

Name \_\_\_\_\_

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## Lesson 1: Graphs of Piecewise Linear Functions

### Exit Ticket

The graph in the Exploratory Challenge is made by combining pieces of nine linear functions (it is a piecewise linear function). Each linear function is defined over an interval of time, represented on the horizontal axis. List those nine time intervals.

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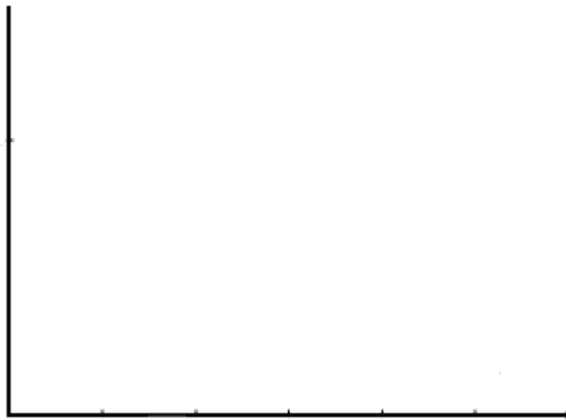
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## Lesson 2: Graphs of Quadratic Functions

### Exit Ticket

If you jumped in the air three times, what might the elevation versus time graph of that story look like?

Label the axes appropriately.



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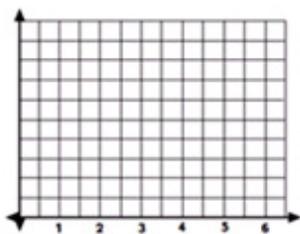
## Lesson 3: Graphs of Exponential Functions

### Exit Ticket

Assume that a bacteria population doubles every hour. Which of the following three tables of data, with  $x$  representing time in hours and  $y$  the count of bacteria, could represent the bacteria population with respect to time? For the chosen table of data, plot the graph of that data. Label the axes appropriately with units.

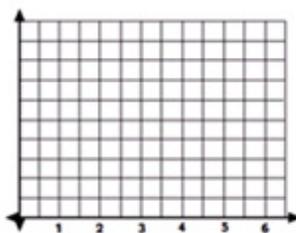
a) 

x	0	1	2	3	4	5	6
y	4	7	10	13	16	19	22



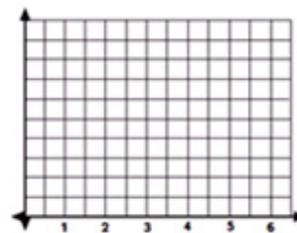
b) 

x	0	1	2	3	4	5	6
y	3	6	12	24	48	96	192



c) 

x	0	1	2	3	4	5	6
y	1	3	7	13	21	31	43





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## Lesson 5: Two Graphing Stories

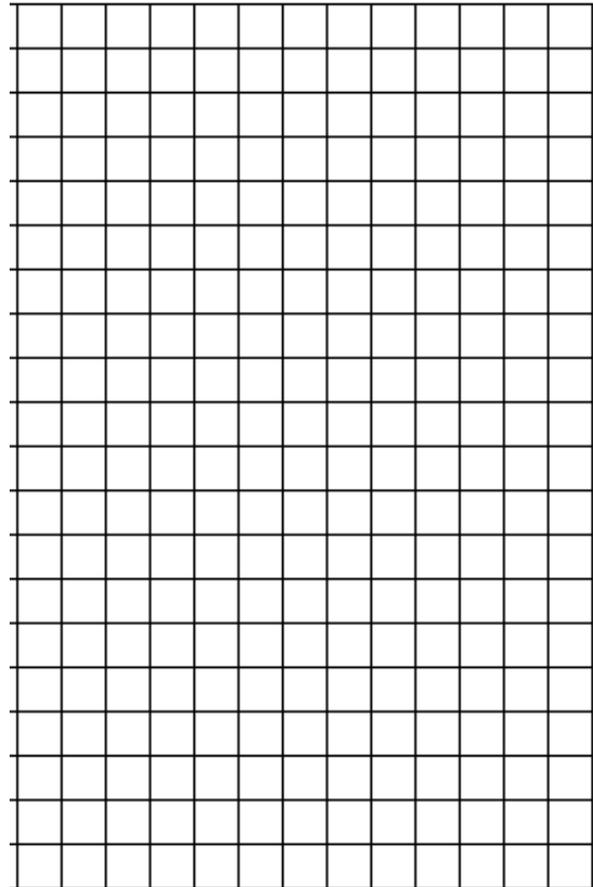
### Exit Ticket

Maya and Earl live at opposite ends of the hallway in their apartment building. Their doors are 50 feet apart. Each person starts at his or her own door and walks at a steady pace towards the other. They stop walking when they meet.

