underline{\hspace{2cm}}$$

$$\frac{4}{9} = \underline{\hspace{2cm}}$$

$$\frac{5}{9} = \underline{\hspace{2cm}}$$

$$\frac{6}{9} = \underline{\hspace{2cm}}$$

$$\frac{7}{9} = \underline{\hspace{2cm}}$$

$$\frac{8}{9} = \underline{\hspace{2cm}}$$

One of these decimal representations is not like the others. Why?

**Enrichment:**

2. Chandler tells Aubrey that the decimal value of  $-\frac{1}{17}$  is not a repeating decimal. Should Aubrey believe him? Explain.

3. Complete the quotients below without using a calculator and answer the questions that follow.

a. Convert each rational number in the table to its decimal equivalent.

$\frac{1}{11} =$	$\frac{2}{11} =$	$\frac{3}{11} =$	$\frac{4}{11} =$	$\frac{5}{11} =$
$\frac{6}{11} =$	$\frac{7}{11} =$	$\frac{8}{11} =$	$\frac{9}{11} =$	$\frac{10}{11} =$

Do you see a pattern? Explain.

b. Convert each rational number in the table to its decimal equivalent.

$\frac{0}{99} =$	$\frac{10}{99} =$	$\frac{20}{99} =$	$\frac{30}{99} =$	$\frac{45}{99} =$
$\frac{58}{99} =$	$\frac{62}{99} =$	$\frac{77}{99} =$	$\frac{81}{99} =$	$\frac{98}{99} =$

Do you see a pattern? Explain.

c. Can you find other rational numbers that follow similar patterns?

## Lesson 15: Multiplication and Division of Rational Numbers

### Classwork

#### Exercise 1

- a. In the space below, create a word problem that involves integer multiplication. Write an equation to model the situation.
- b. Now change the word problem by replacing the integers with non-integer rational numbers (fractions or decimals), and write the new equation.
- c. Was the process used to solve the second problem different from the process used to solve the first? Explain.
- d. The Rules for Multiplying Rational Numbers are the same as the Rules for Multiplying Integers:

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

**Exercise 2**

a. In one year, Melinda’s parents spend \$2,640.90 on cable and internet service. If they spend the same amount each month, what is the resulting monthly change in the family’s income?

b. The Rules for Dividing Rational Numbers are the same as the Rules for Dividing Integers:

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

**Exercise 3**

Use the fundraiser chart to help answer the questions that follow.

**Grimes Middle School Flower Fundraiser**

Customer	Plant Type	Number of Plants	Price per Plant	Total	Paid? Yes or No
Tamara Jones	tulip	2	\$4.25		No
Mrs. Wolff	daisy	1	\$3.75	\$ 3.75	Yes
Mr. Clark	geranium	5	\$2.25		Yes
Susie (Jeremy’s sister)	violet	1	\$2.50	\$ 2.50	Yes
Nana and Pop (Jeremy’s grandparents)	daisy	4	\$3.75	\$15.00	No

Jeremy is selling plants for the school’s fundraiser, and listed above is a chart from his fundraiser order form. Use the information in the chart to answer the following questions. Show your work and represent the answer as a rational number; then, explain your answer in the context of the situation.

- a. If Tamara Jones writes a check to pay for the plants, what is the resulting change in her checking account balance?

Numerical Answer:

Explanation:

- b. Mr. Clark wants to pay for his order with a \$20 bill, but Jeremy does not have change. Jeremy tells Mr. Clark he will give him the change later. How will this affect the total amount of money Jeremy collects? Explain. What rational number represents the change that must be made to the money Jeremy collects?

Numerical Answer:

Explanation:

- c. Jeremy's sister, Susie, borrowed the money from their mom to pay for her order. Their mother has agreed to deduct an equal amount of money from Susie's allowance each week for the next five weeks to repay the loan. What is the weekly change in Susie's allowance?

Numerical Answer:

Explanation:

- d. Jeremy's grandparents want to change their order. They want to order three daisies and one geranium, instead of four daisies. How does this change affect the amount of their order? Explain how you arrived at your answer.
- e. Jeremy approaches three people who do not want to buy any plants; however, they wish to donate some money for the fundraiser when Jeremy delivers the plants one week later. If the people promise to donate a total of \$14.40, what will be the average cash donation?
- f. Jeremy spends one week collecting orders. If 22 people purchase plants totaling \$270, what is the average amount of Jeremy's sale?

**Lesson Summary**

The rules that apply for multiplying and dividing integers apply to rational numbers. We can use the products and quotients of rational numbers to describe real-world situations.

**Problem Set**

1. At lunch time, Benjamin often borrows money from his friends to buy snacks in the school cafeteria. Benjamin borrowed \$0.75 from his friend Clyde five days last week to buy ice cream bars. Represent the amount Benjamin borrowed as the product of two rational numbers; then, determine how much Benjamin owed his friend last week.
2. Monica regularly records her favorite television show. Each episode of the show requires 3.5% of the total capacity of her video recorder. Her recorder currently has 62% of its total memory free. If Monica records all five episodes this week, how much space will be left on her video recorder?

For Problems 3–5, find at least two possible sets of values that will work for each problem.

3. Fill in the blanks with two rational numbers (other than 1 and  $-1$ ).  $\underline{\hspace{1cm}} \times \left(-\frac{1}{2}\right) \times \underline{\hspace{1cm}} = -20$   
What must be true about the relationship between the two numbers you chose?
4. Fill in the blanks with two rational numbers (other than 1 and  $-1$ ).  $-5.6 \times 100 \div 80 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = 700$   
What must be true about the relationship between the two numbers you chose?
5. Fill in the blanks with two rational numbers.  $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = -0.75$   
What must be true about the relationship between the two numbers you chose?

For Problems 6–8, create word problems that can be represented by each expression, and then represent each product or quotient as a single rational number.

6.  $8 \times (-0.25)$
7.  $-6 \div \left(1\frac{1}{3}\right)$
8.  $-\frac{1}{2} \times 12$

## Lesson 16: Applying the Properties of Operations to Multiply and Divide Rational Numbers

### Classwork

#### Example 1: Using the Commutative and Associative Properties to Efficiently Multiply Rational Numbers

- a. Evaluate the expression below.

$$-6 \times 2 \times (-2) \times (-5) \times (-3)$$

- b. What types of strategies were used to evaluate the expressions?
- c. Can you identify the benefits of choosing one strategy versus another?
- d. What is the sign of the product and how was the sign determined?

**Exercise 1**

Find an efficient strategy to evaluate the expression and complete the necessary work.

$$-1 \times (-3) \times 10 \times (-2) \times 2$$

**Exercise 2**

Find an efficient strategy to evaluate the expression and complete the necessary work.

$$4 \times \frac{1}{3} \times (-8) \times 9 \times \left(-\frac{1}{2}\right)$$

**Exercise 3**

What terms did you combine first and why?

**Exercise 4**

Refer to the example and exercises. Do you see an easy way to determine the sign of the product first?

**Example 2: Using the Distributive Property to Multiply Rational Numbers**

Rewrite the mixed number as a sum; then, multiply using the distributive property.

$$-6 \times \left(5\frac{1}{3}\right)$$

**Exercise 5**

Multiply the expression using the distributive property.

$$9 \times \left(-3\frac{1}{2}\right)$$

**Example 3: Using the Distributive Property to Multiply Rational Numbers**

Evaluate using the distributive property.

$$16 \times \left(-\frac{3}{8}\right) + 16 \times \frac{1}{4}$$

**Example 4: Using the Multiplicative Inverse to Rewrite Division as Multiplication**

Rewrite the expression as only multiplication and evaluate.

$$1 \div \frac{2}{3} \times (-8) \times 3 \div \left(-\frac{1}{2}\right)$$

**Exercise 6**

$$4.2 \times \left(-\frac{1}{3}\right) \div \frac{1}{6} \times (-10)$$

### Lesson Summary

Multiplying and dividing using strictly order of operations is not always efficient. The properties of multiplication allow us to manipulate expressions by rearranging and regrouping factors that are easier to compute. Where division is involved, we can easily rewrite division as multiplication to allow the use of these properties. The signs of expressions with products and quotients can be easily determined by checking whether the number of negative terms is even or odd.

### Problem Set

- Evaluate the expression  $-2.2 \times (-2) \div \left(-\frac{1}{4}\right) \times 5$ 
  - Using the order of operations only.
  - Using the properties and methods used in Lesson 16.
  - If you were asked to evaluate another expression, which method would you use, (a) or (b), and why?
- Evaluate the expressions using the distributive property.
  - $\left(2\frac{1}{4}\right) \times (-8)$
  - $\frac{2}{3}(-7) + \frac{2}{3}(-5)$
- Mia evaluated the expression below but got an incorrect answer. Find Mia's error(s), find the correct value of the expression, and explain how Mia could have avoided her error(s).

$$0.38 \times 3 \div \left(-\frac{1}{20}\right) \times 5 \div (-8)$$

$$0.38 \times 5 \times \left(\frac{1}{20}\right) \times 3 \times (-8)$$

$$0.38 \times \left(\frac{1}{4}\right) \times 3 \times (-8)$$

$$0.38 \times \left(\frac{1}{4}\right) \times (-24)$$

$$0.38 \times (-6)$$

$$-2.28$$

## Lesson 17: Comparing Tape Diagram Solutions to Algebraic Solutions

### Classwork

#### Exploratory Challenge: Expenses on Your Family Vacation

John and Ag are summarizing some of the expenses of their family vacation for themselves and their three children, Louie, Missy, and Bonnie. Write an algebraic equation, create a model to determine how much each item will cost using all of the given information, and answer the questions that follow.