Suppose:

- Maya walks at a constant rate of 3 feet every second.
- Earl walks at a constant rate of 4 feet every second.

1. Graph both people’s distance from Maya’s door versus time in seconds.
  
2. According to your graphs, approximately how far will they be from Maya’s door when they meet?





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## Lesson 7: Algebraic Expressions—The Commutative and Associative Properties

### Exit Ticket

Write a mathematical proof of the algebraic equivalence of  $(pq)r$  and  $(qr)p$ .

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## Lesson 8: Adding and Subtracting Polynomials

### Exit Ticket

1. Must the sum of three polynomials again be a polynomial?

2. Find  $(w^2 - w + 1) + (w^3 - 2w^2 + 99)$ .

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## Lesson 9: Multiplying Polynomials

### Exit Ticket

1. Must the product of three polynomials again be a polynomial?

2. Find  $(w^2 + 1)(w^3 - w + 1)$ .

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1. Jacob lives on a street that runs east and west. The grocery store is to the east and the post office is to the west of his house. Both are on the same street as his house. Answer the questions below about the following story:

At 1:00 p.m., Jacob hops in his car and drives at a constant speed of 25 mph for 6 minutes to the post office. After 10 minutes at the post office, he realizes he is late and drives at a constant speed of 30 mph to the grocery store, arriving at 1:28 p.m. He then spends 20 minutes buying groceries.

- a. Draw a graph that shows the distance Jacob's car is from his house with respect to time. Remember to label your axes with the units you chose and any important points (home, post office, grocery store).

- b. On the way to the grocery store, Jacob looks down at his watch and notes the time as he passes his house. What time is it when he passes his house? Explain how you found your answer.
- c. If he drives directly back to his house after the grocery story, what was the total distance he traveled to complete his errands? Show how you found your answer.

2. Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the **ones** place on the water meter display changes too rapidly to read the digit and that the digit in the **tens** place changes every second or so.
- a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at 2:00 a.m. as at noon?) Explain how you arrived at your estimate.
- b. To what level of accuracy can Jason reasonably report a measurement if he takes it at precisely 12:00 p.m.? Explain your answer.
- c. The meter will be checked at regular time spans (for example, every minute, every 10 minutes, and every hour). What is the minimum (or smallest) number of checks needed in a 24-hour period to create a reasonably accurate graph of the water usage **rate** with respect to time? (For example, 24 checks would mean checking the meter every hour; 48 checks would mean checking the meter every half hour.) Defend your choice by describing how the water usage rate might change during the day and how your choice could capture that change.

3. A publishing company orders black and blue ink in bulk for its two-color printing press. To keep things simple with its ink supplier, each time it places an order for blue ink, it buys  $B$  gallons, and each time it places an order for black ink, it buys  $K$  gallons. Over a one-month period, the company places  $m$  orders of blue ink and  $n$  orders of black ink.
- a. What quantities could the following expressions represent in terms of the problem context?

$$m + n$$

$$mB + nK$$

$$\frac{mB + nK}{m + n}$$

- b. The company placed twice as many orders for black ink than for blue ink in January. Give interpretations for the following expressions in terms of the orders placed in January,

$$\frac{m}{m + n} \quad \text{and} \quad \frac{n}{m + n},$$

and explain which expression must be greater using those interpretations.

4. Sam says that he knows a clever set of steps to rewrite the expression

$$(x + 3)(3x + 8) - 3x(x + 3)$$

as a sum of two terms where the steps do not involve multiplying the linear factors first and then collecting like terms. Rewrite the expression as a sum of two terms (where one term is a number and the other is a product of a coefficient and variable) using Sam's steps if you can.

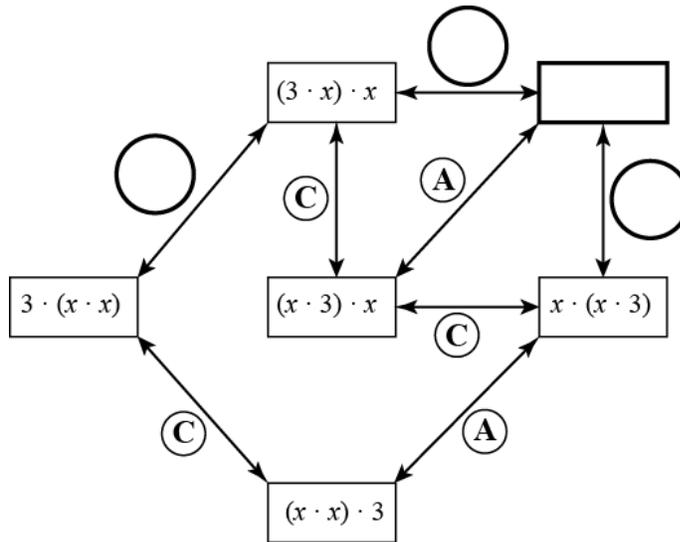
5. Using only the addition and multiplication operations with the numbers 1, 2, 3, and 4 each exactly once, it is possible to build a numeric expression (with parentheses to show the order used to build the expression) that evaluates to 21. For example,  $1 + ((2 + 3) \cdot 4)$  is one such expression.
- a. Build two more numeric expressions that evaluate to 21 using the criteria above. Both must be different from the example given.

- b. In both of your expressions, replace 1 with  $a$ , 2 with  $b$ , 3 with  $c$ , and 4 with  $d$  to get two algebraic expressions. For example,  $a + ((b + c) \cdot d)$  shows the replacements for the example given.

Are your algebraic expressions equivalent? Circle: Yes No

- If they are equivalent, prove that they are using the properties of operations.
- If not, provide **two** examples:
  - (1) Find four different numbers (other than 0, 1, 2, 3, 4) that when substituted for  $a$ ,  $b$ ,  $c$ , and  $d$  into each expression, the expressions evaluate to **different numbers**, and
  - (2) Find four different, non-zero numbers that when substituted into each expression, the expressions evaluate to the **same number**.

6. The diagram below, when completed, shows all possible ways to build equivalent expressions of  $3x^2$  using multiplication. The equivalent expressions are connected by labeled segments stating which property of operations, **A** for associative property and **C** for commutative property, justifies why the two expressions are equivalent. Answer the following questions about  $3x^2$  and the diagram.



- Fill in the empty circles with **A** or **C** and the empty rectangle with the missing expression to complete the diagram.
  - Using the diagram above to help guide you, give *two different* proofs that  $(x \cdot x) \cdot 3 = (3 \cdot x) \cdot x$ .
7. Ahmed learned: “To multiply a whole number by ten, just place a zero at the end of the number.” For example,  $2813 \times 10$ , he says, is 28,130. He doesn't understand why this “rule” is true.
- What is the product of the polynomial,  $2x^3 + 8x^2 + x + 3$ , times the polynomial,  $x$ ?
  - Use part (a) as a hint. Explain why the rule Ahmed learned is true.

- 8.
- a. Find the following products:
- $(x - 1)(x + 1)$
  - $(x - 1)(x^2 + x + 1)$
  - $(x - 1)(x^3 + x^2 + x + 1)$
  - $(x - 1)(x^4 + x^3 + x^2 + x + 1)$
  - $(x - 1)(x^n + x^{n-1} + \dots + x^3 + x^2 + x + 1)$
- b. Substitute  $x = 10$  into each of the products from parts (i) through (iv) and your answers to show how each of the products appears as a statement in arithmetic.
- c. If we substituted  $x = 10$  into the product  $(x - 2)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$  and computed the product, what number would result?

- d. Multiply  $(x - 2)$  and  $(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ , and express your answer in standard form.

Substitute  $x = 10$  into your answer, and see if you obtain the same result that you obtained in part (c).

- e. Francois says  $(x - 9)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$  must equal  $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$  because when  $x = 10$ , multiplying by “ $x - 9$ ” is the same as multiplying by 1.

- i. Multiply  $(x - 9)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ .

- ii. Put  $x = 10$  into your answer.

Is it the same as  $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$  with  $x = 10$ ?

- iii. Was Francois right?

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## Lesson 10: True and False Equations

### Exit Ticket

1. Consider the following equation, where  $a$  represents a real number:  $\sqrt{a+1} = \sqrt{a} + 1$ .

Is this statement a number sentence? If so, is the sentence TRUE or FALSE?