Expenses:

Car and insurance fees: \$400	Airfare and insurance fees: \$875	Motel and tax: \$400
Baseball game and hats: \$103.83	Movies for one day: \$75	Soda and pizza: \$37.95
	Sandals and t-shirts: \$120	

Your Group's Scenario Solution:

After collaborating with all of the groups, summarize the findings in the table below.

Cost of Evening Movie	
Cost of 1 Slice of Pizza	
Cost of the admission ticket to the baseball game	
Cost of 1 T-Shirt	
Cost of 1 Airplane Ticket	
Daily Cost for Car Rental	
Nightly Charge for Motel	

Using the results, determine the cost of the following:

1. A slice of pizza, 1 plane ticket, 2 nights in the motel, and 1 evening movie.

2. One t-shirt, 1 ticket to the baseball game, and 1 day of the rental car.

**Exercise**

The cost of a babysitting service on a cruise is \$10 for the first hour and \$12 for each additional hour. If the total cost of babysitting baby Aaron was \$58, how many hours was Aaron at the sitter?

### Lesson Summary

*Tape diagrams* can be used to model and identify the sequence of operations to find a solution algebraically.

The goal in solving equations algebraically is to isolate the variable.

The process of doing this requires “undoing” addition or subtraction to obtain a 0 and “undoing” multiplication or division to obtain a 1. The additive inverse and multiplicative inverse properties are applied, to get the 0 (the additive identity) and 1 (the multiplicative identity).

The addition and multiplication properties of equality are applied because in an equation,  $A = B$ , when a number is added or multiplied to both sides, the resulting sum or product remains equal.

### Problem Set

1. A taxi cab in Myrtle Beach charges \$2 per mile and \$1 for every person. If a taxi cab ride for two people costs \$12, how far did the taxi cab travel?
2. Heather works as a waitress at her family’s restaurant. She works 2 hours every morning during the breakfast shift and the same number of hours every evening during the dinner shift. In the last four days she worked 28 hours. How many hours did she work during each dinner shift?
3. Jillian exercises 5 times a week. She runs 3 miles each morning and bikes in the evening. If she exercises a total of 30 miles for the week, how many miles does she bike each evening?
4. Marc eats an egg sandwich for breakfast and a big burger for lunch every day. The egg sandwich has 250 calories. If Marc has 5,250 calories for breakfast and lunch for the week in total, how many calories are in one big burger?
5. Jackie won tickets playing the bowling game at the local arcade. The first time, she won 60 tickets. The second time she won a bonus, which was 4 times the number of tickets of the original second prize. All together she won 200 tickets. How many tickets was the original second prize?

## Lesson 18: Writing, Evaluating, and Finding Equivalent

### Expressions with Rational Numbers

#### Classwork

##### Exercise 1

John's father asked him to compare several different cell phone plans and identify which plan will be the least expensive for the family. Use the information contained in the table below to answer the following questions.

##### Cell Phone Plans

Name of Plan	Monthly Fee (Includes 1,500 shared minutes)	Price per Phone Line $x$	Price per line for Unlimited Texting $y$	Price per line for Internet Access $z$
Company A	\$70	\$20	\$15	\$15
Company B	\$90	\$15	\$10	\$20
Company C	\$200	\$10	included in monthly fee	included in monthly fee

All members of the family may not want identical plans; therefore, we will let  $x$  represent the number of phone lines,  $y$  represent the number of phone lines with unlimited texting, and  $z$  represent the number of phone lines with internet access.

Expression

Company A \_\_\_\_\_

Company B \_\_\_\_\_

Company C \_\_\_\_\_

Using the expressions above, find the cost to the family of each company’s phone plan if:

- a. Four people want a phone line, four people want unlimited texting, and the family needs two internet lines.

Company A	Company B	Company C

Which cell phone company should John’s family use? Why?

- b. Four people want a phone line, four people want unlimited texting, and all four people want internet lines.

Company A	Company B	Company C

Which cell phone company should John’s family use? Why?

- c. Two people want a phone line, two people want unlimited texting, and the family needs two internet lines.

Company A	Company B	Company C

Which cell phone company should John's family use? Why?

### Exercise 2

Three friends went to the movies. Each purchased a medium-sized popcorn for  $p$  dollars and a small soft drink for  $s$  dollars.

- Write the expression that represents the total amount of money (in dollars) the three friends spent at the concession stand.
  
  
  
  
  
  
  
  
  
  
- If the concession stand charges \$6.50 for a medium-sized popcorn and \$4.00 for a small soft drink, how much did the three friends spend on their refreshments all together?