2. Suppose we are told that  $b$  has the value 4. Can we determine whether the equation below is TRUE or FALSE? If so, say which it is; if not, state that it cannot be determined. Justify your answer.

$$\sqrt{b+1} = \sqrt{b} + 1$$

3. For what value of  $c$  is the following equation TRUE?

$$\sqrt{c+1} = \sqrt{c} + 1$$

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## Lesson 11: Solution Sets for Equations and Inequalities

### Exit Ticket

1. Here is the graphical representation of a set of real numbers:



- Describe this set of real numbers in words.
  - Describe this set of real numbers in set notation.
  - Write an equation or an inequality that has the set above as its solution set.
2. Indicate whether each of the following equations is sure to have a solution set of all real numbers. Explain your answers for each.
- $3(x + 1) = 3x + 1$
  - $x + 2 = 2 + x$
  - $4x(x + 1) = 4x + 4x^2$
  - $3x(4x)(2x) = 72x^3$

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## Lesson 12: Solving Equations

### Exit Ticket

Determine which of the following equations have the same solution set by recognizing properties, rather than solving.

a.  $2x + 3 = 13 - 5x$

b.  $6 + 4x = -10x + 26$

c.  $6x + 9 = \frac{13}{5} - x$

d.  $0.6 + 0.4x = -x + 2.6$

e.  $3(2x + 3) = \frac{13}{5} - x$

f.  $4x = -10x + 20$

g.  $15(2x + 3) = 13 - 5x$

h.  $15(2x + 3) + 97 = 110 - 5x$

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## Lesson 13: Some Potential Dangers when Solving Equations

### Exit Ticket

1. Solve the equation for  $x$ . For each step, describe the operation and/or properties used to convert the equation.

$$5(2x - 4) - 11 = 4 + 3x$$

2. Consider the equation  $x + 4 = 3x + 2$ .

- a. Show that adding  $x + 2$  to both sides of the equation does not change the solution set.
- b. Show that multiplying both sides of the equation by  $x + 2$  adds a second solution of  $x = -2$  to the solution set.

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## Lesson 14: Solving Inequalities

### Exit Ticket

1. Find the solution set to each inequality. Express the solution in set notation and graphically on the number line.

a.  $6x - 5 < 7x + 4$

b.  $x^2 + 3(x - 1) \geq x^2 + 5$

2. Fergus was absent for today's lesson and asked Mike to explain why the solution to  $-5x > 30$  is  $x < -6$ . Mike said, "Oh! That's easy. When you multiply by a negative, just flip the inequality." Provide a better explanation to Fergus about why the direction of the inequality is reversed.

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## Lesson 15: Solution Sets of Two or More Equations (or Inequalities) Joined by “And” or “Or”

### Exit Ticket

- Solve the system and graph the solution set on a number line.  
 $x - 15 = 5$  or  $2x + 5 = 1$
  - Write a different system of equations that would have the same solution set.
- Swimming pools must have a certain amount of chlorine content. The United States standard for safe levels of chlorine in swimming pools is at least 1 part per million and no greater than 3 parts per million. Write a compound inequality for the acceptable range of chlorine levels.

- Consider each of the following compound sentences:

$$x < 1 \text{ and } x > -1$$

$$x < 1 \text{ or } x > -1$$

Does the change of word from “and” to “or” change the solution set?

Use number-line graphs to support your answer.

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## Lesson 16: Solving and Graphing Inequalities Joined by “And” or “Or”

### Exit Ticket

1. Solve each compound inequality for  $x$  and graph the solution on a number line.

a.  $9 + 2x < 17$  or  $7 - 4x < -9$

b.  $6 \leq \frac{x}{2} \leq 11$

2.

a. Give an example of a compound inequality separated by “or” that has a solution of all real numbers.

b. Take the example from (a) and change the “or” to an “and.” Explain why the solution set is no longer all real numbers. Use a graph on a number line as part of your explanation.

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## Lesson 17: Equations Involving Factored Expressions

### Exit Ticket

- Find the solution set to the equation  $3x^2 + 27x = 0$ .
- Determine if each statement is true or false. If the statement is false, explain why or show work proving that it is false.
  - If  $a = 5$ , then  $ac = 5c$ .
  - If  $ac = 5c$ , then  $a = 5$ .