Exercise 3

Equivalent Expressions			
<p>EXAMPLE: Evaluate <math>x = 2,</math> <math>y = -1</math></p>	$4(x + 2y)$ $4(2 + 2(-1))$ $4(0)$ $0$	$4x + 8y$ $4(2) + 8(-1)$ $8 + (-8)$ $0$	$4x + 4y + 4y$ $4(2) + 4(-1) + 4(-1)$ $8 + (-4) + (-4)$ $0$
<p>1. Evaluate <math>y = 1</math></p>	$5(3 - 4y)$		
<p>2. Evaluate <math>x = 5,</math> <math>y = -2</math></p>	$-3x + 12y$		
<p>3. Evaluate <math>x = -\frac{1}{2},</math> <math>y = 1</math></p>			$-2x + 10x - 6y$

### Lesson Summary

- An expression is a number or a letter, which can be raised to a whole number exponent. An expression can be a product whose factors are any one of the entities described above. An expression can also be the sum or difference of the products described above.
- To evaluate an expression, replace each variable with its corresponding numerical value. Using order of operations, the expression can be written as a single numerical value.
- When numbers are substituted into all the letters in an expression and the results are the same, then the expressions are equivalent.

### Problem Set

1. Sally is paid a fixed amount of money to walk her neighbor's dog every day after school. When she is paid each month, she puts aside \$20 to spend and saves the remaining amount. Write an expression that represents the amount Sally will save in 6 months if she earns  $m$  dollars each month. If Sally is paid \$65 each month, how much will she save in 6 months?
2. A football team scored 3 touchdowns, 3 extra points, and 4 field goals.
  - a. Write an expression to represent the total points the football team scored.
  - b. Write another expression that is equivalent to the one written above.
  - c. If each touchdown is worth 6 points, each extra point is 1 point, and each field goal is 3 points, how many total points did the team score?
3. Write three other expressions that are equivalent to  $8x - 12$ .
4. Profit is defined as earnings less expenses (earnings – expenses). At the local hot air balloon festival, the Ma & Pops Ice Cream Truck sells ice cream pops, which cost them \$0.75 each, but are sold for \$2 each. They also paid \$50 to the festival's organizers for a vendor permit. The table below shows the earnings, expenses, and profit earned when 50, 75, and 100 ice cream pops were sold at the festival.

Number of Pops Sold	Earnings	Expenses	Profit
50	$50(2) = 100$	$50(0.75) + 50$ $37.5 + 50 = 87.5$	$100 - 87.5 = 12.50$
75	$75(2) = 150$	$75(0.75) + 50$ $56.25 + 50 = 106.25$	$150 - 106.25 = 43.75$
100	$100(2) = 200$	$100(0.75) + 50$ $75 + 50 = 125$	$200 - 125 = 75$

- a. Write an expression that represents the profit (in dollars) Ma & Pop earned by selling ice cream pops at the festival.
- b. Write an equivalent expression.
- c. How much did Ma & Pops Ice Cream Truck profit if it sold 20 ice cream pops? What does this mean? Explain why this might be the case?
- d. How much did Ma & Pops Ice Cream truck profit if it sold 75 Ice Cream Pops? What does this mean? Explain why this might be the case?

## Lesson 19: Writing, Evaluating, and Finding Equivalent Expressions with Rational Numbers

### Classwork

#### Example 1: Tic-Tac-Toe Review

Fill in the 9 spaces with one expression from the list below. Use one expression per space. You will use 9 of the expressions:

$$12 - 4x$$

$$8x + 4 - 12x$$

$$8\left(\frac{1}{2}x - 2\right)$$

$$12 - 6x + 2x$$

$$-4x + 4$$

$$x - 2 + 2x - 4$$

$$4x - 12$$

$$4(x - 4)$$

$$3(x - 2)$$

$$0.1(40x) - \frac{1}{2}(24)$$



**Example 2**

Original Price (100%)	Discount Amount (20%) Off	New Price (Pay 80%)	Expression
100			
50			
28			
14.50			
$x$			

**Example 3**

An item that has an original price of  $x$  dollars is discounted 33%.

- Write an expression that represents the amount of the discount.
- Write two equivalent expressions that represent the new, discounted price.
- Use one of your expressions to calculate the new, discounted price if the original price was \$56.
- How would the expressions you created in parts (a) and (b) have to change if the item's price had increased by 33% instead of decreased by 33%?

Example 4

Original Price (100%)	Discount (20% off)	Amount Pay (pay 80%)	Expression	New Price	Sales Tax (8%)	Overall Cost	Expression
100	20	80	$100 - 100(0.20) = 100(0.80)$				
50	10	40	$50 - 50(0.20) = 50(0.80)$				
28	5.60	22.40	$28 - 28(0.20) = 28(0.80)$				
14.50	2.90	11.60	$14.50 - 14.50(0.20)$ or $14.50(0.80)$				
$x$	$0.20x$	$x - 0.20x$	$x - 0.20x$ or $0.80x$				

**Lesson Summary**

- Two expressions are equivalent if they yield the same number for every substitution of numbers for the letters in each expression.
- The expression that allows us to find the cost of an item after the discount has been taken and the sales tax has been added is written by representing the discount price added to the discount price multiplied by the sales tax rate.

**Problem Set**

Solve the following problems. If necessary, round to the nearest penny.

- A family of 12 went to the local Italian restaurant for dinner. Every family member ordered a drink and meal, 3 ordered an appetizer, and 6 people ordered cake for dessert.
  - Write an expression that can be used to figure out the cost of the bill. Include the definitions for the variables the server used.
  - The waitress wrote on her ordering pad the following expression:  $3(4d + 4m + a + 2c)$ . Was she correct? Explain why or why not.
  - What is the cost of the bill if a drink costs \$3, a meal costs \$20, an appetizer costs \$5.50, and a slice of cake costs \$3.75?
  - Suppose the family had a 10% discount coupon for the entire check and then left an 18% tip. What is the total?
- Sally designs web pages for customers. She charges \$135.50 per web page; however, she must pay a monthly rental fee of \$650 for her office. Write an expression to determine her take-home pay after expenses. If Sally designed 5 web pages last month, what was her take-home pay after expenses?
- While shopping, Megan and her friend Rylie find a pair of boots on sale for 25% off of the original price. Megan calculates the final cost of the boots by first deducting the 25%, and then adding the 6% sales tax. Rylie thinks Megan will pay less if she pays the 6% sales tax first and then takes the 25% discount.
  - Write an expression to represent each girl's scenario if the original price of the boots was  $x$  dollars.
  - Evaluate each expression if the boots originally cost \$200.
  - Who was right? Explain how you know.
  - Explain how both girls' expressions are equivalent.

## Lesson 20: Investments—Performing Operations with Rational Numbers

### Classwork

#### Mathematical Modeling Exercise: College Investments

Justin and Adrienne deposited \$20,000 into an investment account for 5 years. They hoped the money invested and the money made on their investment would amount to at least \$30,000 to help pay for their daughter’s college tuition and expenses. The account they chose has several benefits and fees associated with it. Every 6 months, a summary statement is sent to Justin and Adrienne. The statement includes the amount of money either gained or lost. Below are semi-annual (twice a year) statements for a period of 5 years. In addition to the statements, the following information is needed to complete the task:

- Every statement, there is an administrative fee of \$15 to cover costs such as secretarial work, office supplies, postage, etc.
- If there is a withdrawal made, a broker’s fee is deducted from the account. The amount of the broker’s fee is 2% of the transaction amount.

TASK: Using the above information, semi-annual statements, register, and beginning balance, do the following:

1. Record the beginning balance and all transactions from the account statements into the register.
2. Determine the annual gain or loss as well as the overall 5-year gain/loss.
3. Determine if there is enough money in the account after 5 years to cover \$30,000 of college expenses for Justin and Adrienne’s daughter. Write a summary to defend your answer. Be sure to indicate how much money is in excess, or the shortage that exists.
4. Answer the related questions that follow.

**College Investment Fund Semi-Annual Statement**

January 1, 2008 – June 30, 2008

Investment Gain/(Loss): 700.00

**College Investment Fund Semi-Annual Statement**

July 1, 2008 – December 31, 2008

Investment Gain/(Loss): 754.38

**College Investment Fund Semi-Annual Statement**

January 1, 2009 – June 30, 2009

Investment Gain/(Loss): (49.88)

**College Investment Fund Semi-Annual Statement**

July 1, 2009 – December 31, 2009

Withdrawal: 500.00  
Investment Gain/(Loss): (17.41)

**College Investment Fund Semi-Annual Statement**

January 1, 2010 – June 30, 2010

Investment Gain/(Loss): 676.93

**College Investment Fund Semi-Annual Statement**

July 1, 2010 – December 31, 2010

Investment Gain/(Loss): 759.45

**College Investment Fund Semi-Annual Statement**

January 1, 2011 – June 30, 2011

Deposit: 1,500.00  
Investment Gain/(Loss): 880.09

**College Investment Fund Semi-Annual Statement**

July 1, 2011 – December 31, 2011

Investment Gain/(Loss): 922.99

**College Investment Fund Semi-Annual Statement**

January 1, 2012 – June 30, 2012

Deposit: 800.00  
Investment Gain/(Loss): 942.33

**College Investment Fund Semi-Annual Statement**

July 1, 2012 – December 31, 2012

Investment Gain/(Loss): 909.71

5. Register

DATE	DESCRIPTION OF TRANSACTION	WITHDRAWAL	DEPOSIT	BALANCE	EXPRESSION
	Beginning Balance	---	---	\$20,000.00	\$20,000.00
Jan. – June: 2008					
July – Dec.: 2008					
Jan. – June: 2009					
July – Dec.: 2009					
Jan. – June: 2010					
July – Dec.: 2010					
Jan. – June: 2011					
July – Dec.: 2011					
Jan. – June: 2012					
July – Dec.: 2012					