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## Lesson 18: Equations Involving a Variable Expression in the Denominator

### Exit Ticket

1. Rewrite the equation  $\frac{x-2}{x-9} = 2$  as a system of equations. Then, solve for  $x$ .

2. Write an equation that would have the restriction  $x \neq -3$ .

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## Lesson 19: Rearranging Formulas

### Exit Ticket

Given the formula  $x = \frac{1+a}{1-a}$ ,

1. Solve for  $a$  when  $x = 12$ .

2. Rearrange the formula to solve for  $a$ .

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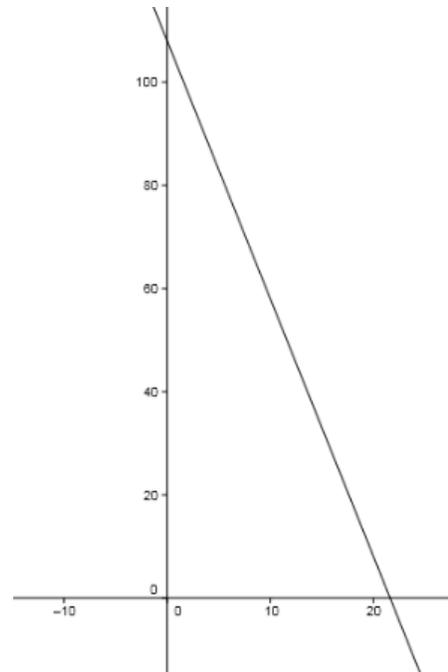
## Lesson 20: Solution Sets to Equations with Two Variables

### Exit Ticket

The Math Club sells hot dogs at a school fundraiser. The club earns \$108 and has a combination of five-dollar and one-dollar bills in its cash box. Possible combinations of bills are listed in the table below. Complete the table.

Number of five-dollar bills	Number of one-dollar bills	Total = \$108
19	13	$5(19) + 1(13) = 108$
16	28	
11	53	
4	88	

- Find one more combination of ones and fives that totals \$108.
- The equation  $5x + 1y = 108$  represents this situation. A graph of the line  $y = -5x + 108$  is shown. Verify that each ordered pair in the table lies on the line.
- What is the meaning of the variables ( $x$  and  $y$ ) and the numbers (1, 5, and 108) in the equation  $5x + 1y = 108$ ?



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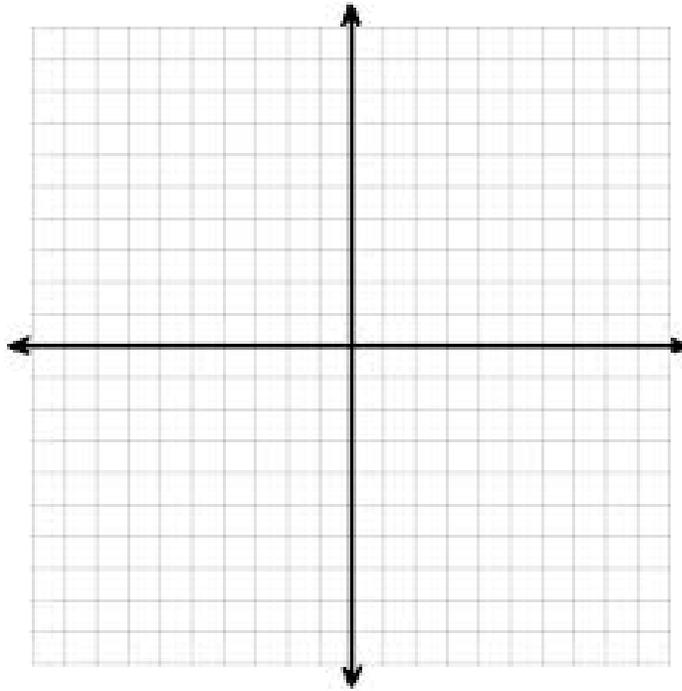
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## Lesson 21: Solution Sets to Inequalities with Two Variables

### Exit Ticket

What pairs of numbers satisfy the statement: The sum of two numbers is less than 10?

Create an inequality with two variables to represent this situation and graph the solution set.