6. Annual Gain/Loss Summary

Year	Total Gain/(Loss)	Numerical Expression
2008		
2009		
2010		
2011		
2012		
5-Year Gain/Loss		

7. Summary

## 8. Related Questions

- a. For the first half of 2009, there was a \$700 gain on the initial investment of \$20,000. Represent the gain as a percentage of the initial investment.
- b. Based on the gains and losses on their investment during this 5-year period, over what period of time was their investment not doing well? How do you know? What factors might contribute to this?
- c. In math class, Jaheim and Frank were working on finding the total amount of the investment after 5 years. As a final step, Jaheim subtracted \$150 for administrative fees from the balance he arrived at after adding in all the deposits and subtracting out the one withdrawal and Broker's fee. For every semi-annual statement, Frank subtracted \$15 from the account balance for the administrative fee. Both boys arrived at the same ending five-year balance. How is this possible? Explain.
- d. Based on the past statements for their investment account, predict what activity you might expect to see on Adrienne and Justin's January–June 2013 account statement. Then record it in the register to arrive at the balance as of June 30, 2013.
- e. Using the answer from part (d), if their daughter's college bill is due in September of 2013, how much money do you estimate will be in their investment account at the end of August 2013 before the college bill is paid? Support your answer.

**Exercise**

Below is a transaction log of a business entertainment account. The transactions are completed and the ending balance in the account is \$525.55. Determine the beginning balance.

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	
12/1/10	Bargain Electronic (I-Pod)	199.99		
12/5/10	Lenny's Drive-Up (Gift Certificate)	75.00		
12/7/10	Check from Customer: Reynolds		200.00	
12/15/10	Pasta House (Dinner)	285.00		
12/20/10	Refund from Clear's Play House		150.00	
12/22/10	Gaffney's Tree Nursery	65.48		525.55

**Lesson Summary**

- Calculations with rational numbers are used when recording investment transactions.
- Deposits are added to an account balance; money is deposited into the account.
- Gains are added to an account balance; they are positive returns on the investment.
- Withdrawals are subtracted from an account balance; money is taken out of the account.
- Losses are subtracted from an account balance; they are negative returns on the investment.
- Fees are subtracted from an account balance; the bank or financial company is charging you for a service.

**Problem Set**

1. You are planning a fundraiser for your student council. The fundraiser is a Glow in the Dark Dance. Solve each entry below and complete the transaction log to determine the ending balance in the student account.
  - a. The cost of admission to the dance is \$7 per person. Write an expression to represent the total amount of money collected for admission. Evaluate the expression if 250 people attended the dance.
  - b. The following expenses were necessary for the dance, and checks were written to each company.
    - DJ for the dance – “Music Madness DJ” costs \$200
    - Glow Sticks for “Glow World Inc.” for the first 100 entrants. Cost of glow sticks were \$0.75 each plus 8% sales tax.

Complete the transaction log below based on this information

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	1,243.56

- c. Write a numerical expression to determine the cost of the glow sticks.

Analyze the results.

- d. Write an algebraic expression to represent the profit earned from the fundraiser. (Profit is the amount of money collected in admissions minus all expenses.)
- e. Evaluate the expression to determine the profit if 250 people attended the dance. Use the variable  $p$  to represent the number of people attending the dance (from part (a)).
- f. Using the transaction log above, what was the amount of the profit earned?

2. The register below shows a series of transactions made to an investment account. Vinnie and Anthony both completed the register in hopes of finding the beginning balance. As you can see, they do not get the same answer. Who was correct? What mistake did the other person make? What was the monthly gain or loss?

Original Register

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	
3/1/11	Broker's Fee	250.00		
3/10/11	Loan Withdrawal	895.22		
3/15/11	Refund – Misc. Fee		50.00	
3/31/11	Investment Results		2,012.22	18,917.00

Vinnie's Work

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	18,000.00
3/1/11	Broker's Fee	250.00		17,750.00
3/10/11	Loan Withdrawal	895.22		16,854.78
3/15/11	Refund – Misc. Fee		50.00	16,904.78
3/31/11	Investment Results		2,012.22	18,917.00

Anthony's Work

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	19,834.00
3/1/11	Broker's Fee	250.00		20,084.00
3/10/11	Loan Withdrawal	895.22		20,979.22
3/15/11	Refund – Misc. Fee		50.00	20,929.22
3/31/11	Investment Results		2,012.22	18,917.00

## Lesson 21: If-Then Moves with Integer Number Cards

### Classwork

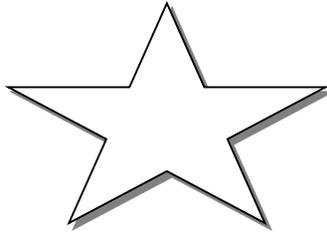
#### Exploratory Challenge: Integer Game Revisited

Let's investigate what happens if a card is added or removed from a hand of integers.

My cards:



My score:



#### Event 1

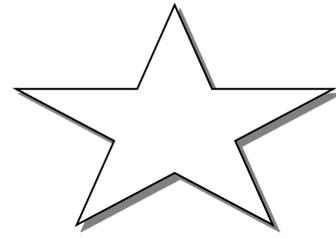
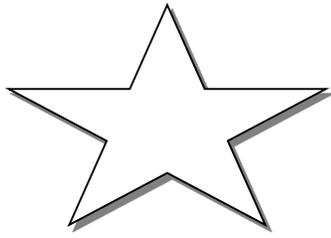
My new score:



Conclusion:

**Event 2**

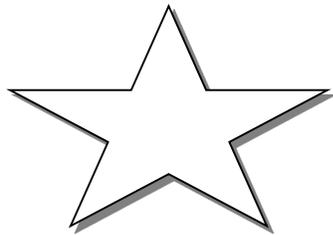
My new score:



Conclusion:

**Event 3**

My new score:



Expression:

Conclusion:

**Event 4**

Expression:

Conclusion:

**Exercises**

- The table below shows two hands from the Integer Game and a series of changes that occurred to each hand. Part of the table is completed for you. Complete the remaining part of the table, then summarize the results.

	Hand 1	Result	Hand 2	Result
Original	$1 + (-4) + 2$		$0 + 5 + (-6)$	
Add 4	$1 + (-4) + 2 + 4$			
Subtract 1	$1 + (-4) + 2 + 4 - 1$			
Multiply by 3				
Divide by 2				

2. Complete the table below using the multiplication property of equality.

	Original expression and result	Equivalent expression and result
	$3 + (-5) =$	
Multiply both expressions by $-3$		
Write a conclusion using if-then		

**Lesson Summary**

- If a number sentence is true,  $a = b$ , and you add or subtract the same number from both sides of the equation, then the resulting number sentence will be true.
- If a number sentence is true,  $a = b$ , and you multiply both sides of the equation by the same number, then the resulting number sentence will be true.
- If a number sentence is true,  $a = b$ , and you divide both sides of the equation by the same non-zero number, then the resulting number sentence will be true.

**Problem Set**

1. Evaluate the following numerical expressions

a.  $2 + (-3) + 7$

b.  $-4 - 1$

c.  $-\frac{5}{2} \times 2$

d.  $-10 \div 2 + 3$

e.  $\left(\frac{1}{2}\right)(8) + 2$

f.  $3 + (-4) - 1$

2. Which expressions from Exercise 1 are equal?

3. If 3 is divided to two of the equivalent expressions from Exercise 1, write an if-then statement using the properties of equality.

4. Write an if-then statement if  $-3$  is multiplied to the following equation:  $-1 - 3 = -4$ .

5. Simplify the expression.

$$5 + 6 - 5 + 4 + 7 - 3 + 6 - 3$$

Using the expression, write an equation.

Rewrite the equation if 5 is added to both expressions.

Write an if-then statement using the properties of equality.

## Lesson 22: Solving Equations Using Algebra

### Classwork

In this lesson, you will transition from solving equations using tape diagrams to solving equations algebraically by *making zero* (using the additive inverse) and *making one* (using the multiplicative inverse). Justify your work by identifying which algebraic property you used for each step in solving the problems. Explain your work by writing out how you solved the equations step by step and relate each step to those used with a tape diagram.

#### Example 1: Yoshiro's New Puppy

Yoshiro has a new puppy. She decides to create an enclosure for her puppy in her back yard. The enclosure is in the shape of a hexagon (six-sided polygon) with one pair of opposite sides running the same distance along the length of two parallel flowerbeds. There are two boundaries at one end of the flowerbeds that are 10 ft. and 12 ft., respectively, and at the other end, the two boundaries are 15 ft. and 20 ft., respectively. If the perimeter of the enclosure is 137 ft., what is the length of each side that runs along the flowerbed?

**Example 2: Swim Practice**

Jenny is on the local swim team for the summer and has swim practice four days per week. The schedule is the same each day. The team swims in the morning and then again for 2 hours in the evening. If she swims 12 hours per week, how long does she swim each morning?

**Exercises**

Solve each equation algebraically using if-then statements to justify each step.

1.  $5x + 4 = 19$

2.  $15x + 14 = 19$

3. Claire's mom found a very good price on a large computer monitor. She paid \$325 for a monitor that was only \$65 more than half the original price. What was the original price?

4.  $2(x + 4) = 18$

5. Ben's family left for vacation after his Dad came home from work on Friday. The entire trip was 600 mi. Dad was very tired after working a long day and decided to stop and spend the night in a hotel after 4 hours of driving. The next morning, Dad drove the remainder of the trip. If the average speed of the car was 60 miles per hour, what was the remaining time left to drive on the second part of the trip? Remember: Distance = rate multiplied by time.