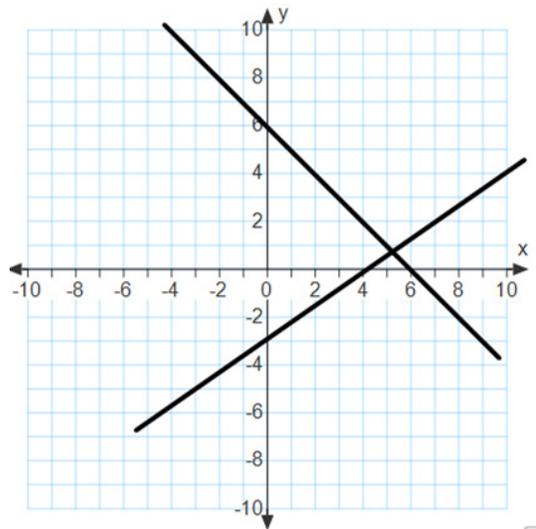
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## Lesson 22: Solution Sets to Simultaneous Equations

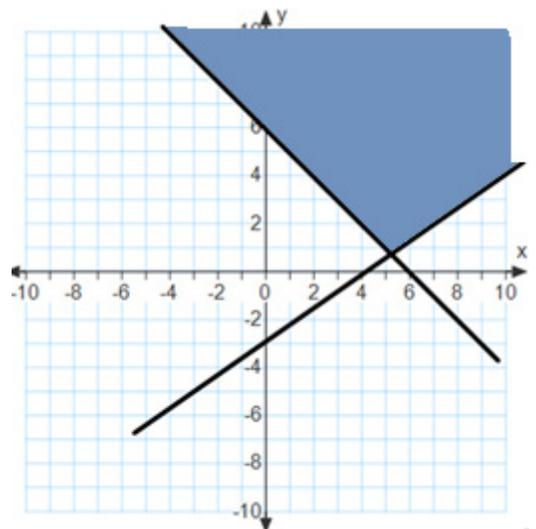
### Exit Ticket

1. Estimate the solution to the system of equations whose graph is shown to the right.



2. Write the two equations for the system of equations and find the exact solution to the system algebraically.

3. Write a system of inequalities that represents the shaded region on the graph shown to the right.





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## Lesson 24: Applications of Systems of Equations and Inequalities

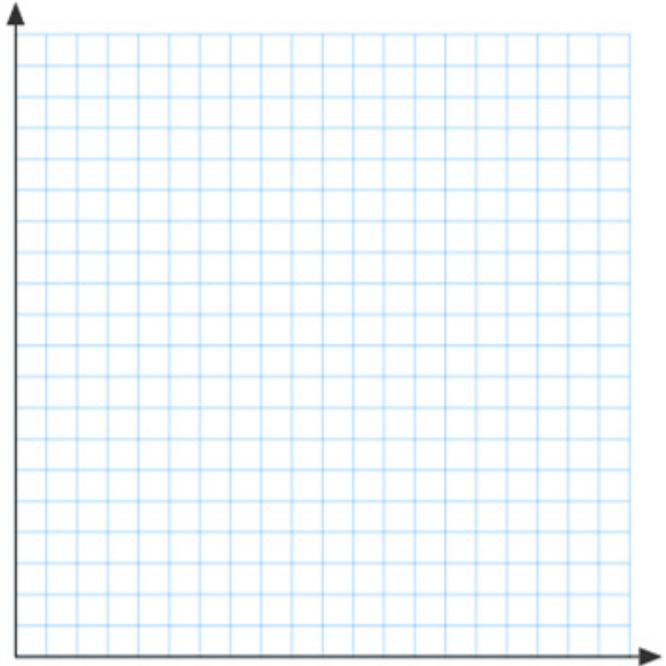
### Exit Ticket

Andy's Cab Service charges a \$6 fee plus \$0.50 per mile. His twin brother Randy starts a rival business where he charges \$0.80 per mile but does not charge a fee.

1. Write a cost equation for each cab service in terms of the number of miles.

2. Graph both cost equations.

3. For what trip distances should a customer use Andy's Cab Service? For what trip distances should a customer use Randy's Cab Service? Justify your answer algebraically, and show the location of the solution on the graph.





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## Lesson 26: Recursive Challenge Problem—The Double and Add 5 Game

### Exit Ticket

The following sequence was generated by an initial value  $a_0$  and recurrence relation  $a_{i+1} = 2a_i + 5$ , for  $i \geq 0$ .

1. Fill in the blanks in the sequence:

(\_\_\_\_\_, 29, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, 539, 1083).

2. In the sequence above, what is  $a_0$ ? What is  $a_5$ ?

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## Lesson 27: Recursive Challenge Problem—The Double and Add 5 Game

### Exit Ticket

Write a *brief* report about the answers you found to the *Double and Add 5* game problems. Include justifications for why your starting numbers are correct.

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## Lesson 28: Federal Income Tax

### Exit Ticket

A famous movie actress made \$10 million last year. She is married and has no children, and her husband does not earn any income. Assume that she computes her taxable income using the following formula:

$$(\text{taxable income}) = (\text{income}) - (\text{exemptions}) - (\text{standard deductions})$$

Find her taxable income, her federal income tax, and her effective federal income tax rate.

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1. Solve the following equations for  $x$ . Write your answer in set notation.

a.  $3x - 5 = 16$

b.  $3(x + 3) - 5 = 16$

c.  $3(2x - 3) - 5 = 16$

d.  $6(x + 3) - 10 = 32$

e. Which two equations above have the same solution set? Write a sentence explaining how the properties of equality can be used to determine the pair without having to find the solution set for each.

2. Let  $c$  and  $d$  be real numbers.
- If  $c = 42 + d$  is true, then which is greater:  $c$  or  $d$ , or are you not able to tell? Explain how you know your choice is correct.
  
  
  
  
  
  
  
  
  
  
  - If  $c = 42 - d$  is true, then which is greater:  $c$  or  $d$ , or are you not able to tell? Explain how you know your choice is correct.

3. If  $a < 0$  and  $c > b$ , circle the expression that is greater:

$$a(b - c) \quad \text{or} \quad a(c - b)$$

Use the properties of inequalities to explain your choice.

4. Solve for  $x$  in each of the equations or inequalities below, and name the property and/or properties used:

a.  $\frac{3}{4}x = 9$

b.  $10 + 3x = 5x$

c.  $a + x = b$

d.  $cx = d$

e.  $\frac{1}{2}x - g < m$

f.  $q + 5x = 7x - r$

g.  $\frac{3}{4}(x + 2) = 6(x + 12)$

h.  $3(5 - 5x) > 5x$

5. The equation  $3x + 4 = 5x - 4$  has the solution set  $\{4\}$ .

a. Explain why the equation  $(3x + 4) + 4 = (5x - 4) + 4$  also has the solution set  $\{4\}$ .

- b. In part (a), the expression  $(3x + 4) + 4$  is equivalent to the expression  $3x + 8$ . What is the definition of equivalent expressions? Why does changing an expression on one side of an equation to an equivalent expression leave the solution set unchanged?

- c. When we square both sides of the original equation, we get the following new equation:

$$(3x + 4)^2 = (5x - 4)^2.$$

Show that 4 is still a solution to the new equation. Show that 0 is also a solution to the new equation but is not a solution to the original equation. Write a sentence that describes how the solution set to an equation may change when both sides of the equation are squared.

- d. When we replace  $x$  by  $x^2$  in the original equation, we get the following new equation:

$$3x^2 + 4 = 5x^2 - 4.$$

Use the fact that the solution set to the original equation is  $\{4\}$  to find the solution set to this new equation.

6. The Zonda Information and Telephone Company (ZI&T) calculates a customer's total monthly cell phone charge using the formula,

$$C = (b + rm)(1 + t),$$

where  $C$  is the total cell phone charge,  $b$  is a basic monthly fee,  $r$  is the rate per minute,  $m$  is the number of minutes used that month, and  $t$  is the tax rate.

Solve for  $m$ , the number of minutes the customer used that month.



8. Alexis is modeling the growth of bacteria for an experiment in science. She assumes that there are  $B$  bacteria in a Petri dish at 12:00 noon. In reality, each bacterium in the Petri dish subdivides into two new bacteria *approximately* every 20 minutes. However, for the purposes of the model, Alexis assumes that each bacterium subdivides into two new bacteria *exactly* every 20 minutes.
- a. Create a table that shows the total number of bacteria in the Petri dish at  $\frac{1}{3}$  hour intervals for 2 hours starting with time 0 to represent 12:00 noon.
- b. Write an equation that describes the relationship between total number of bacteria  $T$  and time  $h$  in hours, assuming there are  $B$  bacteria in the Petri dish at  $h = 0$ .
- c. If Alexis starts with 100 bacteria in the Petri dish, draw a graph that displays the total number of bacteria with respect to time from 12:00 noon ( $h = 0$ ) to 4:00 p.m. ( $h = 4$ ). Label points on your graph at time  $h = 0, 1, 2, 3, 4$ .