**Lesson Summary**

We work backwards to solve an algebraic equation. For example, to find the value of the variable in the equation  $6x - 8 = 40$ :

1. Use the addition property of equality to add the opposite of  $-8$  to each side of the equation to arrive at  $6x - 8 + 8 = 40 + 8$ .
2. Use the additive inverse property to show that  $-8 + 8 = 0$ ; thus,  $6x + 0 = 48$ .
3. Use the additive identity property to arrive at  $6x = 48$ .
4. Then use the multiplication property of equality to multiply both sides of the equation by  $\frac{1}{6}$  to get:  
$$\left(\frac{1}{6}\right)6x = \left(\frac{1}{6}\right)48.$$
5. Then use the multiplicative inverse property to show that  $\frac{1}{6}(6) = 1$ ; thus,  $1x = 8$ .
6. Use the multiplicative identity property to arrive at  $x = 8$ .

**Problem Set**

For each problem below, explain the steps in finding the value of the variable. Then find the value of the variable, showing each step. Write if-then statements to justify each step in solving the equation.

1.  $7(m + 5) = 21$
2.  $-2v + 9 = 25$
3.  $\frac{1}{3}y - 18 = 2$
4.  $6 - 8p = 38$
5.  $15 = 5k - 13$

## Lesson 23: Solving Equations Using Algebra

### Classwork

#### Exercises

1. Youth Group Trip

The youth group is going on a trip to an amusement park in another part of the state. The trip costs each group member \$150, which includes \$85 for the hotel and two one-day combination entrance and meal plan passes.

- Write an equation representing the cost of the trip. Let  $P$  be the cost of the park pass.
- Solve the equation algebraically to find the cost of the park pass. Then write the reason that justifies each step using if-then statements.
- Model the problem using a tape diagram to check your work.

Suppose you want to buy your favorite ice cream bar while at the amusement park and it costs \$2.89. If you purchase the ice cream bar and 3 bottles of water, and pay with a \$10 bill and receive no change, then how much did each bottle of water cost?

- d. Write an equation to model this situation.
- e. Solve the equation to determine the cost of one water bottle. Then, write the reason that justifies each step using if-then statements.
- f. Model the problem using a tape diagram to check your work.



- d. Charlotte's goal is to save \$100 for her beach trip at the end of the summer. Use the amount of weekly allowance you found in part (c) to write an equation to determine the number of weeks that Charlotte must work to meet her goal. Let  $w$  represent the number of weeks.
- e. In looking at your answer to part (d), and based on the story above, do you think it will take Charlotte that many weeks to meet her goal? Why or Why not?

### 3. Travel Baseball Team

Allen is very excited about joining a travel baseball team for the fall season. He wants to determine how much money he should save to pay for the expenses related to this new team. Players are required to pay for uniforms, travel expenses, and meals.

- a. If Allen buys 4 uniform shirts at one time, he gets a \$10.00 discount so that the total cost of 4 shirts would be \$44. Write an algebraic equation that represents the regular price of one shirt. Solve the equation. Write the reason that justifies each step using if-then statements.

b. What is the cost of one shirt without the discount?

c. What is the cost of one shirt with the discount?

d. How much more do you pay per shirt if you buy them one at a time (rather than in bulk)?

Allen's team was also required to buy two pairs of uniform pants and two baseball caps, which total \$68. A pair of pants costs \$12 more than a baseball cap.

e. Write an equation that models this situation. Let  $c$  represent the cost of a baseball cap.

- f. Solve the equation algebraically to find the cost of a baseball cap. Write the reason that justifies each step using if-then statements.
- g. Model the problem using a tape diagram in order to check your work.
- h. What is the cost of one cap?
- i. What is the cost of one pair of pants?

**Lesson Summary**

Equations are useful to model and solve real-world problems. The steps taken to solve an algebraic equation are the same steps used in an arithmetic solution.

**Problem Set**

For Exercises 1–4, solve each equation algebraically using if-then statements to justify your steps.

1.  $\frac{2}{3}x - 4 = 20$

2.  $4 = \frac{-1+x}{2}$

3.  $12(x + 9) = -108$

4.  $5x + 14 = -7$

For Exercises 5–7, write an equation to represent each word problem. Solve the equation showing the steps and then state the value of the variable in the context of the situation.

- A plumber has a very long piece of pipe that is used to run city water parallel to a major roadway. The pipe is cut into two sections. One section of pipe is 12 ft. shorter than the other. If  $\frac{3}{4}$  of the length of the shorter pipe is 120 ft., how long is the longer piece of the pipe?
- Bob's monthly phone bill is made up of a \$10 fee plus \$0.05 per minute. Bob's phone bill for July was \$22. Write an equation to model the situation, using  $m$  to represent the number of minutes. Solve the equation to determine the number of phone minutes Bob used in July.
- Kym switched cell phone plans. She signed up for a new plan that will save her \$3.50 per month compared to her old cell phone plan. The cost of the new phone plan for an entire year is \$294. How much did Kym pay per month under her old phone plan?