- d. For her experiment, Alexis plans to add an anti-bacterial chemical to the Petri dish at 4:00 p.m. that is supposed to kill 99.9% of the bacteria instantaneously. If she started with 100 bacteria at 12:00 noon, how many live bacteria might Alexis expect to find in the Petri dish right after she adds the anti-bacterial chemical?
9. Jack is 27 years older than Susan. In 5 years, he will be 4 times as old as she is.
- a. Find the present ages of Jack and Susan.
- b. What calculations would you do to check if your answer is correct?

10.

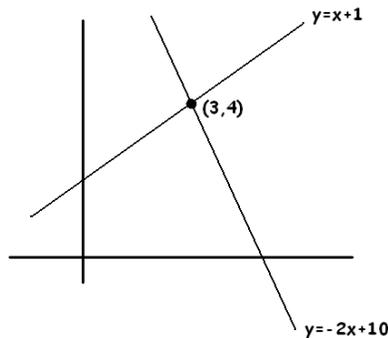
a. Find the product:  $(x^2 - x + 1)(2x^2 + 3x + 2)$ .

b. Use the results of part (a) to factor 21,112 as a product of a two-digit number and a three-digit number.

11. Consider the following system of equations with the solution  $x = 3, y = 4$ .

Equation A1:  $y = x + 1$

Equation A2:  $y = -2x + 10$



a. Write a unique system of two linear equations with the same solution set. This time make both linear equations have positive slope.

Equation B1: \_\_\_\_\_

Equation B2: \_\_\_\_\_

- b. The following system of equations was obtained from the original system by adding a multiple of equation A2 to equation A1.

Equation C1:  $y = x + 1$

Equation C2:  $3y = -3x + 21$

What multiple of A2 was added to A1?

- c. What is the solution to the system given in part (b)?

- d. For any real number  $m$ , the line  $y = m(x - 3) + 4$  passes through the point  $(3,4)$ .

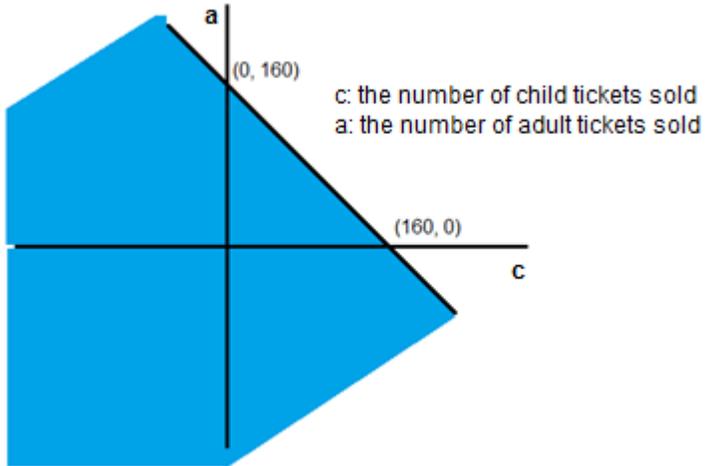
Is it certain, then, that the system of equations

Equation D1:  $y = x + 1$

Equation D2:  $y = m(x - 3) + 4$

has only the solution  $x = 3, y = 4$ ? Explain.

12. The local theater in Jamie's home town has a maximum capacity of 160 people. Jamie shared with Venus the following graph and said that the shaded region represented all the possible combinations of adult and child tickets that could be sold for one show.



- a. Venus objected and said there was more than one reason that Jamie's thinking was flawed. What reasons could Venus be thinking of?

- b. Use equations, inequalities, graphs, and/or words to describe for Jamie the set of all possible combinations of adult and child tickets that could be sold for one show.
- c. The theater charges \$9 for each adult ticket and \$6 for each child ticket. The theater sold 144 tickets for the first showing of the new release. The total money collected from ticket sales for that show was \$1,164. Write a system of equations that could be used to find the number of child tickets and the number of adult tickets sold, and solve the system algebraically. Summarize your findings using the context of the problem.